**( 1 )** A particle executing rectilinear motion travels distance x cm in t seconds where $x = 2t^3 - 9t^2 + 5t + 8$. Find its velocity at a time when its acceleration is 18 cm / $s^2$.

[ Ans: 5 cm / s ]

**( 2 )** River flows from east to west. A sailor, trying to cross the river, tries to row the boat with a velocity four times the velocity of stream of the river in a direction 60° west of north, but due to the drag force of the river travels along the direction making some angle east of north. If he takes 60 minutes to cross the river, what time would he take in moving the distance equal to the width of the river in the direction of the stream ?

[ Ans: 24 minutes ]

**( 3 )** A particle is given four velocities, 3 cm/s towards the east, 8 cm/s towards 30° north of east, 8 cm/s towards 60° west of north and 4 cm/s towards the south directions. Find the resultant velocity of the particle. [ Ans: 5 cm/s ]

**( 4 )** Two boats, A and B, both sailing at 13 km/hr are trying to cross a river flowing at 12 km/hr. Boat A moves along the shortest path and boat B moves along a path of shortest time. Find the ratio of time taken by boat A to that taken by boat B in crossing the river. [ Ans: 2.6 ]

**( 5 )** A boat takes time $t_1$ to travel a distance equal to the width of the river upstream the river, time $t_2$ to travel the same distance downstream the river and time $t_3$ to cross the river. In all the cases, the boat has a constant speed and the river flows with the same velocity. Prove that $t_1 t_2 = t_3^2$.

**( 6 )** The particles A and B are at ( - 5, - 5 ) and ( 5, 0 ) in Cartesian co-ordinate system initially. They start moving simultaneously with velocities ( 1, 3 ) and ( - 2, 3 ) units per second respectively. After what time will they be closest to each other and what is the shortest distance between them. $\left[ \text{Ans: } \frac{10}{3} \text{ seconds, } 5 \text{ units} \right]$

**( 7 )** A rod moves in a vertical plane. One of its ends rests along a wall and the other end is on the ground. Prove that the magnitude of velocity of any end at any instant is directly proportional to the distance of the other end from the point of intersection of ground and wall at that instant.

( 8 ) Prove that the magnitude of acceleration of a particle executing rectilinear motion is inversely proportional to its distance from a fixed point if $t = ax^2 + bx + c$.

( 9 ) Two cars moving with uniform accelerations a and b cover the same distance in the same time and have final velocities u and v respectively. Prove that the common distance covered by both is $\dfrac{(u - v)(va - ub)}{(a - b)^2}$.

( 10 ) A particle executes rectilinear motion with constant acceleration. Its distances from a fixed point O are $d_1$, $d_2$, $d_3$ respectively at time $t_1$, $t_2$, and $t_3$. Also $t_1$, $t_2$, and $t_3$ are in A. P. with common difference d. $d_1$, $d_2$, $d_3$ are in G. P. Prove that the acceleration is $\dfrac{(\sqrt{d_1} - \sqrt{d_3})^2}{d^2}$.

( 11 ) For a particle moving on a rectilinear path covering distance x in time t according to the equation $x^n = t$, prove that its acceleration is proportional to $v^{\frac{2n - 1}{n - 1}}$.

( 12 ) A particle is projected in the vertically upward direction with velocity 25 m/s. After time t, another particle is projected upwards with velocity 7 m/s, If they meet in the shortest possible time period, prove that $t \approx 4.3$ seconds.

( 13 ) An object falls from the tower of height of 200 m. At the same time another object is projected vertically upwards from the bottom of the tower. They meet at the height of 77.5 m. Find the initial velocity of the second object. [ Ans: 40 m/s ]

( 14 ) A body falling freely from the top of a tower covers half its height in 2 seconds prior to the last second of its reaching the ground. Find the height of the tower.
[ Ans: 78.4 m ]

( 15 ) A particle projected vertically upwards attains height some height at times $t_1$ and $t_2$ seconds. Prove that $| t_1 - t_2 | = \dfrac{2v}{g}$, where v is the magnitude of velocity at the given height and g is the acceleration due to gravity.

**( 16 )** An object is so projected that it attains the maximum range while grazing the top of two walls of height 10 m. If the nearest wall is at a distance of 15 m from the point of projection, how far is the farthest wall ? [ Ans: 30 m from the point of projection ]

**( 17 )** Find horizontal range R of the projectile in terms of its maximum height h and initial speed u.

$$\left[ \text{Ans : } 2\sqrt{\frac{2h}{g}\left(u^2 - 2hg\right)} \right]$$

**( 18 )** Two bodies are projected from the same point with the same speed in two different directions. Horizontal ranges for both are R and R' and the times of flight are t and t' respectively. Prove that $4\left(\frac{R^2}{t^2} - \frac{R'^2}{t'^2}\right) = g^2\left(t'^2 - t^2\right)$.

**( 19 )** A ball just passes over two walls of height 10 m. The walls are 20 m apart. The initial velocity of the ball is 20 m/s. Prove that the latus rectum of the parabolic path is 20 m and that the ball takes 2 sec. to pass over the walls. [ Take g = 10 $m/s^2$. ]

**( 20 )** An object just passes over a wall with height 8 m at a distance 12 m and strikes the ground at a distance 36 m from the point of projection. Prove that the angle of projection is 45° and the projection velocity u = 14 m/s.

**( 21 )** In vertical motion under gravity, the speed at the highest point of a particle is $\sqrt{\frac{6}{7}}$ times its speed at half the height of the maximum height. Find the measure of angle of projection.

$$\left[ \text{Ans : } \frac{\pi}{6} \right]$$

**( 22 )** A particle executing rectilinear motion with uniform acceleration covers successive equal distances in times $t_1$, $t_2$ and $t_3$ respectively. Prove that $\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$.

**( 23 )** Two particles are projected vertically upwards with the same velocity of 30 m/s with a time gap of 2 seconds. When and where will they meet ? [ Take g = 10 $m/s^2$. ]

[ Ans: 2 seconds after the second particle is projected at a height of 40 m. ]

**( 24 )** Velocity of a particle executing linear motion is v. Its distance from the origin is x and $v^2 = 12x - 3x^4$. If the acceleration is a, prove that $48v^6 = (6 - a)(18 + a)^3$.

**( 25 )** The projectile has initial velocity u, maximum height h and horizontal range R. Prove that $u^2 = \dfrac{(R^2 + 16h^2)g}{8h}$.

**( 26 )** A shell bursts on contact with the ground and the pieces from it fly off in all directions with all speeds upto 20 m/s. Find the minimum safe distance from the point of bursting of the shell. Also find the duration of danger period for a man standing 20 m away. [ Take g = 10 m/s$^2$ ]. [ Ans: 40 m, $2\sqrt{2}$ s ]

**( 27 )** A rifle bullet can reach a maximum distance of 700 m. If a bullet is fired at the same angle from a car travelling at 36 km/h in the direction of the target, then show that the range of the bullet will be increased by nearly 120 m.

**( 28 )** A fighter plane, flying upwards at an angle of 60° with the vertical, drops a bomb at an altitude of 1000 m. The bomb strikes the ground 20 s after its release. Find
( i ) the velocity of the plane at the time of dropping of the bomb,
( ii ) the maximum height attained by the bomb,
( iii ) the horizontal distance covered by the bomb before striking the ground and
( iv ) the velocity of the bomb when it strikes the ground.

[ Ans: ( i ) 100 m/s, ( ii ) 1125 m, ( iii ) 1732 m, ( iv ) 150 m/s. ]

**( 29 )** A rifle is aimed at a cocoanut hanging on a 30 m tall palm tree 40 m away from the shooter. The cocoanut starts dropping at the same time when the bullet is fired. What should be the minimum velocity with which the bullet should be fired to ensure hitting the cocoanut. What happens if the velocity is less or more ? [ Take g = 10 m/s$^2$ ].

[ Ans: 20.5 m/s; it will fall short of the tree if the velocity is less and will hit the cocoanut at some height above the ground with higher velocity. ]

**( 30 )** Two projectiles having initial velocities u and v and angles of projection with the horizontal $\alpha$ and $\beta$ graze a perpendicular wall in their path in the same time. Prove that the ratios of their ranges and maximum heights are independent of their initial velocities.