MATHEMATICS

Duration: Three Hours ***Maximum Marks: 150***

- **Read the following instructions carefully.]**

1. This question paper contains all objective questions. Q.1 to 20 carry one mark each and Q. 21 to Q. 85 carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (ORS) by darkening the appropriate bubble (marked A, B,C,D) using HB pencil against the question number on the left hand side of the ORS. Each question has only one correct answer. In case you wish to change an answer erase the old answer completely.
4. Wrong answers will carry Negative marks. In Q. 1 to Q. 20, 0.25 mark will be deduced for each wrong answer. In Q. 21 to Q. 76, Q. 78, Q.80, Q82 and in Q.84, 0.5 mark will be deduced for each wrong answer. However, there is no negative marking in Q. 77, Q.79, Q.81, Q.83 and in Q. 85. More than one answer bubbled against a question will be taken as an incorrect response.
5. Write your registration number, your name and name of the examination centre at the specified locations on the right half of the ORS.
6. Using HB pencil, darken the appropriate bubble under each digit of your registration number and the letters corresponding to your paper code.
7. Calculator is allowed in the examination hall.
8. Charts, graph sheets or tables are Not allowed in the examination hall.
9. Rough work can be done on the question paper itself. Additionally bank pages are given at the end of the question paper for rough work.
10. This question paper contains 24 printed pages including pages for rough work. Please check all pages and report, if there is any discrepancy.

ONE MARKS QUESTIONS (1-20)

1. The dimension of the subspace $\{(x_1, x_2, x_3, x_4, x_5) : 3x_1 - x_2 + x_3 = 0\}$ of R^5 is

(a.) 1

(b.) 2

(c.) 3

(d.) 4

2. Let the linear transformations S and $T : R^3 \to R^3$ be defined by

$s(x, y, z) = (2x, 4x - y, 2x + 3y - z)$

$T(x, y, z) = (x\cos\theta - y\sin\theta, \sin\theta + y\cos\theta, z)$ where $0 < \theta < \pi / 2$. Then

(a.) S is one to one but not T

(b.) T is one to one but not S

(c.) Both S and T are one to one

(d.) Neither S nor T is one to one

3. Let E a non-measurable subset of $[0,1]$. If $f : [0,1] \to R$ is defined by

$$f(x)=\begin{cases}\dfrac{-1}{2} & x\in E\\ 0 & \text{Otherwise}\end{cases}$$

Then

(a.) f is measurable but not $|f|$

(b.) $|f|$ is measurable but not f

(c.) Both f and $|f|$ are measurable

(d.) Neither f nor $|f|$ is measurable

4. Let $L^2([0,1])$ denote the space of all square integrable functions on $[0,1]$.

Define $f_1, f_2 : [0,1] \to R$ by

$$f_1(t)=\begin{cases}t^{-1/3}, & 0<t\le 1\\ 0, & t=0\end{cases} \quad f_2(t)=\begin{cases}t^{-2/3}, & 0<t\le 1\\ 0, & t=0\end{cases}$$ Then,

(a.) f_1 belongs to $L^2([0,1])$ but Not f_2

(b.) f_2 belongs to $L^2([0,1])$ but Not f_1

(c.) Both f_1 and f_2 belong to $L^2([0,1])$

(d.) Neither f_1 nor f_2 belongs to $L^2([0,1])$

5. For the ordinary differential equation

$(x-1)\dfrac{d^2y}{dx^2}+(\cot \pi x)\dfrac{dy}{dx}+(\operatorname{cosec}^2 \pi x)\,y=0$ which of the following statements is true?

(a.) 0 is regular and 1 is irregular

(b.) 0 is irregular and 1 is regular

(c.) Both 0 and 1 are regular

(d.) Both 0 and 1 are irregular

6. For the n-th Legendre polynomial $c_n \dfrac{d^n y}{dx^n}(x^2-1)^n$, the value of C_n is

(a.) $\dfrac{1}{(n!2^n)}$

(b.) $\dfrac{n!}{(2^n)}$

(c.) $(n!)\,2^n$

(d.) $\frac{2^n}{(n!)}$

7. Let G be a cyclic group of order 8, then its group of automorphisms has order

(a.) 2

(b.) 4

(c.) 6

(d.) 8

8. Let $M_3(R)$ be the ring of all 3×3 real matrices. If I, $J \subseteq M_3$ (R) are defined as

$$I = \left\{ \begin{pmatrix} a & b & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} | \, a,b,c \in R \right\}, \quad J = \left\{ \begin{pmatrix} a & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \end{pmatrix} | \, a,b,c \in R \right\}$$

Then

(a.) I is a right ideal and J a left ideal

(b.) I and J are both left ideals

(c.) I and J are both right ideals

(d.) I is a left ideal and J a right ideal

9. Consider the Hilbert space

$l^2 = \left\{ (x_1, x_2, ...) \,|\, x_i \in R, \, i = 1, 2, ... \quad and \quad \sum_{i=1}^{\infty} x_i^2, < \infty \right\}$ under the inner product $\langle (x_1, x_2, ...), (y_1, y_2, ...) \rangle = \sum_{i=1}^{\infty} x_i y_i$.

Let $S = \left\{ (x_1, x_2, ...) \in l^2 \,|\, \sum_{n=1}^{\infty} \frac{x_n}{n} = 0 \right\}$. Then the number of interior points of S is

(a.) 0

(b.) Non zero by finite

(c.) Count ably infinite

(d.) Un count ably infinite

10. Let $C([0,1])$ be the space of all real valued continuous functions on $[0,1]$ with the norm $\|f\|_\infty = \{|f(x)| : x \in [0,1]\}$. Sup consider the subspace $P_n([0,1])$ of all polynomials of degree less than or equal to n and the subspace $P([0,1])$ of all polynomials on $[0,1]$. Then,

(a.) $P_n([0,1])$ is closed in $C([0,1])$ but not $P([0,1])$

(b.) $P([0,1])$ is closed in $C([0,1])$ but not $P_n([0,1])$

(c.) Both $P([0,1])$ and $P_n([0,1])$ are closed in $C([0,1])$

(d.) Neither $P([0,1])$ nor $P_n([0,1])$ is closed in $C([0,1])$

11. In the region $x>0, y>0$, the partial differential equation

$$\left(x^2-y^2\right)\frac{\partial^2 u}{\partial x^2}+2\left(x^2+y^2\right)\frac{\partial^2 u}{\partial x\partial y}+\left(x^2-y^2\right)\frac{\partial^2 u}{\partial y^2}=0$$

(a.) Changes type

(b.) Is elliptic

(c.) Is parabolic

(d.) Is hyperbolic

12. Consider the partial differential equation $\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=0$ satisfying the initial condition $u(x,0)=\alpha+\beta x$. If $u(x,t)=1$ along the characteristic $x=t+1$, then

(a.) $\alpha=1, \beta=1$

(b.) $\alpha=2, \beta=0$

(c.) $\alpha=0, \beta=0$

(d.) $\alpha=0, \beta=1$

13. Consider the usual topology on R. Let

$S=\{U\subseteq R: U$ is either bounded open or empty or $R\}$ and $T=\{U\subseteq R: U$ is either unbounded open or empty or $R\}$.

Then, on R

(a.) S is a topology but y not T

(b.) T is a topology but not S

(c.) Both S and T are topologies

(d.) Neither S nor T is a topology

14. Let X, Y and Z be events which are mutually independent, with probabilities a, b respectively. Let the random variable N denote the number of X, Y or Z which occur. Then, the probability that N = 2 is

(a.) $ab+bc+ca-abc$

(b.) $ab+bc+ca-3abc$

(c.) $2(a+b+c)-abc$

(d.) $ab+bc+ca$

15. Assume that 45 percent of the population favours a certain candidate in an election. If a random sample of size 200 is chosen, then the standard deviation of the number of members of the sample that favours the candidate is

(a.) 6.12

(b.) 5.26

(c.) 8.18

(d.) 7.04

16. Let X and Y be independent Poisson random variables with parameters 1 and 2 respectively.

Then, P is $\left(X = 1 | \frac{X+Y}{2} = 2 \right)$

(a.) 0.426

(b.) 0.293

(c.) 0.395

(d.) 0.512

17. For a linear programming primal maximization problem P with dual Q, which of the following statements is correct?

(a.) The optimal values of P and Q exist and are the same

(b.) Both optimal values exist and the optimal value of P is less than the optimal value of Q

(c.) P will have an optimal solution, if and only if Q also has an optimal solution

(d.) Both P and Q cannot be infeasible

18. Let a convex set in 9-dimenstional space be given by the solution set of the following system of linear inequalities

$$\sum_{j=1}^{3} x_{ij} = 1, \quad i = 1,2,3$$

$$\sum_{j=1}^{3} x_{ij} = 1, \quad j = 1,2,3$$

$$x_{ij} \geq 0, \quad i, j = 1,2,3$$

Then, the number of extreme points of this set is

(a.) 3

(b.) 4

(c.) 9

(d.) 6

19. Let I be the functional defined by

$$I\left(y(x)\right) = \int_0^{\pi/2} \left\{ \left(\frac{dy}{dx}\right)^2 - y^2 \right\} dx; \; y(0) = 0$$

$$y(\pi/2) = 1$$

Where the unknown function $y(x)$ possesses two derivatives every where in $(0, \pi/2)$. Then

(a.) The functional has an extremum which can not be achieved in the class of continuous functions (b.) The corresponding Euler's equation does not have a unique solution satisfying the given boundary conditions

(c.) I is not linear

(d.) I is linear

20. Solution of the initial value problem

$$\frac{d^2y}{dx^2}+a_1(x)\frac{dy}{dx}+a_2(x)y=F(x),\ 0\le x\le 1\ \ y(0)=c_0,\left(\frac{dy}{dx}=x\right)_{x=0}=c_{1'}$$

Where $a_1(x),a_2(x)$ and $F(x)$ are continuous functions on $\{0,1\}$, may be reduced, in general to a solution of some linear

(a.) Fredholm integral equation of first kind

(b.) Volterra's integral equation of first kind

(c.) Fredholm integral equation of second king

(d.) Volterra's integral equation of second kind

TWO MARKS QUESTIONS (21–75)

21. Let V be the vector space of all real polynomials. Consider the subspace W spanned by

$t^2+t+2,\ \ t^2+2t+5,\ \ 5t^2+3t+4$ and $2t^2+2t+4$.

Then the dimension of W is

(a.) 4

(b.) 3

(c.) 2

(d.) 1

22. Consider the inner product space $P([0,1])$ with the inner product $\langle f,g\rangle=\int_0^1 f(x)g(x)\,dx$ and $V=span\{t^2\}$. Let $h(t)\in V$ be such that

$\|(2t-1)-h(t)\|\le\|(2t-1)-x(t)\|$ for $x(t)\in V$. Then, $h(t)$ is

(a.) $\frac{5}{6}t^2$

(b.) $\frac{5}{3}t^2$

(c.) $\frac{5}{12}t^2$

(d.) $\frac{5}{24}t^2$

23. Let $M=\begin{pmatrix}1 & a & b\\ 0 & 2 & c\\ 0 & 0 & 1\end{pmatrix}$, $a,b,c\in R$.

Then, M is diagonalizable, if and only if

(a.) $a=bc$

(b.) $b=ac$

(c.) $c=ab$

(d.) $a=b=c$

24. Let M be the real 5×5 matrix having all of its entries equal to 1. Then,

(a.) M is not diagonalizable

(b.) M is idempotent

(c.) M is nilpotent

(d.) The minimal polynomial and the characteristic polynomial of M are not equal

25. Let $\{v_1,v_2,.....,v_{16}\}$ be an ordered basis for $V=C^{16}$. If T is a linear transformation on V defined by $T(v_1)=v_i+1$ for $1\le i\le 15$ and $T(v_{16})=-(v_1+v_2+.......+v_{16})$.

Then,

(a.) R is singular with rational eigen values

(b.) T is singular but has no rational eigen values

(c.) T is regular (invertible) with rational eigen values

(d.) T is regular but has no rational eigen values

26. The value of $\int_0^{2\pi}\exp\left(e^{i\theta}-i\theta\right)d\theta$ equals

(a.) $2\pi i$

(b.) 2π

(c.) π

(d.) $i\pi$

27. The sum of the residues at all the poles of $f(z)=\frac{\cot\pi z}{(z+a)^2}$, where a is a constant, $(a\ne 0,\pm1,\pm2,......)$ is

(a.) $\frac{1}{\pi}\sum_{n=\infty}^{\infty}\frac{1}{(n+a)^2}-\pi\cos ec^2\pi a$

(b.) $-\frac{1}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{(n+a)^2}-\pi\operatorname{cosec}^2\pi a$

(c.) $\frac{1}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{(n+a)^2}-\pi\operatorname{cosec}^2\pi a$

(d.) $\frac{1}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{(n+a)^2}-\pi\operatorname{cosec}^2\pi a$

28. Let $f(z)$ be an entire function such that for some constant, $\alpha, |f(z)|\le\alpha|z|^3$ for $|z|\ge 1$ and $f(z)=f(iz)$ for all $z\in C$. Then

(a.) $f(z)=\alpha z^3$ for all $z\in C$

(b.) $f(z)$ is a constant

(c.) $f(z)$ is a quadratic polynomial

(d.) No such $f(z)$ exists

29. Which of the following is not the real part of an analytic function?

(a.) x^2-y^2

(b.) $\frac{1}{1+x^2+y^2}$

(c.) $\cos x\cosh y$

(d.) $x+\frac{x}{x^2+y^2}$

30. The radius of convergence of $\sum_{n=0}^{\infty}\frac{\left(1+\frac{1}{n}\right)^{n^2}}{n^3}z^n$ is

(a.) e

(b.) 1/e

(c.) 1

(d.) ∞

31. Let, $S,T\subseteq R^2$ be given by

$S=\left\{\left(s,\sin\frac{1}{2}\right):0<x\le 1\right\}\cup\{(0,0)\}$ and $T=\left\{\left(x,\sin\frac{1}{2}\right):0<x\le 1\right\}\cup\{(0,0)\}$. Then, under the usual metric on R^2,

(a.) S is compact but not T

(b.) T is compact but not S

(c.) Both S and T are compact

(d.) Neither S nor T is compact

32. Let $S,T \subseteq R$ be given by $S=\left\{x\in R: 2x^2\cos\frac{1}{x}=1\right\}$ and $T=\left\{x\in R: 2x^2\cos\frac{1}{x}\le 1\right\}\cup\{0\}$. Then, under the usual metric on R,

(a.) S is complete but not T

(b.) T is complete but not S

(c.) Both S and T are complete

(d.) Neither S nor T is complete

33. Let $f: R\to R$ be defined by

$f(x)=\begin{cases} n, & \text{If} \quad x=n, \quad x\in N \\ 0, & \text{Otherwise} \end{cases}$ and $T=N\cup\left\{n+\frac{1}{n}: n\in N\right\}$. Then, under the usual metric on R, f is uniformly continuous on

(a.) N but Not T

(b.) T but not N

(c.) Both N and T

(d.) Neither N nor T

34. For each $n\in N$ and $n>1$ define $f_n:[0,1]\to R$ by

$$f_n(x)=\begin{cases} |nx-1| & \text{for} \quad 0\le x<\frac{2}{n} \\ 1 & \text{for} \quad \frac{2}{n}\le x\le 1 \end{cases}$$

Let $g_1, g_2:[0,1]\to R$ be defined by

$g_1(x)=\begin{cases} 1 & \text{for} \quad 0<x\le 1 \\ 0 & \text{for} \quad x=0 \end{cases}$ and $g_2(x)=1$ for $0\le x\le 1$.

Then, on $[0,1]$

(a.) $f_n\to g_1$ point wise but not uniformly

(b.) $f_n\to g_2$ point wise but not uniformly

(c.) $f_n\to g_1$ uniformly

(d.) $f_n\to g_2$ uniformly

35. Let $f_n, g_n:[0,1]\to R$ be defined by

$f_n(x)=x^2\left(1-x^2\right)^{n-1}$ and $g_n(x)=\dfrac{1}{n^2\left(1+x^2\right)}$ for $n\in N$.

Then, on $[0,1]$

(a.) $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly but not $\sum_{n=1}^{\infty} g_n(x)$

(b.) $\sum_{n=1}^{\infty} g_n(x)$ converges uniformly but not $\sum_{n=1}^{\infty} f_n(x)$

(c.) Both $\sum_{n=1}^{\infty} f_n(x)$ and $\sum_{n=1}^{\infty} g_n(x)$ converge uniformly

(d.) Neither $\sum_{n=1}^{\infty} f_n(x)$ nor $\sum_{n=1}^{\infty} g_n(x)$ converges uniformly

36. The function $f:[0,\infty] \to R$ defined by

$$f(x) = \int_0^x \left(2\sin^4 t \cos^2 t\right) dt$$ is

(a.) Not continuous

(b.) Continuous but not uniformly

(c.) Uniformly continuous but not Lipschitz continuous

(d.) Lipschitz continuous

37. Let S be a non-measurable subset of R and T be measurable subset of R such that $S \subset T$. Denote the outer measure of a set U by m*(U). Then,

(a.) $m^*(T/S) = 0$ and $m^*(S) = 0$

(b.) $m^*(T/S) > 0$ and $m^*(S) > 0$

(c.) $m^*(T/S) > 0$ and $m^*(S) = 0$

(d.) $m^*(T/S) = 0$ and $m^*(S) > 0$

38. Let $f: R^2 \to R$ be defined by

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} & for \quad (x,y) \neq (0,0) \\ 0 & for \quad (x,y) = (0,0) \end{cases}$$

Then, the directional derivative of f at $(0,0)$ in the direction of the vector $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is

(a.) $\frac{1}{\sqrt{2}}$

(b.) $\frac{1}{2}$

(c.) $\frac{1}{2\sqrt{2}}$

(d.) $\frac{1}{4\sqrt{2}}$

39. Consider the hemisphere $x^2 + y^2 + (z-2)^2 = 9, \quad 2 \le z \le 5$ and the vector field $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + (z-2)\vec{k}$. The surface integral $\iint(\vec{F} \bullet \vec{n})\, d\sigma$, evaluated over the hemisphere with $\vec{n}$ denoting the unit outward normal is

(a.) 9π

(b.) 27π

(c.) 54π

(d.) 162π

40. Let $y_1(x)$ and $y_2(x)$ be two solutions of

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (\sec x)\, y = 0$$

With Wronskin $W(x)$. If $y_1(0) = 1$, $\left(\frac{dy_1}{dx}\right)_{x=0} = 0$ and $W\left(\frac{1}{2}\right) = \frac{1}{3}$, then $\left(\frac{dy_2}{dx}\right)_{x=0}$ equals

(a.) 1/4

(b.) 1

(c.) 3/4

(d.) 4/3

41. If $y(x)$ is the solution of the differential equation $\frac{dy}{dx} = 2(1+y)\sqrt{y}$ satisfying $y(0) = 0;\ y(\pi/2) = 1$, then the largest interval (to the right of origin) on which the solution exists is

(a.) $[0, 3\pi/4)$

(b.) $[0, \pi)$

(c.) $[0, 2\pi)$

(d.) $[0, 2\pi/3)$

42. A particular solution of $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + \frac{y}{4} = \frac{1}{\sqrt{x}}$ is

(a.) $\frac{1}{2\sqrt{x}}$

(b.) $\frac{\log x}{2\sqrt{x}}$

(c.) $\frac{(\log x)^2}{2\sqrt{x}}$

(d.) $\dfrac{(\log x)\sqrt{x}}{2}$

43. The initial value problem $x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}+xy=0;\ y(0)=1,\left(\dfrac{dy}{dx}\right)_{x=0}=0$ has

(a.) A unique solution

(b.) No solution

(c.) Infinitely many solutions

(d.) Two linearly independent solutions

44. An integrating factor for $(\cos y\sin 2x)\,dx+(\cos^2 y-\cos^2 x)\,dy=0$ is

(a.) $\sec^2 y+\sec y\tan y$

(b.) $\tan^2 y+\sec y\tan y$

(c.) $1/(\sec^2 y+\sec y\tan y)$

(d.) $1/(\tan^2 y+\sec y\tan y)$

45. Let F_4, F_8 and F_{16} be finite fields of 4,8 and 16 elements respectively. Then,

(a.) F_4 is isomorphic to a subfield of F_8

(b.) F_9 is isomorphic to a subfield of F_{16}

(c.) F_4 is isomorphic to a subfield of F_{16}

(d.) None of the above

46. Let G be the group with the generators a and b given by

$G=\langle a,b:a^4=b^2=1, ba=a^{-1}b\rangle$.

If $Z(G)$ denotes the centre of G, then G/Z(G) isomorphic to

(a.) The trivial group

(b.) C_2, the cyclic group of order 2

(c.) $C_2\times C_2$

(d.) C_4

47. Let I denote the ideal generated by $x^4+x^3+x^2+x+1$ in $Z_2[x]$ and $F=Z_2[x]/I$. Then,

(a.) F is an infinite field

(b.) F is a finite field of 4 elements

(c.) F is a finite field of 8 elements

(d.) F us a finite field of 16 elements

48. Let bijections f and $g:R/\{0,1\}\to R/\{0,1\}$ be defined by $f(x)=1/(1-x)$ and $g(x)=x/(x-1)$, and let G be the group generated by f and g under composition of mappings. It is given that G has order 6. Then,

(a.) G and its automorphisms group are both Abelian

(b.) G and its automorphisms group are both non-Abelian

(c.) G is abelian but its automorphisms group is non-abelian

(d.) G is non-abelian but its automorphisms group is Abelian

49. Let $R=\{\alpha_0+\alpha_1 i+\alpha_2 j+\alpha_3 k:\alpha_0,\alpha_1,\alpha_2,\alpha_3\in Z_3\}$ be the ring of quaternions over Z_3 where $i^2=j^2=k^2=ijk=-1; ij=-ji=k;\ ki=-ik=j$. Then

(a.) R is field

(b.) R is a division ring

(c.) R has zero divisors

(d.) None of the above

50. Consider the sequence of continuous linear operators $T_n:l^2\to l^2$ defined by

$T_n(x)=(0,0,......,0,x_{n+1},x_{n+2},x_{n+3},.....)$ for every $x=(x_1,x_2,.....)\in l^2$ and $n\in N$. Then, for every $x\neq 0$ in l^2

(a.) Both $\|T_n\|$ and $\|T_n(x)\|$ converge to 0

(b.) Neither $\|T_n\|$ nor $\|T_n(x)\|$ converges to 0

(c.) $\|T_n\|$ converges to 0 but not $\|T_n(x)\|$

(d.) $\|T_n(x)\|$ converges to 0 but not $\|T_n\|$

51. Let the continuous linear operator $T:l^2\to l^2$ defined by

$T(x_1,x_2,.....)=(0,x_1,0,x_3,0,x_5,0...)$. Then

(a.) T is compact but not T^2

(b.) T2 is compact but not T

(c.) Both T and T^2 are compact

(d.) Neither T nor T^2 is compact

52. Let $f(x)$ be differentiable function such that $\frac{d^3 f}{dx^3}=1$ for all $x\in[0,3]$. If $p(x)$ is the quadratic polynomial which interpolates $f(x)$ at $x=0$, $x=2$ and $x=3$, then $f(1)-p(1)$ equals

(a.) 0

(b.) 1/3

(c.) 1/6

(d.) 2/3

53. Let $h(x)$ be twice continuously differentiable function on [1,2] with fixed point α. Then, the sequence of iterates $x_{n+1}=h(x_n)$ converges to α quadratic ally, provided

(a.) $\frac{dh}{dx}(\alpha)\neq 0$

(b.) $\frac{dh}{dx}(\alpha)\neq 0,\ \frac{d^2h}{dx^2}(\alpha)=0$

(c.) $\frac{dh}{dx}(\alpha)\neq 0,\frac{d^2h}{dx^2}(\alpha)\neq 0$

(d.) $\frac{dh}{dx}(\alpha)=0,\frac{d^2h}{dx^2}(\alpha)\neq 0$

54. Consider the initial value problem (IVP):

$$\frac{dy}{dx}=f\left(x,y(x)\right),y(x_0)=y_0\,.$$

Let $y_1=y_0w_1k_1+3k_1$ approximate the solution of the above IVP at $x_1=x_0+h$ with $k_1=hf\left(x_0,y_0\right),k_2=hf\left(x_0+(h/6),y_0+(k_1/6)\right)$ and h being the step-size. If the formula for y_1 yields a second order method, then the value of w_1 is

(a.) –1

(b.) –2

(c.) 3

(d.) 1/6

55. Let $u(x,t)$ be the solution of the initial value problem

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}=0;\ u(x,0)=\sin x;\ \frac{\partial u}{\partial u}(x,0)=1.$$

Then $u(\pi,\pi/2)$ equals

(a.) $\pi/2$

(b.) 1/2

(c.) –1

(d.) 1

56. Let $u(x,t)$ be the bounded of $\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}=0$ with $u(x,0)=\frac{e^{2x}-1}{e^{2x}+1}$. Then $\lim\limits_{t\to+\infty}u(1,t)$ equals

(a.) –1/2

(b.) 1/2

(c.) –1

(d.) 1

57. Let $u(x,y)$ be a solution of Laplace's equation on $x^2+y^2\leq 1$. If

$$u(\cos\theta,\sin\theta)=\begin{cases}\sin\theta & for\quad 0\leq\theta\leq\pi\\ 0 & for\quad \pi\leq\theta\leq 2\pi\end{cases}$$

Then $u(0,0)$ equals

(a.) $1/\pi$

(b.) $2/\pi$

(c.) $1/(2\pi)$

(d.) $\pi/2$

58. Let PQRS be a rectangle in the first quadrant whose adjacent sides PQ and QR have slopes 1 and –1 respectively. If $u(x,t)$ is a solution of $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial z^2} = 0$ and $u(P)=1,\ u(Q)=-1/2,\ u(R)=1/2$, then u(S) equals

(a.) 2

(b.) 1

(c.) 1/2

(d.) –1/2

59. In the motion of a two-particle system, if the two particles are connected by a rigid weightless rod of constant length l, then the number of degree of freedom of the system is

(a.) 2

(b.) 3

(c.) 5

(d.) 6

60. A particle of unit mass moves in the xy-plane under the influence of a central force depending only on its distance from the origin. If (r,θ) be the polar coordinates of the particle at a given instant and V(r) the potential due to the given force, then the Lagrangian for such a system is

(a.) $\frac{1}{2}\dot{r}^2 - V(r)$

(b.) $\frac{1}{2}(\dot{r}^2 + \dot{\theta}^2) + V(r)$

(c.) $\frac{1}{2}(\dot{r}^2 + r\dot{\theta}) + V(r)$

(d.) $\frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$

61. Let $q, \dot{q}$ and p denote respectively the generalized coordinates, and the corresponding velocity and momenta of a one-dimensional system with the Hamiltonian $H = \frac{1}{2}\left(p^2 - \frac{1}{q^2}\right)$. Then the Lagrangian of the system is

(a.) $\frac{1}{2}\dot{q}^2 - \frac{1}{q}$

(b.) $\frac{1}{2}\dot{q}^2+\frac{1}{q}$

(c.) $\frac{1}{2}\left(\dot{q}^2+\frac{1}{q^2}\right)$

(d.) $\frac{1}{2}\left(\dot{q}^2-\frac{1}{q^2}\right)$

62. Let τ_1 be the usual topology on R. Define another topology τ_2 on R by $\tau_2=\{U\subseteq R\,|\,U^c$ is either finite or empty or whole of $R\}$ where U^c denotes the component of U in R. If $I:(R,\tau_1)\to(R,\tau_2)$ is the identity map, then

(a.) I is continuous but not I^{-1}

(b.) I^{-1} is continuous but not I

(c.) Both I and I^{-1} are continuous

(d.) Neither I nor I^{-1} continuous

63. Let τ_1 be the usual topology on R. Define another topology τ_2 on R by

$\tau_2=\{U\subseteq R\,|\,U^c$ is either countable or empty or whole of $R\}$.

Then, Z is

(a.) Closed in (R,τ_1) but not in (R,τ_2)

(b.) Closed in (R,τ_2) but not in (R,τ_1)

(c.) Closed in both (R,τ_1) and (R,τ_2)

(d.) Closed neither in (R,τ_1) nor in (R,τ_2)

64. Consider R^2 with the usual topology. The complement of $N\times N$ is

(a.) Open but not connected

(b.) Connected but not open

(c.) Both open and connected

(d.) Neither open nor connected

65. Let T denote the number of times we have to roll a fair dice before each face appearing in the first six rolls. Ten E(T|N =3) is

(a.) 9

(b.) 15

(c.) 16

(d.) 17

66. Let there be three types of light bulbs with lifetimes X, Y and Z having exponential distributions with mean $\theta, 2\theta$ and 3θ respectively. Then, the maximum link hood estimator of θ based on the observation X, Y and Z is

(a.) $(X+2Y+3Z)/3$

(b.) $3(X+2Y+3Z)$

(c.) $\frac{1}{3}\left(X+\frac{Y}{2}+\frac{Z}{3}\right)$

(d.) $\frac{1}{6}\left(X+\frac{Y}{2}+\frac{Z}{3}\right)$

67. Let Z be the vertical coordinate, between –1 and 1, of a point chosen uniformly at random on the surface of a unit sphere in R^3. Then, $P\left(-\frac{1}{2}\le Z\le\frac{1}{2}\right)$ is

(a.) 5/6

(b.) $\left(\sqrt{3}\right)/2$

(c.) 3/4

(d.) 1/2

68. Let the marks obtained in the half-yearly and final examinations in a large class have an approximately bivariate normal distribution with the following parameters

	Mean	Deviation
Marks (half yearly)	60	18
Marks (final exam)	55	20
Correlation :	0.75	

Then, estimate of the average final examination score of students who were above average on the half-yearly examination is

(a.) 60

(b.) 67

(c.) 70

(d.) 72

69. Let $V_1, V_2, \ldots\ldots\ldots, V_5$ be 5 independent uniform (0,1) variables and let $V_{(1)} < V_{(2)} < \ldots\ldots < V_{(5)}$ be their order statistics. Then, for $0 < x < y < 1$, the joint density $f(x, y)$ of $\left(V_{(2)}, V_{(4)}\right)$ is given by

(a.) $(5!)\,xy(1-x)(1-y)$

(b.) $x(y-x)(1-y)/(5!)$

(c.) $(5!)\,x(y-x)(1-y)$

(d.) $xy(1-x)(1-y)/(5!)$

70. Consider the linear programming problem

$$\max c_1x_1 + c_2x_2 + c_3x_3$$
$$\begin{aligned} s.t \quad & x_1 + x_2 + x_3 \le 4 \\ & x_1 \le 2 \\ & x_3 \le 3 \\ & 3x_1 + x_3 \le 7 \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

If $(1,0,3)$ optimal solution, then

(a.) $c_1 \le c_2 \le c_3$

(b.) $c_3 \le c_1 \le c_2$

(c.) $c_2 \le c_3 \le c_1$

(d.) $c_2 \le c_1 \le c_3$

71. Let the convex set S be given by the solution set of the following system of linear inequalities in the sixteen variables $\{x_{ij} : i, j = 1, \ldots, 4\}$.

$$\sum_{J=1}^{4} x_{ij} = 3, \; i = 1, \ldots, 4$$

$$\sum_{J=1}^{4} x_{ij} = 3, \qquad j = 1, \ldots 4$$

$$x_{ij} \ge 0, \qquad i, j = 1, \ldots, 4$$

Then, the dimension of S is equal to

(a.) 4

(b.) 9

(c.) 8

(d.) 12

72. Let $I(y(x)) = \int_0^1 F\left(x, y, \frac{dy}{dx}\right) dx$, satisfying $y(0) = 0,\ y(1) = 1$

Where F has continuous second order derivatives with respect to its arguments, and the unknown function $y(x)$ possess two derivatives every where in (0,1). If the function F depends only on x and $\frac{dy}{dx}$, then the Euler's equation is an ordinary differential equation in y which, in general, is

(a.) First order linear

(b.) First order nonlinear

(c.) Second order linear

(d.) Second order nonlinear

73. The functional

$$I(y(x)) = \int_0^1 \left(y + \frac{d^2y}{dx^2} \right) dx$$

Defined on the set of functions $C^2([0,1])$ satisfying

$$y(0)=1, \quad y(1)=1, \left(\frac{dy}{dx}\right)_{x=0} = 0 \text{ and } \left(\frac{dy}{dx}\right)_{x=1} = -1$$

(a.) Only one extremal

(b.) Exactly two extremals

(c.) Infinite number of extremals

(d.) 1No extremals

74. Which of the following functions is a solution of the Volterra type integral equation

$$f(x) = x + \int_0^x (\sin(x-t)\, f(t))\, dt$$

(a.) $x + \frac{x^3}{3}$

(b.) $x - \frac{x^3}{3}$

(c.) $x + \frac{x^3}{6}$

(d.) $x - \frac{x^3}{6}$

75. Which of the following functions is a solution of the Fredholm type equation

$$f(x) = x + \int_0^1 (xtf(t))\, dt$$

(a.) $2x/3$

(b.) $3x/2$

(c.) $3x/4$

(d.) $4x/3$

TWO MARKS QUESTIONS (76–85)

Statement for Linked Answer Questions 76 & 77:

Let $T: C^3 \to C^3$ be defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, -x_1 - x_2, -x_1 - x_3)$ and M be its matrix with respect to the standard ordered basis.

76. The eigen values of M are

(a.) $-1, i-i$

(b.) $1, i, -i$

(c.) $1, i, i$

(d.) $-1, -i, -i$

77. The matrix M is similar to a matrix which is

(a.) Unitary

(b.) Hermitian

(c.) Skew Hermitian

(d.) Having trace 0

Statement for Linked Answer Questions 78 & 79:

Let H be an infinite dimensional Hilbert space and f be a continuous linear functional on H such that $\|f\|=1$. Define $W=\{x\in H : f(x)=1\}$. Then interior and the boundary of the closed unit ball U of H are denoted by U° and ∂U respectively.

78. Which of the following is correct?

(a.) $U^\circ \cap W = \phi$ and $W \cap \partial U = \phi$

(b.) $U^\circ \cap W \neq \phi$ and $W \cap \partial U = \phi$

(c.) $U^\circ \cap W = \phi$ and $W \cap \partial U \neq \phi$

(d.) $U^\circ \cap W \neq \phi$ and $W \cap \partial U \neq \phi$

79. The number of points in $W \cap U$ is

(a.) 0

(b.) 1

(c.) Not one but countable

(d.) Uncountable

Statement for Linked Answer Questions 80 and 81:

Consider the partial differential equation

$$x\frac{\partial u}{\partial y} - y\frac{\partial u}{\partial x} = u$$

80. The characteristic curves for the above equation in the (x, y) plane are

(a.) Straight line with slopes 1

(b.) Straight lines with slopes –1

(c.) Circles with centre at the origin

(d.) Circles touching y axis and centered on x–axis

81. If $u(x,y)$ is a solution to the above equation with $u(x,0)=\sin\left(\frac{\pi}{4}x\right)$, then $u\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ equals

(a.) $\frac{1}{\sqrt{2}}e^{\frac{\pi}{4}}$

(b.) $\frac{\pi}{4}e^{\frac{\pi}{\sqrt{2}}}$

(c.) $\frac{1}{\sqrt{2}}e^{\frac{1}{\sqrt{2}}}$

(d.) $\frac{\pi}{4}e^{\frac{\pi}{4}}$

Statement for Linked Answer Questions 82 & 83:

Let $p(x)=c_0+c_1x$ minimize $\langle f(x)-p(x), f(x)-p(x)\rangle=\int_{-1}^{1}\left(f(x)-p(x)\right)^2dx$ over all polynomials of degree less than or equal to 1.

82. The best choice of coefficients c_0, c_1 is

(a.) $\langle f,1\rangle,\langle f,x\rangle$

(b.) $\langle f,1\rangle,\frac{2}{3}\langle f,x\rangle$

(c.) $\frac{1}{2}\langle f,1\rangle,\frac{3}{2}\langle f,x\rangle$

(d.) $\frac{2}{3}\langle f,1\rangle,\frac{2}{3}\langle f,x\rangle$

83. If $f(x)=x^2+x$, then $p(x)$ is given by

(a.) $\left(1+\frac{x}{3}\right)$

(b.) $\frac{1}{3}(1+3x)$

(c.) $\frac{1}{3}(1+x)$

(d.) $\frac{2}{3}(1+x)$

Statement for Linked Answer Questions 84 & 85:

Consider the Linear Programming Problem P:

max $\quad c_1x_1 + c_2x_2 + ... + c_nx_n$

s.t. $\quad \sum_{j=1}^{n} a_{ij}x_j \leq b_i,\ \ i = 1,.....,m$

$x_j \geq 0,\ \ j = 1,.....,n,$

With m constants in n non-negative variables.

84. Let $x^* = \left(x_1^*, x_2^*, ..., x_n^*\right)$ be an optimal extreme point solution to P with $x_1^*, x_2^*, x_3^*, ..., x_n^* > 0$. Then out of the m constraints $\sum_{j=1}^{n} a_{ij}x_j \leq b_i \ \ i = 1,....,m$ the number of constraints not satisfied with equality at x* is

(a.) At most m–4

(b.) At most n–4

(c.) Equal to m–3

(d.) Equal to m–2

85. Treat $c_1^{'}s, a_{ij}$ ’s fixed and consider the problem P for different values of $b_i{}'s$. Let P be unbounded for some set of parameters $b_1, b_2, ..., b_m$. Then

(a.) n > m

(b.) P is either unbounded or infeasible every choice of $b_i^{'}s$

(c.) m > n

(d.) P has an optimal solution for some choice of $b_i^{'}s$