GATE EC 2010

Q. No. 1 - 25 Carry One Mark Each

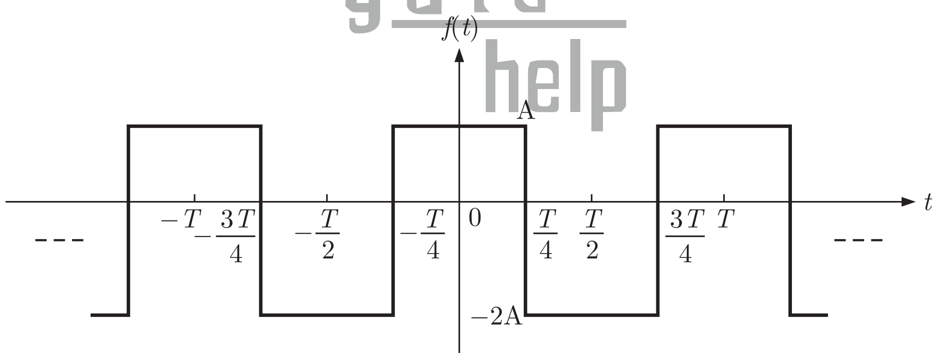
MCQ 1.1 The eigen values of a skew-symmetric matrix are

(A) always zero
(B) always pure imaginary
(C) either zero or pure imaginary
(D) always real

SOL 1.1 Eigen value of a Skew-symmetric matrix are either zero or pure imaginary in conjugate pairs.

Hence (C) is correct option.

MCQ 1.2 The trigonometric Fourier series for the waveform $f(t)$ shown below contains



(A) only cosine terms and zero values for the dc components
(B) only cosine terms and a positive value for the dc components
(C) only cosine terms and a negative value for the dc components
(D) only sine terms and a negative value for the dc components

SOL 1.2 For a function $x(t)$ trigonometric fourier series is

$$x(t) = A_o + \sum_{n=1}^{\infty}[A_n \cos n\omega t + B_n \sin n\omega t]$$

Where, $A_o = \frac{1}{T_0}\int_{T_0} x(t)\,dt$ $\qquad T_0 \rightarrow$ fundamental period

$$A_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega t\,dt$$

$$B_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega t\, dt$$

For an even function $x(t), B_n = 0$

Since given function is even function so coefficient $B_n = 0$, only cosine and constant terms are present in its fourier series representation.

Constant term :

$$A_0 = \frac{1}{T}\int_{-T/4}^{3T/4} x(t)\,dt$$

$$= \frac{1}{T}\left[\int_{-T/4}^{T/4} A\,dt + \int_{T/4}^{3T/4} -2A\,dt\right]$$

$$= \frac{1}{T}\left[\frac{TA}{2} - 2A\frac{T}{2}\right] = -\frac{A}{2}$$

Constant term is negative.

Hence (C) is correct option.

MCQ 1.3 A function $n(x)$ satisfied the differential equation $\frac{d^2n(x)}{dx^2} - \frac{n(x)}{L^2} = 0$ where L is a constant. The boundary conditions are : $n(0) = K$ and $n(\infty) = 0$. The solution to this equation is

(A) $n(x) = K\exp(x/L)$ (B) $n(x) = K\exp(-x/\sqrt{L})$

(C) $n(x) = K^2\exp(-x/L)$ (D) $n(x) = K\exp(-x/L)$

SOL 1.3 Given differential equation

$$\frac{d^2n(x)}{dx^2} - \frac{n(x)}{L^2} = 0$$

Let $n(x) = Ae^{\lambda x}$

So, $$A\lambda^2 e^{\lambda x} - \frac{Ae^{\lambda x}}{L^2} = 0$$

$$\lambda^2 - \frac{1}{L^2} = 0 \Rightarrow \lambda = \pm\frac{1}{L}$$

Boundary condition, $n(\infty) = 0$ so take $\lambda = -\frac{1}{L}$

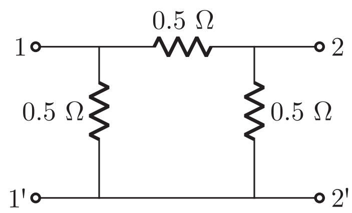
$$n(x) = Ae^{-\frac{x}{L}}$$

$$n(0) = Ae^0 = K \Rightarrow A = K$$

So, $$n(x) = Ke^{-(x/L)}$$

Hence (D) is correct option.

MCQ 1.4 For the two-port network shown below, the short-circuit admittance parameter matrix is



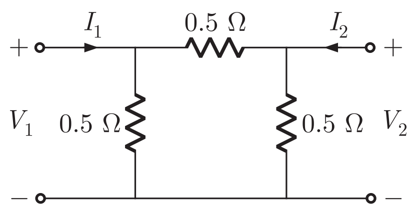
(A) $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$ S

(B) $\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$ S

(C) $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$ S

(D) $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ S

SOL 1.4 Given circuit is as shown below



By writing node equation at input port

$$I_1 = \frac{V_1}{0.5} + \frac{V_1 - V_2}{0.5} = 4V_1 - 2V_2 \quad ...(1)$$

By writing node equation at output port

$$I_2 = \frac{V_2}{0.5} + \frac{V_2 - V_1}{0.5} = -2V_1 + 4V_2 \quad ...(2)$$

From (1) and (2), we have admittance matrix

$$Y = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$

Hence (A) is correct option.

MCQ 1.5 For parallel RLC circuit, which one of the following statements is NOT correct ?

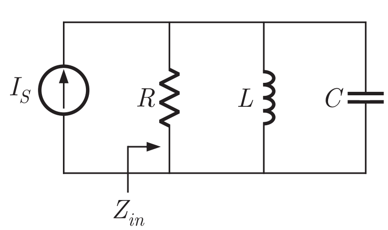
(A) The bandwidth of the circuit decreases if R is increased

(B) The bandwidth of the circuit remains same if L is increased

(C) At resonance, input impedance is a real quantity

(D) At resonance, the magnitude of input impedance attains its minimum values.

SOL 1.5 A parallel RLC circuit is shown below :



Input impedance $Z_{in} = \dfrac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C}$

At resonance $\dfrac{1}{\omega L} = \omega C$

So, $Z_{in} = \dfrac{1}{1/R} = R$ (maximum at resonance)

Thus (D) is not true.

Furthermore bandwidth is ω_B i.e $\omega_B \propto \dfrac{1}{R}$ and is independent of L,
Hence statements A, B, C, are true.
Hence (D) is correct option.

MCQ 1.6 At room temperature, a possible value for the mobility of electrons in the inversion layer of a silicon n-channel MOSFET is

(A) 450 cm^2/V-s (B) 1350 cm^2/V-s

(C) 1800 cm^2/V-s (D) 3600 cm^2/V-s

SOL 1.6 At room temperature mobility of electrons for Si sample is given $\mu_n = 1350\ cm^2/Vs$. For an n-channel MOSFET to create an inversion layer of electrons, a large positive gate voltage is to be applied. Therefore, induced electric field increases and mobility decreases.
So, Mobility $\mu_n < 1350\ cm^2/Vs$ for n-channel MOSFET
Hence (A) is correct option.

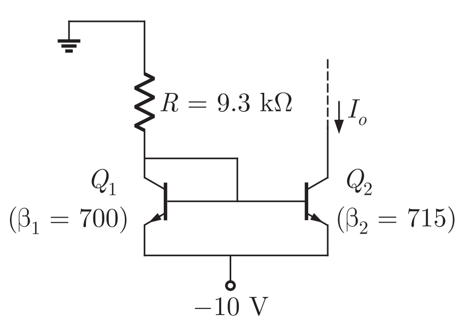
MCQ 1.7 Thin gate oxide in a CMOS process in preferably grown using

(A) wet oxidation (B) dry oxidation

(C) epitaxial oxidation (D) ion implantation

SOL 1.7 Dry oxidation is used to achieve high quality oxide growth.
Hence (B) is correct option.

MCQ 1.8 In the silicon BJT circuit shown below, assume that the emitter area of transistor Q_1 is half that of transistor Q_2



The value of current I_o is approximately

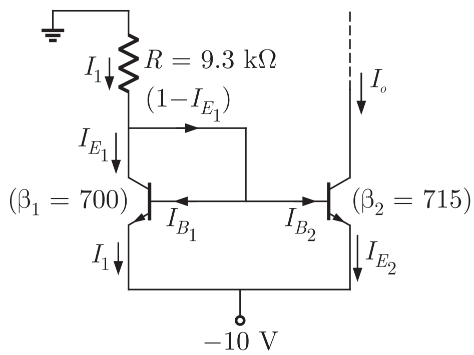
(A) 0.5 mA (B) 2 mA

(C) 9.3 mA (D) 15 mA

SOL 1.8 Since, emitter area of transistor Q_1 is half of transistor Q_2, so current

$$I_{E_1} = \frac{1}{2} I_{E_2} \text{ and } I_{B_1} = \frac{1}{2} I_{B_2}$$

The circuit is as shown below :



$$V_B = -10 - (-0.7) = -9.3 \text{ V}$$

Collector current

$$I_1 = \frac{0 - (-9.3)}{(9.3 \text{ k}\Omega)} = 1 \text{ mA}$$

$$\beta_1 = 700 \text{ (high), So } I_C \approx I_{E_1}$$

Applying KCL at base we have

$$1 - I_E = I_{B_1} + I_{B_2}$$

$$1 - (\beta_1 + 1) I_{B_1} = I_{B_1} + I_{B_2}$$

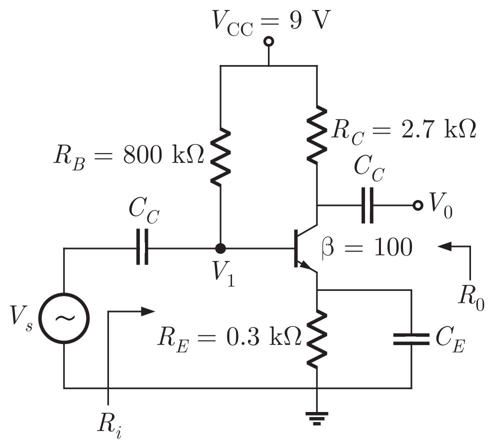
$$1 = (700 + 1 + 1)\frac{I_{B_2}}{2} + I_{B_2}$$

$$I_{B_2} \approx \frac{2}{702}$$

$$I_0 = I_{C_2} = \beta_2 \cdot I_{B_2} = 715 \times \frac{2}{702} \approx 2 \text{ mA}$$

Hence (B) is correct option.

MCQ 1.9 The amplifier circuit shown below uses a silicon transistor. The capacitors C_C and C_E can be assumed to be short at signal frequency and effect of output resistance r_0 can be ignored. If C_E is disconnected from the circuit, which one of the following statements is true



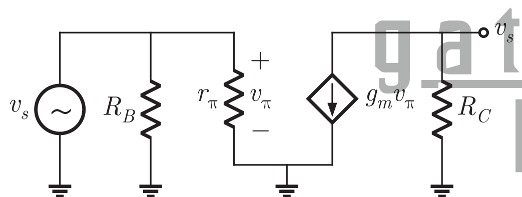
(A) The input resistance R_i increases and magnitude of voltage gain A_V decreases

(B) The input resistance R_i decreases and magnitude of voltage gain A_V increases

(C) Both input resistance R_i and magnitude of voltage gain A_V decreases

(D) Both input resistance R_i and the magnitude of voltage gain A_V increases

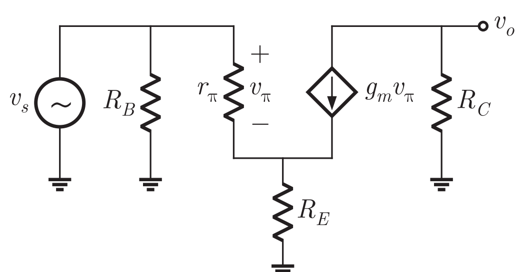
SOL 1.9 The equivalent circuit of given amplifier circuit (when C_E is connected, R_E is short-circuited)



Input impedance $R_i = R_B \,||\, r_\pi$

Voltage gain $A_V = g_m R_C$

Now, if C_E is disconnected, resistance R_E appears in the circuit



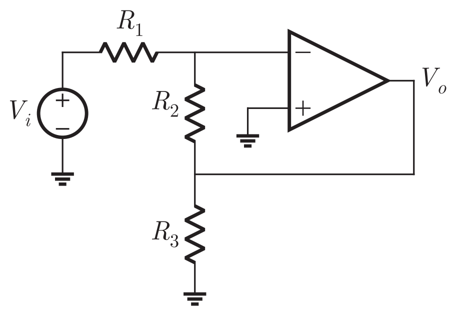
Input impedance $R_{in} = R_B \,||\, [r_\pi + (\beta + 1)]\, R_E$

Input impedance increases

Voltage gain $A_V = \dfrac{g_m R_C}{1 + g_m R_E}$ Voltage gain decreases.

Hence (A) is correct option.

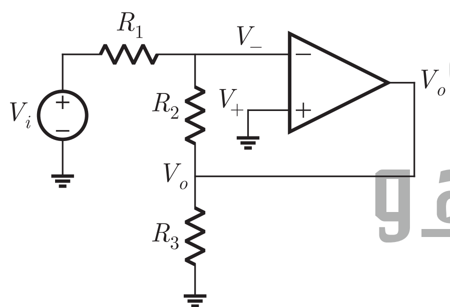
MCQ 1.10 Assuming the OP-AMP to be ideal, the voltage gain of the amplifier shown below is



(A) $-\frac{R_2}{R_1}$ (B) $-\frac{R_3}{R_1}$

(C) $-\frac{R_2 \,||\, R_3}{R_1}$ (D) $-\left(\frac{R_2 + R_3}{R_1}\right)$

SOL 1.10 The circuit is as shown below :



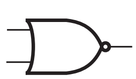
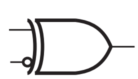
So, $$\frac{0 - V_i}{R_1} + \frac{0 - V_o}{R_2} = 0$$

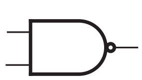
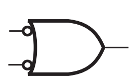
or $$\frac{V_o}{V_i} = -\frac{R_2}{R_1}$$

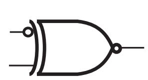
Hence (A) is correct option.

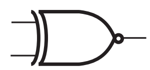
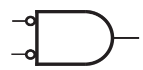
MCQ 1.11 Match the logic gates in **Column A** with their equivalents in **Column B**

Column A **Column B**

P  1 

Q  2 

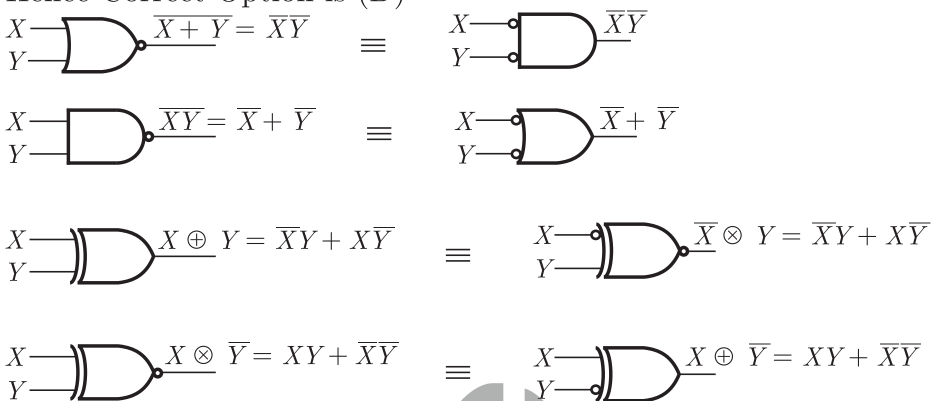
R  3 

S  4 

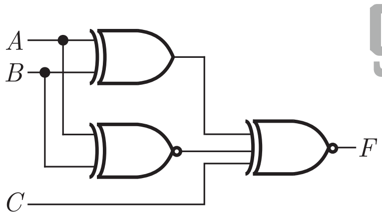
(A) P-2, Q-4, R-1, S-3 (B) P-4, Q-2, R-1, S-3

(C) P-2, Q-4, R-3, S-1 (D) P-4, Q-2, R-3, S-1

SOL 1.11 Hence Correct Option is (D)



MCQ 1.12 For the output F to be 1 in the logic circuit shown, the input combination should be



(A) $A = 1, B = 1, C = 0$ (B) $A = 1, B = 0, C = 0$

(C) $A = 0, B = 1, C = 0$ (D) $A = 0, B = 0, C = 1$

SOL 1.12 In the circuit $F = (A \oplus B) \odot (A \odot B) \odot C$

For two variables $A \oplus B = \overline{A \odot B}$

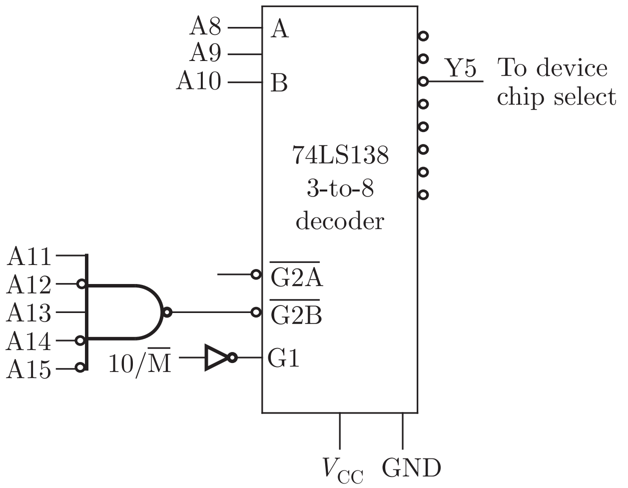
So, $(A \oplus B) \odot (A \odot B) = 0$ (always)

$$F = 0 \odot C = 0 \cdot C + 1 \cdot \overline{C} = \overline{C}$$

So, $F = 1$ when $\overline{C} = 1$ or $C = 0$

Hence (A) (B) (C) are correct options.

MCQ 1.13 In the circuit shown, the device connected Y5 can have address in the range



(A) 2000-20FF (B) 2D00-2DFF

(C) 2E00-2EFF (D) FD00-FDFF

SOL 1.13 Since $\overline{G_2}$ is active low input, output of NAND gate must be 0

$$\overline{G_2} = \overline{\overline{A_{15}} \cdot \overline{A_{14}} A_{13} \overline{A_{12}} A_{11}} = 0$$

So, $A_{15} A_{14} A_{13} A_{12} A_{11} = 00101$

To select Y_5 Decoder input

$$ABC = A_8 A_9 A_{10} = 101$$

Address range

$A_{15} A_{14} A_{13} A_{12} A_{11} A_{10} A_9 A_8 \ldots\ldots\ldots\ldots\ldots A_0$

$\underbrace{001}_{2}\underbrace{1101}_{D}\ldots\ldots\ldots A_0$

$(2D00 - 2DFF)$

Hence (B) is correct option.

MCQ 1.14 Consider the z-transform $x(z) = 5z^2 + 4z^{-1} + 3; 0 < |z| < \infty$. The inverse z-transform $x[n]$ is

(A) $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$ (B) $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$

(C) $5u[n+2] + 3u[n] + 4u[n-1]$ (D) $5u[n-2] + 3u[n] + 4u[n+1]$

SOL 1.14 Hence (A) is correct option. Hence (A) is correct option.

We know that $\alpha Z^{\pm a} \xleftrightarrow{\text{Inverse Z-transform}} \alpha\delta[n \pm a]$

Given that $X(z) = 5z^2 + 4z^{-1} + 3$

Inverse z-transform $x[n] = 5\delta[n+2] + 4\delta[n-1] + 3\delta[n]$

MCQ 1.15 Two discrete time system with impulse response $h_1[n] = \delta[n-1]$ and $h_2[n] = \delta[n-2]$ are connected in cascade. The overall impulse response of the cascaded system is

(A) $\delta[n-1] + \delta[n-2]$ (B) $\delta[n-4]$

(C) $\delta[n-3]$ (D) $\delta[n-1]\delta[n-2]$

SOL 1.15 Hence (C) is correct option

We have $h_1[n] = \delta[n-1]\, or\, H_1[Z] = Z^{-1}$

and $h_2[n] = \delta[n-2]\, or\, H_2(Z) = Z^{-2}$

Response of cascaded system

$$H(z) = H_1(z) \cdot H_2(z) = z^{-1} \cdot z^{-2} = z^{-3}$$

or, $h[n] = \delta[n-3]$

MCQ 1.16 For a N-point FET algorithm $N = 2^m$ which one of the following statements is TRUE ?

(A) It is not possible to construct a signal flow graph with both input and output in normal order

(B) The number of butterflies in the m^{th} stage in N/m

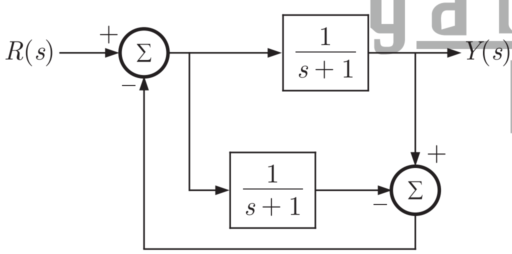
(C) In-place computation requires storage of only 2N data

(D) Computation of a butterfly requires only one complex multiplication.

SOL 1.16 For an N-point FET algorithm butterfly operates on one pair of samples and involves two complex addition and one complex multiplication.

Hence (D) is correct option.

MCQ 1.17 The transfer function $Y(s)/R(s)$ of the system shown is



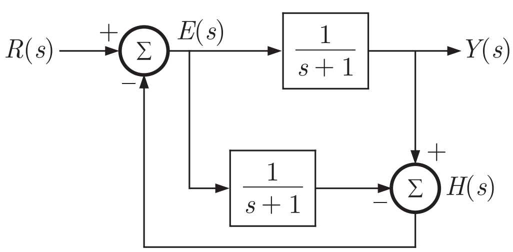
(A) 0

(B) $\frac{1}{s+1}$

(C) $\frac{2}{s+1}$

(D) $\frac{2}{s+3}$

SOL 1.17 From the given block diagram



$$H(s) = Y(s) - E(s) \cdot \frac{1}{s+1}$$

$$E(s) = R(s) - H(s)$$

$$= R(s) - Y(s) + \frac{E(s)}{(s+1)}$$

$$E(s)\left[1 - \frac{1}{s+1}\right] = R(s) - Y(s)$$

$$\frac{sE(s)}{(s+1)} = R(s) - Y(s) \qquad ...(1)$$

$$Y(s) = \frac{E(s)}{s+1} \qquad ...(2)$$

From (1) and (2) $\quad sY(s) = R(s) - Y(s)$

$$(s+1)\,Y(s) = R(s)$$

Transfer function

$$\frac{Y(s)}{R(s)} = \frac{1}{s+1}$$

Hence (B) is correct option.

MCQ 1.18 A system with transfer function $\frac{Y(s)}{X(s)} = \frac{s}{s+p}$ has an output $y(t) = \cos(2t - \frac{\pi}{3})$ for the input signal $x(t) = p\cos(2t - \frac{\pi}{2})$. Then, the system parameter p is

(A) $\sqrt{3}$

(B) $\frac{2}{\sqrt{3}}$

(C) 1

(D) $\frac{\sqrt{3}}{2}$

SOL 1.18 Transfer function is given as

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{s+p}$$

$$H(j\omega) = \frac{j\omega}{j\omega + p}$$

Amplitude Response

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + p^2}}$$

Phase Response $\quad \theta_h(\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega}{p}\right)$

Input $\quad x(t) = p\cos\left(2t - \frac{\pi}{2}\right)$

Output $\quad y(t) = |H(j\omega)|x(t - \theta_h) = \cos\left(2t - \frac{\pi}{3}\right)$

$$|H(j\omega)| = p = \frac{\omega}{\sqrt{\omega^2 + p^2}}$$

$$\frac{1}{p} = \frac{2}{\sqrt{4 + p^2}}, \qquad (\omega = 2 \text{ rad/sec})$$

or $\quad 4p^2 = 4 + p^2 \Rightarrow 3p^2 = 4$

or $\quad p = 2/\sqrt{3}$

Alternative :

$$\theta_h = \left[-\frac{\pi}{3} - \left(-\frac{\pi}{2}\right)\right] = \frac{\pi}{6}$$

So, $$\frac{\pi}{6} = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{p}\right)$$

$$\tan^{-1}\left(\frac{\omega}{p}\right) = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

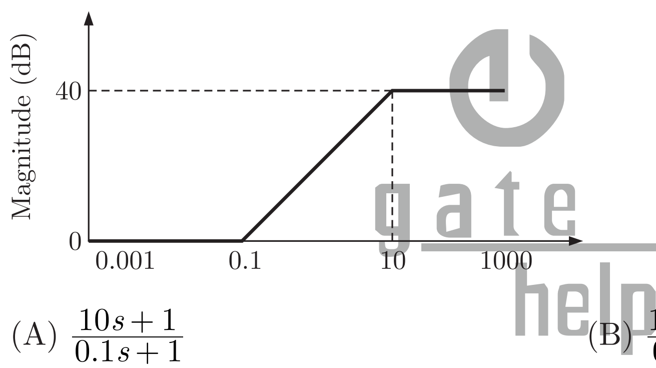
$$\frac{\omega}{p} = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\frac{2}{p} = \sqrt{3}, \quad (\omega = 2\,\text{rad/sec})$$

or $$p = 2/\sqrt{3}$$

Hence (B) is correct option

MCQ 1.19 For the asymptotic Bode magnitude plot shown below, the system transfer function can be



(A) $\frac{10s+1}{0.1s+1}$ (B) $\frac{100s+1}{0.1s+1}$

(C) $\frac{100s}{10s+1}$ (D) $\frac{0.1s+1}{10s+1}$

SOL 1.19 Initial slope is zero, so $K = 1$

At corner frequency $\omega_1 = 0.5\,\text{rad/sec}$, slope increases by $+20$ dB/decade, so there is a zero in the transfer function at ω_1

At corner frequency $\omega_2 = 10\,\text{rad/sec}$, slope decreases by -20 dB/decade and becomes zero, so there is a pole in transfer function at ω_2

Transfer function $$H(s) = \frac{K\left(1 + \frac{s}{\omega_1}\right)}{\left(1 + \frac{s}{\omega_2}\right)}$$

$$= \frac{1\left(1 + \frac{s}{0.1}\right)}{\left(1 + \frac{s}{0.1}\right)} = \frac{(1+10s)}{(1+0.1s)}$$

Hence (A) is correct option

MCQ 1.20 Suppose that the modulating signal is $m(t) = 2\cos(2\pi f_m t)$ and the carrier signal is $x_C(t) = A_C\cos(2\pi f_C t)$, which one of the following is a conventional AM signal without over-modulation

(A) $x(t) = A_C m(t)\cos(2\pi f_C t)$

(B) $x(t) = A_C[1 + m(t)]\cos(2\pi f_C t)$

(C) $x(t) = A_C\cos(2\pi f_C t) + \frac{A_C}{4}m(t)\cos(2\pi f_C t)$

(D) $x(t) = A_C\cos(2\pi f_m t)\cos(2\pi f_C t) + A_C\sin(2\pi f_m t)\sin(2\pi f_C t)$

SOL 1.20 Conventional AM signal is given by

$$x(t) = A_C[1 + \mu m(t)]\cos(2\pi f_C t)$$

Where $\mu < 1$, for no over modulation.

In option (C)

$$x(t) = A_C\left[1 + \frac{1}{4}m(t)\right]\cos(2\pi f_C t)$$

Thus $\mu = \frac{1}{4} < 1$ and this is a conventional AM-signal without over-modulation

Hence (C) is correct option.

MCQ 1.21 Consider an angle modulated signal

$$x(t) = 6\cos[2\pi \times 10^6 t + 2\sin(800\pi t)] + 4\cos(800\pi t)$$

The average power of $x(t)$ is

(A) 10 W

(B) 18 W

(C) 20 W

(D) 28 W

SOL 1.21 Hence (B) is correct option.

Power $P = \frac{(6)^2}{2} = 18$ W

MCQ 1.22 If the scattering matrix $[S]$ of a two port network is

$[S] = \begin{bmatrix} 0.2\angle 0^\circ & 0.9\angle 90^\circ \\ 0.9\angle 90^\circ & 0.1\angle 90^\circ \end{bmatrix}$, then the network is

(A) lossless and reciprocal

(B) lossless but not reciprocal

(C) not lossless but reciprocal

(D) neither lossless nor reciprocal

SOL 1.22 For a lossless network

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

For the given scattering matrix

$$S_{11} = 0.2\angle 0^\circ,\ S_{12} = 0.9\angle 90^\circ$$
$$S_{21} = 0.9\angle 90^\circ,\ S_{22} = 0.1\angle 90^\circ$$

Here, $(0.2)^2 + (0.9)^2 \neq 1$ (not lossless)

Reciprocity :

$$S_{12} = S_{21} = 0.9\angle 90^\circ \text{ (Reciprocal)}$$

Hence (C) is correct option.

MCQ 1.23 A transmission line has a characteristic impedance of $50\,\Omega$ and a resistance of $0.1\,\Omega/\text{m}$. If the line is distortion less, the attenuation constant(in Np/m) is

(A) 500 (B) 5

(C) 0.014 (D) 0.002

SOL 1.23 For distortion less transmission line characteristics impedance

$$Z_0 = \sqrt{\frac{R}{G}}$$

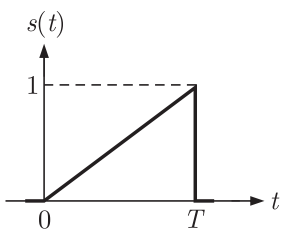
Attenuation constant

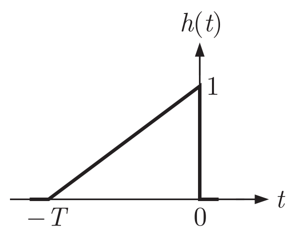
$$\alpha = \sqrt{RG}$$

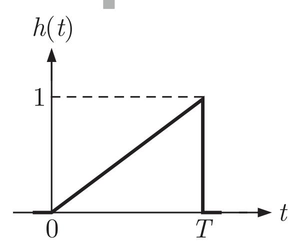
So, $$\alpha = \frac{R}{Z_0} = \frac{0.1}{50} = 0.002$$

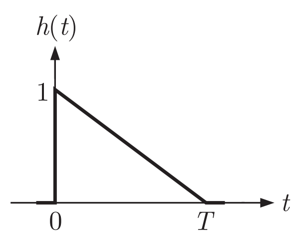
Hence (D) is correct option.

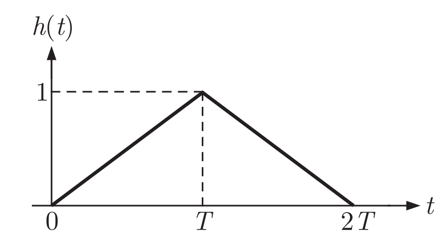
MCQ 1.24 Consider the pulse shape $s(t)$ as shown. The impulse response $h(t)$ of the filter matched to this pulse is



(A)


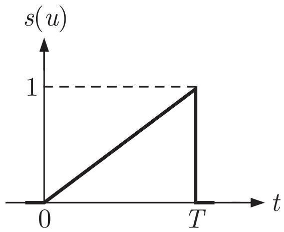
(B)


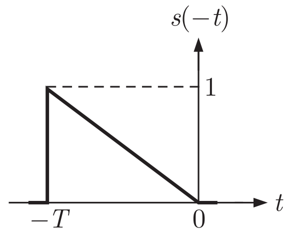
(C)


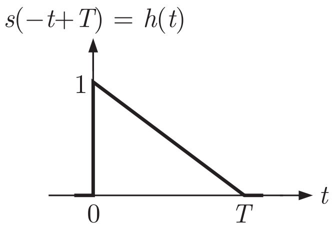
(D)


SOL 1.24 Impulse response of the matched filter is given by

$$h(t) = S(T - t)$$







Hence (C) is correct option.

MCQ 1.25 The electric field component of a time harmonic plane EM wave traveling in a nonmagnetic lossless dielectric medium has an amplitude of 1 V/m. If the relative permittivity of the medium is 4, the magnitude of the time-average power density vector (in W/m^2) is

(A) $\frac{1}{30\pi}$ (B) $\frac{1}{60\pi}$

(C) $\frac{1}{120\pi}$ (D) $\frac{1}{240\pi}$

SOL 1.25 Intrinsic impedance of EM wave

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{120\pi}{2} = 60\pi$$

Time average power density

$$P_{av} = \frac{1}{2}EH = \frac{1}{2}\frac{E^2}{\eta} = \frac{1}{2\times 60\pi} = \frac{1}{120\pi}$$

Hence (C) is correct option.

Q. No. 26-51 carry two marks each :

MCQ 1.26 If $e^y = x^{1/x}$, then y has a

(A) maximum at $x = e$ (B) minimum at $x = e$

(C) maximum at $x = e^{-1}$ (D) minimum at $x = e^{-1}$

SOL 1.26 Hence (A) is correct option.

Given that $e^y = x^{\frac{1}{x}}$

or $\ln e^y = \ln x^{\frac{1}{x}}$

or $y = \frac{1}{x}\ln x$

Now $\frac{dy}{dx} = \frac{1}{x}\frac{1}{x} + \ln x(-x^{-\frac{1}{x^2}}) = \frac{1}{x^2} - \frac{\ln}{x^2}$

For maxima and minima :

$$\frac{dy}{dx} = \frac{1}{x^2}(1 - \ln x) = 0$$

$$\ln x = 1 \rightarrow x = e^1$$

Now $\frac{d^2y}{dx^2} = -\frac{2}{x^3} - \ln x\left(-\frac{2}{x^3}\right) - \frac{1}{x^2}\left(\frac{1}{x}\right)$

$$= -\frac{2}{x^2} + \frac{2\ln x}{x^3} - \frac{1}{x^3}$$

$$\left.\frac{d^2x}{dy^2}\right|_{\text{at } x = e^1} = \frac{-2}{e^2} + \frac{2}{e^3} - \frac{1}{e^3} < 0$$

So, y has a maximum at $x = e^1$

MCQ 1.27 A fair coin is tossed independently four times. The probability of the event "the number of time heads shown up is more than the number of times tail shown up"

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$

(C) $\frac{1}{4}$ (D) $\frac{5}{16}$

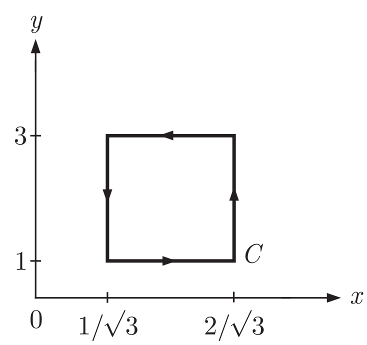
SOL 1.27 According to given condition head should comes 3 times or 4 times

$$P(\text{Heads comes 3 times or 4 times}) = {}^4C_4\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$$

$$= 1 \cdot \frac{1}{16} + 4 \cdot \frac{1}{8} \cdot \frac{1}{2} = \frac{5}{16}$$

Hence (D) is correct option.

MCQ 1.28 If $\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$, then $\oint_C \vec{A}.d\vec{l}$ over the path shown in the figure is



(A) 0 (B) $\frac{2}{\sqrt{3}}$

(C) 1 (D) $2\sqrt{3}$

SOL 1.28 Hence (C) is correct option

$$\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$$

$$\begin{aligned}
\vec{d}l &= dx\hat{a}_x + dy\hat{a}_y \\
\oint_C \vec{A} \cdot \vec{d}l &= \oint_C (xy\hat{a}_x + x^2\hat{a}_y) \cdot (dx\hat{a}_x + dy\hat{a}_y) \\
&= \oint_C (xydx + x^2 dy) \\
&= \int_{1/\sqrt{3}}^{2/\sqrt{3}} xdx + \int_{2/\sqrt{3}}^{1/\sqrt{3}} 3xdx + \int_1^3 \frac{4}{3} dy + \int_3^1 \frac{1}{3} dy \\
&= \frac{1}{2}\left[\frac{4}{3} - \frac{1}{3}\right] + \frac{3}{2}\left[\frac{1}{3} - \frac{4}{3}\right] + \frac{4}{3}[3-1] + \frac{1}{3}[1-3] \\
&= 1
\end{aligned}$$

MCQ 1.29 The residues of a complex function $x(z) = \frac{1-2z}{z(z-1)(z-2)}$ at its poles are

(A) $\frac{1}{2}, -\frac{1}{2}$ and 1

(B) $\frac{1}{2}, -\frac{1}{2}$ and -1

(C) $\frac{1}{2}, -1$ and $-\frac{3}{2}$

(D) $\frac{1}{2}, -1$ and $\frac{3}{2}$

SOL 1.29 Hence (C) is correct option.

Given function

$$X(z) = \frac{1-2z}{z(z-1)(z-2)}$$

Poles are located at $z = 0, z = 1$, and $z = 2$

At $Z = 0$ residues is

$$R_0 = z \cdot X(z)\big|_{Z=0} = \frac{1-2\times 0}{(0-1)(0-2)} = \frac{1}{2}$$

at $z = 1$, $R_1 = (Z-1) \cdot X(Z)\big|_{Z=1}$

$$= \frac{1-2\times 1}{1(1-2)} = 1$$

At $z = 2$, $R_2 = (z-2) \cdot X(z)\big|_{z=2}$

$$= \frac{1-2\times 2}{2(2-1)} = -\frac{3}{2}$$

MCQ 1.30 Consider differential equation $\frac{dy(x)}{dx} - y(x) = x$, with the initial condition $y(0) = 0$. Using Euler's first order method with a step size of 0.1, the value of $y(0.3)$ is

(A) 0.01

(B) 0.031

(C) 0.0631

(D) 0.1

SOL 1.30 Hence (B) is correct option.

Taking step size $h = 0.1,\ y(0) = 0$

x	y	$\frac{dy}{dx} = x + y$	$y_{i+1} = y_i + h\frac{dy}{dx}$
0	0	0	$y_1 = 0 + 0.1(0) = 0$

x	y	$\frac{dy}{dx} = x + y$	$y_{i+1} = y_i + h\frac{dy}{dx}$
0.1	0	0.1	$y_2 = 0 + 0.1(0.1) = 0.01$
0.2	0.01	0.21	$y_3 = 0.01 + 0.21 \times 0.1 = 0.031$
0.3	0.031		

From table, at $x = 0.3, y(x = 0.3) = 0.031$

MCQ 1.31 Given $f(t) = L^{-1}\left[\frac{3s+1}{s^3 + 4s^2 + (k-3)s}\right]$. If $\lim_{t \to \infty} f(t) = 1$, then the value of k is

(A) 1 (B) 2

(C) 3 (D) 4

SOL 1.31 Hence (D) is correct option.

We have $$f(t) = \mathcal{L}^{-1}\left[\frac{3s+1}{s^3 + 4s^2 + (k-3)s}\right]$$

and $$\lim_{t \to \infty} f(t) = 1$$

By final value theorem

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s) = 1$$

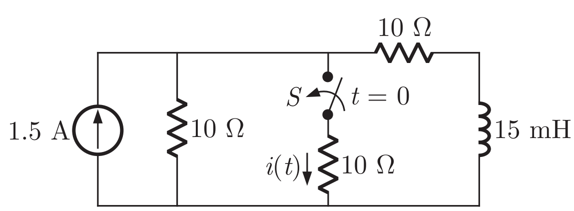
or $$\lim_{s \to 0} \frac{s.(3s+1)}{s^3 + 4s^2 + (k-3)s} = 1$$

or $$\lim_{s \to 0} \frac{s(3s+1)}{s[s^2 + 4s + (k-3)]} = 1$$

$$\frac{1}{k-3} = 1$$

or $$k = 4$$

MCQ 1.32 In the circuit shown, the switch S is open for a long time and is closed at $t = 0$. The current $i(t)$ for $t \geq 0^+$ is



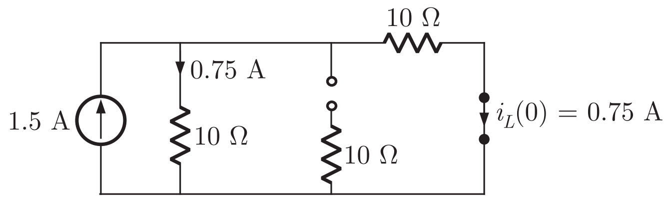
(A) $i(t) = 0.5 - 0.125e^{-1000t}$ A (B) $i(t) = 1.5 - 0.125e^{-1000t}$ A

(C) $i(t) = 0.5 - 0.5e^{-1000t}$ A (D) $i(t) = 0.375e^{-1000t}$ A

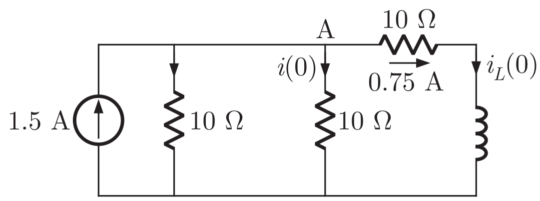
SOL 1.32 Hence (A) is correct option.

Let the current $i(t) = A + Be^{-t/\tau}$ $\tau \to$ Time constant

When the switch S is open for a long time before $t < 0$, the circuit is



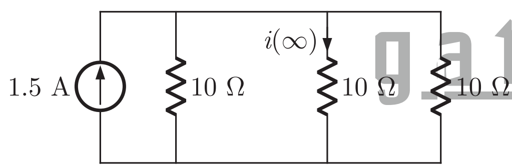
At $t = 0$, inductor current does not change simultaneously, So the circuit is



Current is resistor (AB)

$$i(0) = \frac{0.75}{2} = 0.375 \text{ A}$$

Similarly for steady state the circuit is as shown below



$$i(\infty) = \frac{15}{3} = 0.5 \text{ A}$$

$$\tau = \frac{L}{R_{eq}} = \frac{15 \times 10^{-3}}{10 + (10 \,||\, 10)} = 10^{-3} \text{ sec}$$

$$i(t) = A + Be^{-\frac{t}{1 \times 10^{-3}}} = A + Be^{-100t}$$

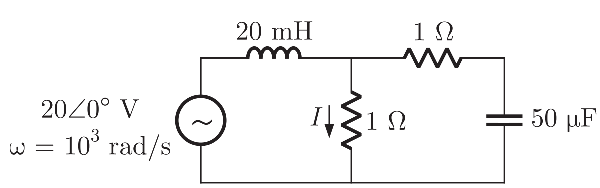
Now $\quad i(0) = A + B = 0.375$

and $\quad i(\infty) = A = 0.5$

So, $\quad B = 0.375 - 0.5 = -0.125$

Hence $\quad i(t) = 0.5 - 0.125e^{-1000\,t}$ A

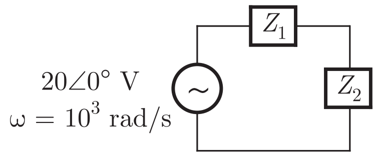
MCQ 1.33 The current I in the circuit shown is



(A) $-j1$ A (B) $j1$ A

(C) 0 A (D) 20 A

SOL 1.33 Circuit is redrawn as shown below



Where,
$$Z_1 = j\omega L = j \times 10^3 \times 20 \times 10^{-3} = 20j$$
$$Z_2 = R \,||\, X_C$$
$$X_C = \frac{1}{j\omega C} = \frac{1}{j \times 10^3 \times 50 \times 10^{-6}} = -20j$$
$$Z_2 = \frac{1(-20j)}{1-20j} \qquad R = 1\,\Omega$$

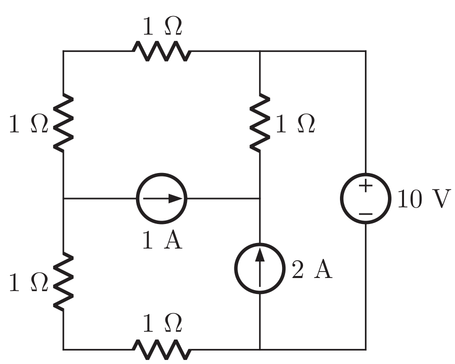
Voltage across Z_2

$$V_{Z_2} = \frac{Z_2}{Z_1 + Z_2} \cdot 20\underline{/0} = \frac{\left(\frac{-20j}{1-20j}\right)}{\left(20j - \frac{20j}{1-20j}\right)} \cdot 20$$
$$= \left(\frac{(-20j)}{20j + 400 - 20j}\right) \cdot 20 = -j$$

Current in resistor R is
$$I = \frac{V_{Z_2}}{R} = -\frac{j}{1} = -j\text{ A}$$

Hence (A) is correct option.

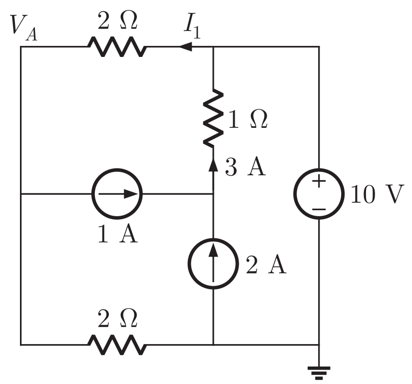
MCQ 1.34 In the circuit shown, the power supplied by the voltage source is



(A) 0 W (B) 5 W

(C) 10 W (D) 100 W

SOL 1.34 The circuit can be redrawn as



Applying nodal analysis

$$\frac{V_A - 10}{2} + 1 + \frac{V_A - 0}{2} = 0$$

$$2V_A - 10 + 2 = 0 = V_4 = 4 \text{ V}$$

Current, $I_1 = \frac{10 - 4}{2} = 3 \text{ A}$

Current from voltage source is

$$I_2 = I_1 - 3 = 0$$

Since current through voltage source is zero, therefore power delivered is zero.
Hence (A) is correct option.

MCQ 1.35 In a uniformly doped BJT, assume that N_E, N_B and N_C are the emitter, base and collector doping in atoms/cm^3, respectively. If the emitter injection efficiency of the BJT is close unity, which one of the following condition is TRUE

(A) $N_E = N_B = N_C$ (B) $N_E >> N_B$ and $N_B > N_C$

(C) $N_E = N_B$ and $N_B < N_C$ (D) $N_E < N_B < N_C$

SOL 1.35 Emitter injection efficiency is given as

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}}$$

To achieve $\gamma = 1, N_E >> N_B$

Hence (B) is correct option.

MCQ 1.36 Compared to a p-n junction with $N_A = N_D = 10^{14}/\text{cm}^3$, which one of the following statements is TRUE for a p-n junction with $N_A = N_D = 10^{20}/\text{cm}^3$?

(A) Reverse breakdown voltage is lower and depletion capacitance is lower

(B) Reverse breakdown voltage is higher and depletion capacitance is lower

(C) Reverse breakdown voltage is lower and depletion capacitance is higher

(D) Reverse breakdown voltage is higher and depletion capacitance is higher

SOL 1.36 Reverse bias breakdown or Zener effect occurs in highly doped PN junction through tunneling mechanism. In a highly doped PN junction, the conduction and valence bands on opposite sides of the junction are sufficiently close during reverse bias that electron may tunnel directly from the valence band on the p-side into the conduction band on n-side.

Breakdown voltage $V_B \propto \frac{1}{N_A N_D}$

So, breakdown voltage decreases as concentration increases

Depletion capacitance

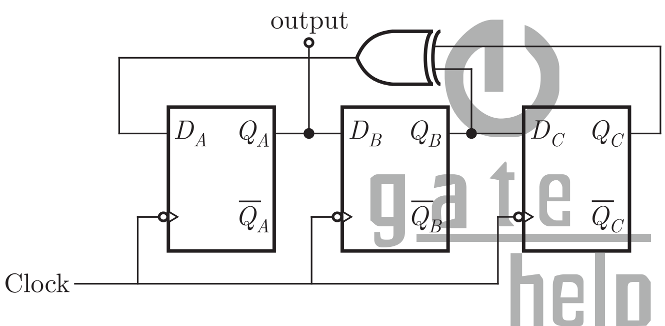
$$C = \left\{\frac{e\varepsilon_s N_A N_D}{2(V_{bi} + V_R)(N_A + N_D)}\right\}^{1/2}$$

Thus $\quad C \propto N_A N_D$

Depletion capacitance increases as concentration increases

Hence (C) is correct option.

MCQ 1.37 Assuming that the flip-flop are in reset condition initially, the count sequence observed at Q_A, in the circuit shown is



(A) 0010111... (B) 0001011...

(C) 0101111... (D) 0110100....

SOL 1.37 Let $Q_A(n), Q_B(n), Q_C(n)$ are present states and $Q_A(n+1), Q_B(n+1), Q_C(n+1)$ are next states of flop-flops.

In the circuit

$$Q_A(n+1) = Q_B(n) \odot Q_C(n)$$
$$Q_B(n+1) = Q_A(n)$$
$$Q_C(n+1) = Q_B(n)$$

Initially all flip-flops are reset

1^{st} clock pulse

$$Q_A = 0 \odot 0 = 1$$
$$Q_B = 0$$
$$Q_C = 0$$

2^{nd} clock pulse

$$Q_A = 0 \odot 0 = 1$$
$$Q_B = 1$$
$$Q_C = 0$$

3^{rd} clock pulse

$$Q_A = 1 \odot 0 = 0$$
$$Q_B = 1$$
$$Q_C = 1$$

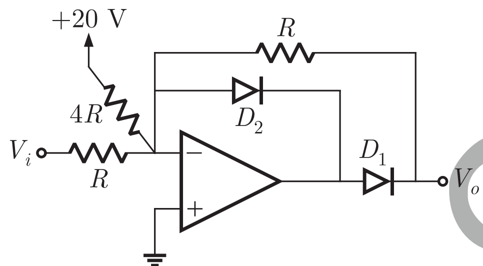
4^{th} clock pulse

$$Q_A = 1 \odot 1 = 1$$
$$Q_B = 0$$
$$Q_C = 1$$

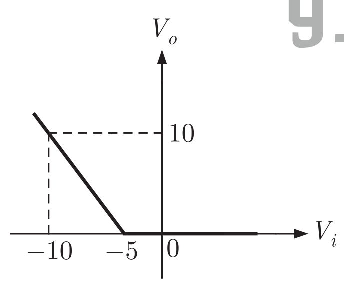
So, sequence $Q_A = 01101.......$

Hence (D) is correct option.

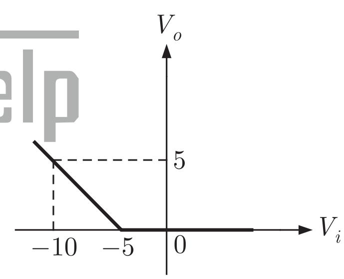
MCQ 1.38 The transfer characteristic for the precision rectifier circuit shown below is (assume ideal OP-AMP and practical diodes)



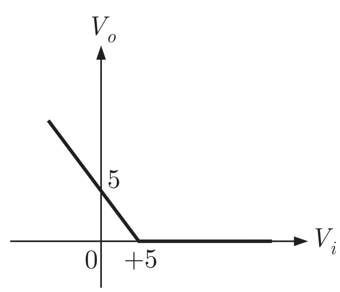
(A)



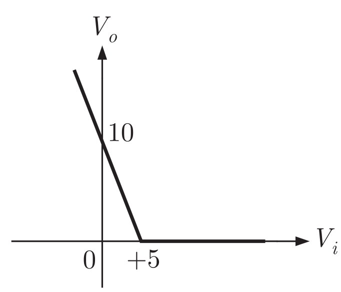
(B)



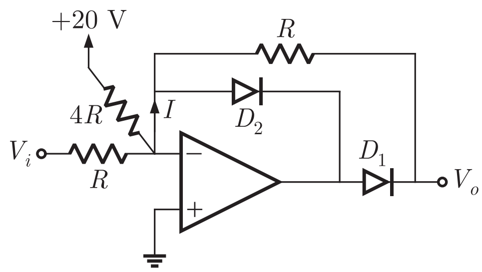
(C)



(D)



SOL 1.38 The circuit is as shown below

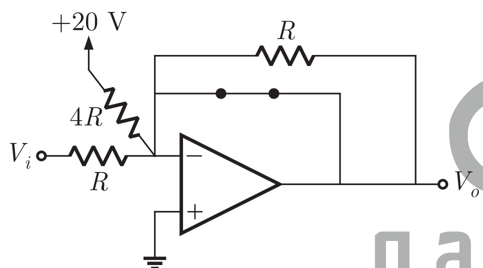


Current $$I = \frac{20-0}{4R} + \frac{V_i - 0}{R} = \frac{5+V_i}{R}$$

If $I > 0$, diode D_2 conducts

So, for $\frac{5+V_I}{2} > 0 \Rightarrow V_I > -5, D_2$ conducts

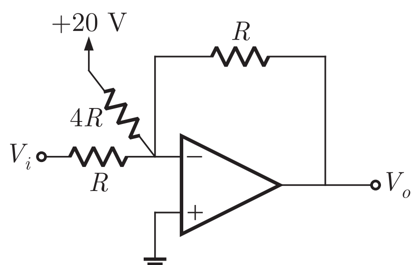
Equivalent circuit is shown below



Output is $V_o = 0$. If $I < 0$, diode D_2 will be off

$$\frac{5+V_I}{R} < 0 \Rightarrow V_I < -5, D_2 \text{ is off}$$

The circuit is shown below



$$\frac{0-V_i}{R} + \frac{0-20}{4R} + \frac{0-V_o}{R} = 0$$

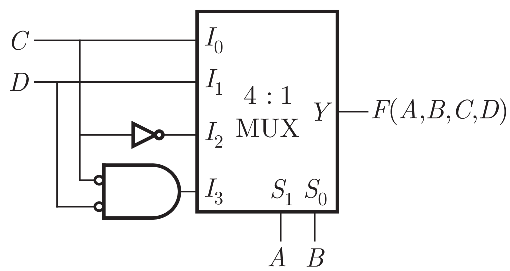
or $$V_o = -V_i - 5$$

At $V_i = -5$ V, $V_o = 0$

At $V_i = -10$ V, $V_o = 5$ V

Hence (B) is correct option.

MCQ 1.39 The Boolean function realized by the logic circuit shown is



(A) $F = \Sigma m(0,1,3,5,9,10,14)$ (B) $F = \Sigma m(2,3,5,7,8,12,13)$

(C) $F = \Sigma m(1,2,4,5,11,14,15)$ (D) $F = \Sigma m(2,3,5,7,8,9,12)$

SOL 1.39 Output of the MUX can be written as

$$F = I_0 \overline{S_0}\,\overline{S_1} + I_1 \overline{S_0} S_1 + I_2 S_0 \overline{S_1} + I_3 S_0 S_1$$

Here, $I_0 = C, I_1 = D, I_2 = \overline{C}, I_3 = \overline{CD}$

and $S_0 = A, S_1 = B$

So, $$F = C\,\overline{A}\,\overline{B} + D\,\overline{A}\,B + \overline{C}\,A\,\overline{B} + \overline{C}\,\overline{D}\,A\,\overline{B}$$

Writing all SOP terms

$$F = \underbrace{\overline{A}\,\overline{B}\,C\,\overline{D}}_{m_3} + \underbrace{\overline{A}\,\overline{B}\,C\,\overline{D}}_{m_2} + \underbrace{\overline{A}\,BCD}_{m_7} + \underbrace{\overline{A}\,B\,\overline{C}\,D}_{m_5}$$
$$+ \underbrace{A\,\overline{B}\,\overline{C}\,D}_{m_9} + \underbrace{A\,\overline{B}\,\overline{C}\,\overline{D}}_{m_8} + \underbrace{AB\overline{C}\,\overline{D}}_{m_{12}}$$

$$F = \sum m(2,3,5,7,8,9,12)$$

Hence (D) is correct option.

MCQ 1.40 For the 8085 assembly language program given below, the content of the accumulator after the execution of the program is

```
3000  MVI   A,   45H
3002  MOV   B,   A
3003  STC
3004  CMC
3005  RAR
3006  XRA   B
```

(A) 00H (B) 45H

(C) 67H (D) E7H

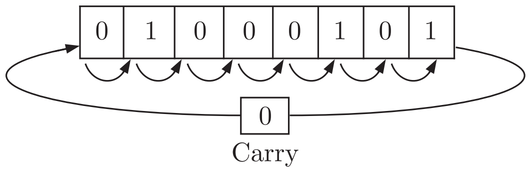
SOL 1.40 By executing instruction one by one

MVI A, 45 H $\Rightarrow$ MOV 45 H into accumulator, $A = 45$ H

STC $\Rightarrow$ Set carry, $C = 1$

CMC $\Rightarrow$ Complement carry flag, $C = 0$

RAR $\Rightarrow$ Rotate accumulator right through carry



$$A = 00100010$$

XRA B⇒ XOR A and B

$$A = A \oplus B = 00100010 \oplus 01000101 = 01100111 = 674$$

Hence (C) is correct option.

MCQ 1.41 A continuous time LTI system is described by

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$$

Assuming zero initial conditions, the response $y(t)$ of the above system for the input $x(t) = e^{-2t}u(t)$ is given by

(A) $(e^t - e^{3t})\,u(t)$

(B) $(e^{-t} - e^{-3t})\,u(t)$

(C) $(e^{-t} + e^{-3t})\,u(t)$

(D) $(e^t + e^{3t})\,u(t)$

SOL 1.41 System is described as

$$\frac{d^2y(t)}{dt^2} + 4\frac{dt(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$$

Taking laplace transform on both side of given equation

$$s^2Y(s) + 4sY(s) + 3Y(s) = 2sX(s) + 4X(s)$$

$$(s^2 + 4s + 3)\,Y(s) = 2(s+2)\,X(s)\,s$$

Transfer function of the system

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2(s+2)}{s^2+4s+3} = \frac{2(s+2)}{(s+3)(s+1)}$$

Input $\quad x(t) = e^{-2t}u(t)$

or, $\quad X(s) = \frac{1}{(s+2)}$

Output $\quad Y(s) = H(s) \cdot X(s)$

$$Y(s) = \frac{2(s+2)}{(s+3)(s+1)} \cdot \frac{1}{(s+2)}$$

By Partial fraction

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+3}$$

Taking inverse laplace transform

$$y(t) = (e^{-t} - e^{-3t})\,u(t)$$

Hence (B) is correct option.

MCQ 1.42 The transfer function of a discrete time LTI system is given by

$$H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Consider the following statements:

S1: The system is stable and causal for ROC: $|z| > 1/2$

S2: The system is stable but not causal for ROC: $|z| < 1/4$

S3: The system is neither stable nor causal for ROC: $1/4 < |z| < 1/2$

Which one of the following statements is valid ?

(A) Both S1 and S2 are true (B) Both S2 and S3 are true

(C) Both S1 and S3 are true (D) S1, S2 and S3 are all true

SOL 1.42 Hence (C) is correct option.

We have

$$H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

By partial fraction $H(z)$ can be written as

$$H(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})} + \frac{1}{(1 - \frac{1}{4}z^{-1})}$$

For ROC : $|z| > 1/2$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n],\ n > 0 \qquad \frac{1}{1 - z^{-1}} = a^n u[n], |z| > a$$

Thus system is causal. Since ROC of $H(z)$ includes unit circle, so it is stable also. Hence S_1 is True

For ROC : $|z| < \frac{1}{4}$

$$h[n] = -\left(\frac{1}{2}\right)^n u[-n-1] + \left(\frac{1}{4}\right)^n u(n), |z| > \frac{1}{4}, |z| < \frac{1}{2}$$

System is not causal. ROC of $H(z)$ does not include unity circle, so it is not stable and S_3 is True

MCQ 1.43 The Nyquist sampling rate for the signal

$s(t) = \frac{\sin(500\pi t)}{\pi t} \times \frac{\sin(700)\pi t}{\pi t}$ is given by

(A) 400 Hz (B) 600 Hz

(C) 1200 Hz (D) 1400 Hz

SOL 1.43 Hence(C) is correct option.

$$S(t) = \sin c(500t) \sin c(700t)$$

$S(f)$ is convolution of two signals whose spectrum covers $f_1 = 250$ Hz and $f_2 = 350$ Hz . So convolution extends

$$f = 25 + 350 = 600 \text{ Hz}$$

Nyquist sampling rate

$$N = 2f = 2 \times 600 = 1200 \text{ Hz}$$

MCQ 1.44 A unity negative feedback closed loop system has a plant with the transfer function $G(s) = \frac{1}{s^2+2s+2}$ and a controller $G_c(s)$ in the feed forward path. For a unit set input, the transfer function of the controller that gives minimum steady state error is

(A) $G_c(s) = \dfrac{s+1}{s+2}$

(B) $G_c(s) = \dfrac{s+2}{s+1}$

(C) $G_c(s) = \dfrac{(s+1)(s+4)}{(s+2)(s+3)}$

(D) $G_c(s) = 1 + \dfrac{2}{s} + 3s$

SOL 1.44 Steady state error is given as

$$e_{SS} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)\,G_C(s)}$$

$$R(s) = \frac{1}{s} \qquad \text{(unit step unit)}$$

$$e_{SS} = \lim_{s \to 0} \frac{1}{1 + G(s)\,G_C(s)}$$

$$= \lim_{s \to 0} \frac{1}{1 + \dfrac{G_C(s)}{s^2 + 2s + 2}}$$

e_{SS} will be minimum if $\lim_{s \to 0} G_C(s)$ is maximum

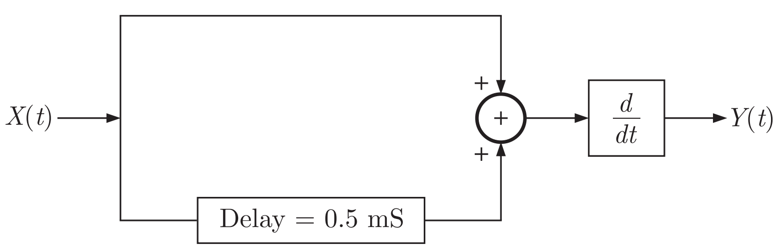
In option (D)

$$\lim_{s \to 0} G_C(s) = \lim_{s \to 0} 1 + \frac{2}{s} + 3s = \infty$$

So, $$e_{SS} = \lim_{s \to 0} \frac{1}{\infty} = 0 \text{ (minimum)}$$

Hence (D) is correct option.

MCQ 1.45 $X(t)$ is a stationary process with the power spectral density $S_x(f) > 0$, for all f. The process is passed through a system shown below



Let $S_y(f)$ be the power spectral density of $Y(t)$. Which one of the following statements is correct

(A) $S_y(f) > 0$ for all f

(B) $S_y(f) = 0$ for $|f| > 1$ kHz

(C) $S_y(f) = 0$ for $f = nf_0$, $f_0 = 2$ kHz kHz, n any integer

(D) $S_y(f) = 0$ for $f = (2n+1)f_0 = 1\,\text{kHz}$, n any integer

SOL 1.45 For the given system, output is written as

$$y(t) = \frac{d}{dt}[x(t) + x(t-0.5)]$$

$$y(t) = \frac{dx(t)}{dt} + \frac{dx(t-0.5)}{dt}$$

Taking laplace on both sides of above equation

$$Y(s) = sX(s) + se^{-0.5s}X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = s(1 + e^{-0.5s})$$

$$H(f) = jf(1 + e^{-0.5 \times 2\pi f}) = jf(1 + e^{-\pi f})$$

Power spectral density of output

$$S_Y(f) = |H(f)|^2 S_X(f) = f^2(1 + e^{-\pi f})^2 S_X(f)$$

For $S_Y(f) = 0$, $\quad 1 + e^{-\pi f} = 0$

$$f = (2n+1)f_0$$

or $\quad f_0 = 1\,\text{KHz}$

Hence (D) is correct option.

MCQ 1.46 A plane wave having the electric field components $\vec{E}_i = 24\cos(3 \times 10^8 - \beta y)\hat{a}_x$ V/m and traveling in free space is incident normally on a lossless medium with $\mu = \mu_0$ and $\varepsilon = 9\varepsilon_0$ which occupies the region $y \geq 0$. The reflected magnetic field component is given by

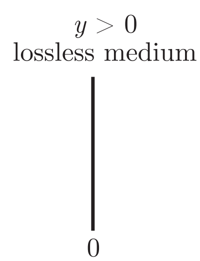
(A) $\frac{1}{10\pi}\cos(3 \times 10^8 t + y)\,\hat{a}_x$ A/m

(B) $\frac{1}{20\pi}\cos(3 \times 10^8 t + y)\,\hat{a}_x$ A/m

(C) $-\frac{1}{20\pi}\cos(3 \times 10^8 t + y)\,\hat{a}_x$ A/m

(D) $-\frac{1}{10\pi}\cos(3 \times 10^8 t + y)\,\hat{a}_x$ A/m

SOL 1.46 In the given problem



$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$$

$$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{9\varepsilon_0}} = \frac{120}{3} = 40\pi$$

Reflection coefficient

$$\tau = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{400\pi - 120\pi}{40\pi + 120\pi} = -\frac{1}{2}$$

τ is negative So magnetic field component does not change its direction Direction of incident magnetic field

$$\hat{a}_E \times \hat{a}_H = \hat{a}_K$$
$$\hat{a}_Z \times \hat{a}_H = \hat{a}_y$$
$$\hat{a}_H = \hat{a}_x \ (+x \text{ direction})$$

So, reflection magnetic field component

$$H_r = \left|\frac{\tau \times 24}{\eta}\right| \cos(3 \times 10^8 + \beta y)\,\hat{a}_x, \ y \geq 0$$

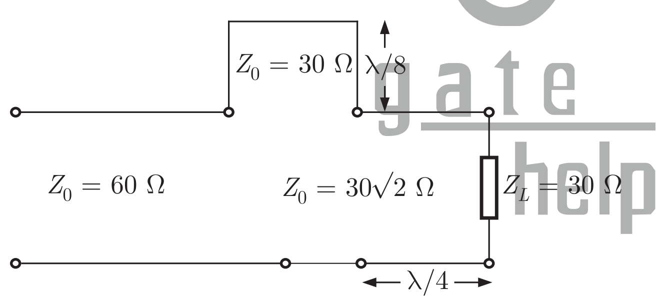
$$= \left|\frac{1 \times 24}{2 \times 120\pi}\right| \cos(3 \times 10^8 + \beta y)\,\hat{a}_x, \ \ y \geq 0$$

$$\beta = \frac{\omega}{v_C} = \frac{3 \times 10^8}{3 \times 10^8} = 1$$

So, $H_r = \frac{1}{10\pi}\cos(3 \times 10^8 + y)\,\hat{a}_x, y \geq 0$

Hence (A) is correct option.

MCQ 1.47 In the circuit shown, all the transmission line sections are lossless. The Voltage Standing Wave Ration(VSWR) on the 60 Ω line is



(A) 1.00 (B) 1.64

(C) 2.50 (D) 3.00

SOL 1.47 For length of $\lambda/4$ transmission line

$$Z_{\text{in}} = Z_o\left[\frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}\right]$$

$Z_L = 30\,\Omega$, $\quad Z_o = 30\,\Omega,\ \beta = \frac{2\pi}{\lambda},\ l = \frac{\lambda}{4}$

So, $\quad \tan\beta l = \tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\right) = \infty$

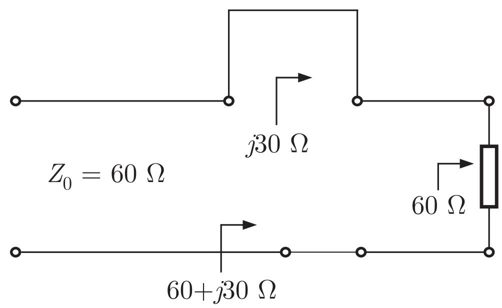
$$Z_{\text{in}} = Z_o\left[\frac{\frac{Z_L}{\tan\beta l} + jZ_o}{\frac{Z_o}{\tan\beta l} + jZ_L}\right] = \frac{Z_0^2}{Z_L} = 60\,\Omega$$

For length of $\lambda/8$ transmission line

$$Z_{\text{in}} = Z_o\left[\frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}\right]$$

$$Z_o = 30\,\Omega,\ Z_L = 0\ \text{(short)}$$
$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{8}\right) = 1$$
$$Z_{\text{in}} = jZ_o \tan\beta l = 30j$$

Circuit is shown below.



Reflection coefficient

$$\tau = \left|\frac{Z_L - Z_o}{Z_L + Z_o}\right| = \left|\frac{60 + 3j - 60}{60 + 3j + 60}\right| = \frac{1}{\sqrt{17}}$$

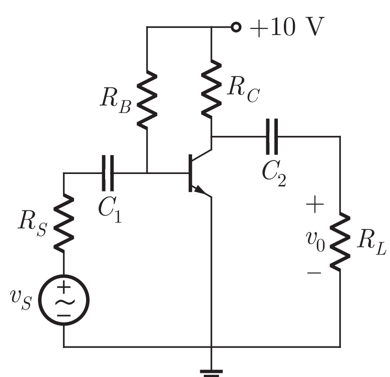
$$\text{VSWR} = \frac{1 + |\tau|}{1 - |\tau|} = \frac{1 + \sqrt{17}}{1 - \sqrt{17}} = 1.64$$

Hence (B) is correct option.

Common Data Questions: 48 & 49 :

Consider the common emitter amplifier shown below with the following circuit parameters:

$\beta = 100, g_m = 0.3861\ \text{A/V}, r_0 = 259\,\Omega, R_S = 1\,\text{k}\Omega, R_B = 93\,\text{k}\Omega,$
$R_C = 250\,\text{k}\Omega, R_L = 1\,\text{k}\Omega, C_1 = \infty$ and $C_2 = 4.7\,\mu\text{F}$

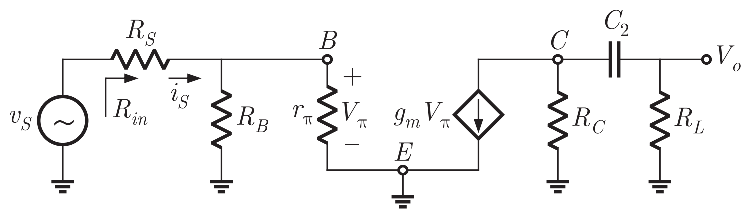


MCQ 1.48 The resistance seen by the source v_S is

(A) 258 Ω (B) 1258 Ω

(C) 93 kΩ (D) ∞

SOL 1.48 By small signal equivalent circuit analysis



Input resistance seen by source v_s

$$R_{\text{in}} = \frac{v_s}{i_s} = R_s + R_s \,||\, r_s$$

$$= (1000\,\Omega) + (93\,\text{k}\Omega \,||\, 259\,\Omega) = 1258\,\Omega$$

Hence (B) is correct option.

MCQ 1.49 The lower cut-off frequency due to C_2 is

(A) 33.9 Hz (B) 27.1 Hz

(C) 13.6 Hz (D) 16.9 Hz

SOL 1.49 Cut-off frequency due to C_2

$$f_o = \frac{1}{2\pi (R_C + R_L) C_2}$$

$$f_o = \frac{1}{2 \times 3.14 \times 1250 \times 4.7 \times 10^{-6}} = 271\,\text{Hz}$$

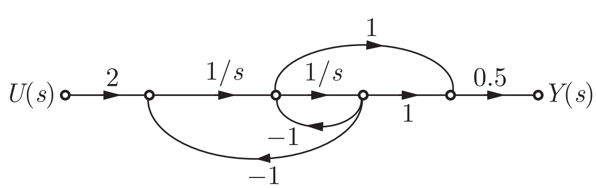
Lower cut-off frequency

$$f_L \approx \frac{f_o}{10} = \frac{271}{10} = 27.1\,\text{Hz}$$

Hence (B) is correct option.

Common Data Question : 50 & 51 :

The signal flow graph of a system is shown below:



MCQ 1.50 The state variable representation of the system can be

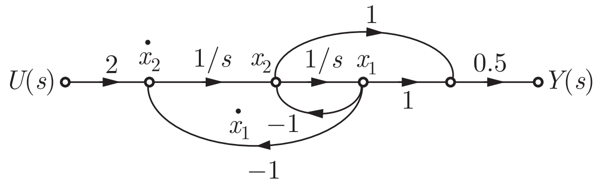
(A) $\dot{x} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$
$\dot{y} = [0 \;\; 0.5] x$

(B) $\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$
$\dot{y} = [0 \;\; 0.5] x$

(C) $\dot{x} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$
$\dot{y} = [0.5 \;\; 0.5] x$

(D) $\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$
$\dot{y} = [0.5 \;\; 0.5] x$

SOL 1.50 Assign output of each integrator by a state variable



$$\dot{x}_1 = -x_1 + x_2$$
$$\dot{x}_2 = -x_1 + 2u$$
$$y = 0.5x_1 + 0.5x_2$$

State variable representation

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$
$$\dot{y} = [0.5 \;\; 0.5]\, x$$

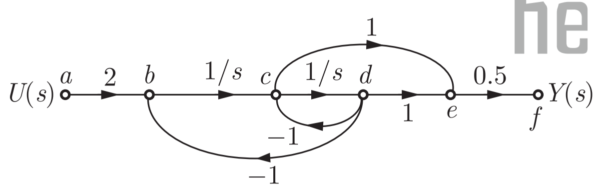
Hence (D) is correct option.

MCQ 1.51 The transfer function of the system is

(A) $\frac{s+1}{s^2+1}$ (B) $\frac{s-1}{s^2+1}$

(C) $\frac{s+1}{s^2+s+1}$ (D) $\frac{s-1}{s^2+s+1}$

SOL 1.51 By masson's gain formula



Transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{\sum P_K \Delta_K}{\Delta}$$

Forward path given

$$P_1(abcdef) = 2 \times \frac{1}{s} \times \frac{1}{s} \times 0.5 = \frac{1}{s^2}$$

$$P_2(abcdef) = 2 \times \frac{1}{3} \times 1 \times 0.5$$

Loop gain $L_1(cdc) = -\frac{1}{s}$

$$L_2(bcdb) = \frac{1}{s} \times \frac{1}{s} \times -1 = \frac{-1}{s^2}$$

$$\Delta = 1 - [L_1 + L_2] = 1 - \left[-\frac{1}{s} - \frac{1}{s^2}\right] = 1 + \frac{1}{s} + \frac{1}{s^2}$$

$$\Delta_1 = 1,\; \Delta_2 = 2$$

So, $$H(s) = \frac{Y(s)}{U(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

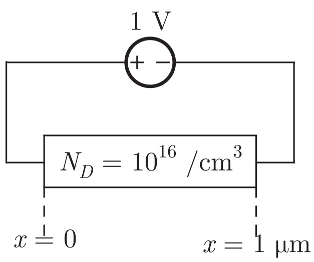
$$= \frac{\frac{1}{s^2}\cdot 1 + \frac{1}{s}\cdot 1}{1 + \frac{1}{s} + \frac{1}{s^2}} = \frac{(1+s)}{(s^2+s+1)}$$

Hence (C) is correct option.

Linked Answer Questions: Q. 52 to Q. 55

Statements for Linked Answer Question : 52 & 53 :

The silicon sample with unit cross-sectional area shown below is in thermal equilibrium. The following information is given: $T = 300\,\text{K}$ electronic charge $= 1.6\times10^{-19}\,\text{C}$, thermal voltage $= 26\,\text{mV}$ and electron mobility $= 1350\,\text{cm}^2/\text{V-s}$



MCQ 1.52 The magnitude of the electric field at $x = 0.5\,\mu\text{m}$ is

(A) 1 kV/cm (B) 5 kV/cm

(C) 10 kV/cm (D) 26 kV/cm

SOL 1.52 Sample is in thermal equilibrium so, electric field

$$E = \frac{1}{1\,\mu\text{m}} = 10\,\text{kV/cm}$$

Hence (C) is correct option.

MCQ 1.53 The magnitude of the electron of the electron drift current density at $x = 0.5\,\mu\text{m}$ is

(A) $2.16\times10^4\ \text{A/cm}^2$ (B) $1.08\times10^4\ \text{A/m}^2$

(C) $4.32\times10^3\ \text{A/cm}^2$ (D) $6.48\times10^2\ \text{A/cm}^2$

SOL 1.53 Electron drift current density

$$J_d = N_D\mu_n eE$$
$$= 10^{16}\times1350\times1.6\times10^{-19}\times10\times10^{13}$$
$$= 2.16\times10^4\ \text{A/cm}^2$$

Hence (A) is correct option.

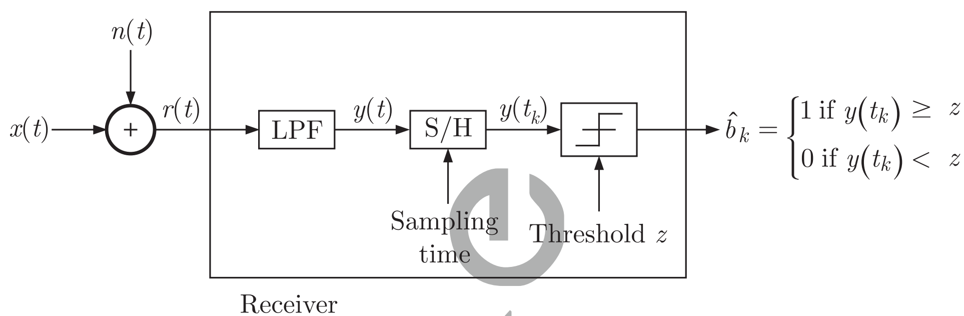
Statement for linked Answer Question : 54 & 55 :

Consider a baseband binary PAM receiver shown below. The additive channel noise $n(t)$ is with power spectral density $S_n(f) = N_0/2 = 10^{-20}$ W/Hz. The low-pass filter is ideal with unity gain and cut-off frequency 1 MHz. Let Y_k represent the random variable $y(t_k)$.

$Y_k = N_k$, if transmitted bit $b_k = 0$

$Y_k = a + N_k$ if transmitted bit $b_k = 1$

Where N_k represents the noise sample value. The noise sample has a probability density function, $P_{Nk}(n) = 0.5\alpha e^{-\alpha|n|}$ (This has mean zero and variance $2/\alpha^2$). Assume transmitted bits to be equiprobable and threshold z is set to $a/2 = 10^{-6}$ V.



MCQ 1.54 The value of the parameter α (in V^{-1}) is

(A) 10^{10}

(B) 10^{7}

(C) 1.414×10^{-10}

(D) 2×10^{-20}

SOL 1.54 Let response of LPF filters

$$H(f) = \begin{cases} 1, & |f| < 1\,\text{MHz} \\ 0, & \text{elsewhere} \end{cases}$$

Noise variance (power) is given as

$$P = \sigma^2 = \int_0^{f_o} |H(f)|^2 N_o\, df = \frac{2}{\alpha^2} \text{ (given)}$$

$$\int_0^{1\times 10^6} 2 \times 10^{-20}\, df = \frac{2}{\alpha^2}$$

$$2 \times 10^{-20} \times 10^6 = \frac{2}{\alpha^2}$$

$$\alpha^2 = 10^{14}$$

or $$\alpha = 10^{7}$$

Hence (B) is correct option.

MCQ 1.55 The probability of bit error is

(A) $0.5 \times e^{-3.5}$

(B) $0.5 \times e^{-5}$

(C) $0.5 \times e^{-7}$

(D) $0.5 \times e^{-10}$

SOL 1.55 Probability of error is given by

$$P_e = \frac{1}{2}[P(0/1) + P(1/0)]$$

$$P(0/1) = \int_{-\infty}^{\alpha/2} 0.5 e^{-\alpha|n-a|} dn = 0.5 e^{-10}$$

where $a = 2 \times 10^{-6}$ V and $\alpha = 10^7 V^{-1}$

$$P(1/0) = \int_{a/2}^{\infty} 0.5 e^{-\alpha|n|} dn = 0.5 e^{-10}$$

$$P_e = 0.5 e^{-10}$$

Hence (D) is correct option.

Q. No. 56 - 60 Carry One Mark Each :

MCQ 1.56 Which of the following options is closest in meaning to the world below:

(A) Cyclic (B) Indirect

(C) Confusing (D) Crooked

SOL 1.56 Circuitous means round about or not direct. Indirect is closest in meaning to this circuitous

(A) Cyclic : Recurring in nature

(B) Indirect : Not direct

(C) Confusing : lacking clarity of meaning

(D) Crooked : set at an angle; not straight

Hence (B) is correct option.

MCQ 1.57 The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

Unemployed: Worker

(A) fallow : land (B) unaware: sleeper

(C) wit : jester (D) renovated : house

SOL 1.57 A worker may by unemployed. Like in same relation a sleeper may be unaware.

Hence (B) is correct option.

MCQ 1.58 Choose the most appropriate word from the options given below to complete the following sentence;

If we manage to ____ our natural resources, we would leave a better planet for our children.

(A) uphold (B) restrain

(C) Cherish (D) conserve

SOL 1.58 Here conserve is most appropriate word.

Hence (D) is correct option.

MCQ 1.59 Choose the most appropriate word from the options given below to complete the following sentence:

His rather casual remarks on politics ___ his lack of seriousness about the subject

(A) masked (B) belled

(C) betrayed (D) suppressed

SOL 1.59 Betrayed means reveal unintentionally that is most appropriate.

Hence (C) is correct option.

MCQ 1.60 25 persons are in a room, 15 of them play hockey, 17 of them football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is ;

(A) 2 (B) 17

(C) 13 (D) 3

SOL 1.60 Hence (D) is correct option.

Number of people who play hockey $n(A) = 15$

Number of people who play football $n(B) = 17$

Persons who play both hockey and football $n(A \cap B) = 10$

Persons who play either hockey or football or both :

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 15 + 17 - 10 = 22 \end{aligned}$$

Thus people who play neither hockey nor football $= 25 - 22 = 3$

Q. No. 61-65 Carry Two Marks Each

MCQ 1.61 Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.

Which of the following statements best sums up the meaning of the above passage :

(A) Modern warfare has resulted in civil strife.

(B) Chemical agents are useful in modern warfare.

(C) Use of chemical agents in warfare would be undesirable

(D) People in military establishment like to use agents in war

SOL 1.61 Hence (D) is correct option.

MCQ 1.62 If $137 + 276 = 435$ how much is $731 + 672$?

(A) 534 (B) 1403

(C) 1623 (D) 1513

SOL 1.62 Since $7 + 6 = 13$ but unit digit is 5 so base may be 8 as 5 is the remainder when 13 is divided by 8. Let us check.

$$\begin{array}{r} 137_8 \\ \underline{276_8} \\ 435 \end{array} \quad \text{Thus here base is 8. Now} \quad \begin{array}{r} 731_8 \\ \underline{672_8} \\ 1623 \end{array}$$

Hence (C) is correct option.

MCQ 1.63 5 skilled workers can build a wall in 20 days; 8 semi-killed worker can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 killed, 6 semi-killed and 5 unskilled workers, how long will it take to build the wall

(A) 20 days (B) 18 days

(C) 16 days (D) 15 days

SOL 1.63 Hence (D) is correct option.

Let W be the total work.

Per day work of 5 skilled workers $= \frac{W}{20}$

Per day work of one skill worker $= \frac{W}{5 \times 20} = \frac{W}{100}$

Similarly per day work of 1 semi-skilled workers $= \frac{W}{8 \times 25} = \frac{W}{200}$

Similarly per day work of one semi-skill worker $= \frac{W}{10 \times 30} = \frac{W}{300}$

Thus total per day work of 2 skilled, 6 semi-skilled and 5 unskilled workers is

$$= \frac{2W}{100} + \frac{6W}{200} + \frac{5W}{300} = \frac{12W + 18W + 10W}{600} = \frac{W}{15}$$

Therefore time to complete the work is 15 days.

MCQ 1.64 Given digits 2, 2, 3, 3, 4, 4, 4 how many distinct 4 digit numbers greater than 3000 can be formed

(A) 50 (B) 51

(C) 52 (D) 54

SOL 1.64 As the number must be greater than 3000, it must be start with 3 or 4. Thus we have two case:

Case (1) If left most digit is 3 an other three digits are any of 2, 2, 3, 3, 4, 4, 4, 4.

(1) Using 2, 2, 3 we have 3223, 3232, 3322 i.e. $\frac{3!}{2!} = 3$ no.

(2) Using 2,2,4 we have 3224, 3242, 3422 i.e. $\frac{3!}{2!} = 3$ no.

(3) Using 2,3,3 we have 3233,3323,3332 i.e. $\frac{3!}{2!} = 3$ no.

(4) Using 2,3,4 we have $3! = 6$ no.

(5) Using 2,4,4 we have 3244, 3424, 3442 i.e. $\frac{3!}{2!} = 3$ no.

(6) Using 3,3,4 we have 3334, 3343, 3433 i.e. $\frac{3!}{2!} = 3$ no.

(7) Using 3,4,4 we have 3344,3434,3443 i.e. $\frac{3!}{2!} = 3$ no.

(8) Using 4,4,4 we have 3444 i.e. $\frac{3!}{3!} = 1$ no.

Total 4 digit numbers in this case is
$1 + 3 + 3 + 3 + 6 + 3 + 3 + 3 + 1 = 25$
Case 2 : If left most is 4 and other three digits are any of 2, 2, 3, 3, 3, 4, 4, 4.
(1) Using 2,2,3 we have 4223, 4232, 4322 i.e. . $\frac{3!}{2!} = 3$ no

(2) Using 2,2,4 we have 4224, 4242, 4422 i.e. . $\frac{3!}{2!} = 3$ no

(3) Using 2,3,3 we have 4233, 4323, 4332 i.e. . $\frac{3!}{2!} = 3$ no

(4) Using 2,3,4 we have i.e. . $3! = 6$ no
(5) Using 2,4,4 we have 4244, 4424, 4442 i.e. . $\frac{3!}{2!} = 3$ no

(6) Using 3,3,3 we have 4333 i.e $\frac{3!}{3!} = 1$. no.

(7) Using 3,3,4 we have 4334, 4343, 4433 i.e. . $\frac{3!}{2!} = 3$ no

(8) Using 3,4,4 we have 4344, 4434, 4443 i.e. . $\frac{3!}{2!} = 3$ no

(9) Using 4,4,4 we have 4444 i.e. $\frac{3!}{3!} = 1$. no

Total 4 digit numbers in 2nd case $= 3 + 3 + 3 + 6 + 3 + 3 + 1 + 3 + 1 = 26$
Thus total 4 digit numbers using case (1) and case (2) is $= 25 + 26 = 51$
Hence (B) is correct option.

MCQ 1.65 Hari(H), Gita(G), Irfan(I) and Saira(S) are sibilings (i.e. brothers and sisters). All were born on I[st] January. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:

(i) Hari's age + Gita's age > Irfan's age + Saira's age

(ii) The age difference between Gita and Saira is 1 year. However, Gita is not the oldest and Saira is not the youngest

(iii) There are not twins.

In what order were they born (oldest first)

(A) HSIG (B) SGHI

(C) IGSH (D) IHSG

SOL 1.65 Let H, G, S and I be ages of Hari, Gita, Saira and Irfan respectively.
Now from statement (1) we have $H+G>I+S$
Form statement (2) we get that $G-S=1$ or $S-G=1$
As G can't be oldest and S can't be youngest thus either GS or SG possible.
From statement (3) we get that there are no twins
(A) HSIG : There is I between S and G which is not possible
(B) SGHI : SG order is also here and $S>G>H>I$ and $G+H>S+I$ which is possible.
(C) IGSH : This gives $I>G$ and $S>H$ and adding these both inequalities we have $I+S>H+G$ which is not possible.
(D) IHSG : This gives $I>H$ and $S>G$ and adding these both inequalities we have $I+S>H+G$ which is not possible.
Hence (B) is correct option.

Answer Sheet

1.	(C)	13.	(B)	25.	(C)	37.	(D)	49.	(B)	61.	(D)
2.	(C)	14.	(A)	26.	(A)	38.	(B)	50.	(D)	62.	(C)
3.	(D)	15.	(C)	27.	(D)	39.	(D)	51.	(C)	63.	(D)
4.	(A)	16.	(D)	28.	(C)	40.	(C)	52.	(C)	64.	(B)
5.	(D)	17.	(B)	29.	(C)	41.	(B)	53.	(A)	65.	(B)
6.	(A)	18.	(B)	30.	(B)	42.	(C)	54.	(B)		
7.	(B)	19.	(A)	31.	(D)	43.	(C)	55.	(D)		
8.	(B)	20.	(C)	32.	(A)	44.	(D)	56.	(B)		
9.	(A)	21.	(B)	33.	(A)	45.	(D)	57.	(B)		
10.	(A)	22.	(C)	34.	(A)	46.	(A)	58.	(D)		
11.	(D)	23.	(D)	35.	(B)	47.	(B)	59.	(C)		
12.	(*)	24.	(C)	36.	(C)	48.	(B)	60.	(D)		