MATHEMATICS

ONE MARKS QUESTIONS (1-20)

1. The dimension of the vector space $V = \left\{ A = \left(a_{ij}\right)_{n\times n}; a_{ij} \in C, a_{ij} = -a_{ji} \right\}$ over field R is

(a.) n^2

(b.) $n^2 - 1$

(c.) $n^2 - n$

(d.) $\frac{n^2}{2}$

2. The minimal polynomial associated with the matrix $\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ is

(a.) $x^3 - x^2 - 2x - 3$

(b.) $x^3 - x^2 + 2x - 3$

(c.) $x^3 - x^2 - 3x - 3$

(d.) $x^3 - x^2 + 3x - 3$

3. For the function $f(z) = \sin\left(\frac{1}{\cos(1/z)}\right)$, the point z = 0 is

(a.) a removable singularity

(b.) a pole

(c.) an essential singularity

(d.) a non-isolated singularity

4. Let $f(z) = \sum_{n=0}^{15} z^n$ *for* $z \in C$. If $C : |z - i| = 2$ then $\oint_C \frac{f(z)\,dz}{(z-i)^{15}} =$

(a.) $2\pi i(1+15i)$

(b.) $2\pi i(1-15i)$

(c.) $4\pi i(1+15i)$

(d.) $2\pi i$

5. For what values of α and β, the quadrature formula $\int_{-1}^{1} f(x)dx \approx \alpha f(-1) + f(\beta)$ is exact for all polynomials of degree ≤ 1?

(a.) $\alpha = 1, \beta = 1$

(b.) $\alpha = -1, \beta = 1$

(c.) $\alpha = 1, \beta = -1$

(d.) $\alpha = -1, \beta = -1$

6. Let $f:[0,4] \to R$ be a three times continuously differentiable function. Then the value of $f[1,2,3,4]$ is

(a.) $\frac{f''(\xi)}{3}$ for some $\xi \in (0,4)$

(b.) $\frac{f''(\xi)}{6}$ for some $\xi \in (0,4)$

(c.) $\frac{f'''(\xi)}{3}$ for some $\xi \in (0,4)$

(d.) $\frac{f'''(\xi)}{6}$ for some $\xi \in (0,4)$

7. Which one of the following is TRUE?

(a.) Every linear programming problem has a feasible solution.

(b.) If a linear programming problem has an optimal solution then it is unique.

(c.) The union of two convex sets is necessarily convex.

(d.) Extreme points of the disk $x^2 + y^2 \leq 1$ are the point on the circle $x^2 + y^2 = 1$.

8. The dual of the linear programming problem:

Minimize $c^T x$ subject to $Ax \geq b$ and $x \geq 0$ is

(a.) Maximize $b^T w$ subject to $A^T w \geq c$ and $w \geq 0$

(b.) Maximize $b^T w$ subject to $A^T w \leq c$ and $w \geq 0$

(c.) Maximize $b^T w$ subject to $A^T w \leq c$ and w is unrestricted

(d.) Maximize $b^T w$ subject to $A^T w \geq c$ and w is unrestricted

9. The resolvent kernel for the integral equation $u(x) = F(x) + \int_{\log 2}^{x} f^{(t-x)} u(t)\, dt$ is

(a.) $\cos(x-t)$

(b.) 1

(c.) e^{t-x}

(d.) $e^{2(t-x)}$

10. Consider the metrics $d_2(f,g)=\left(\int_a^b |f(t)-g(t)|^2\,dt\right)^{1/2}$ and $d_\infty(f,g)=\sup_{t\in[a,b]}|f(t)-g(t)|$ on the space $X=C[a,b]$ of all real valued continuous functions on $[a,b]$. Then which of the following is TRUE?

(a.) Both (X,d_2) and (X,d_∞) are complete.

(b.) (X,d_2) is complete but (X,d_∞) is NOT complete.

(c.) (X,d_∞) is complete but (X,d_2) is NOT complete.

(d.) Both (X,d_2) and (X,d_∞) are NOT complete.

11. A function $f:R\to R$ need NOT be Lebesgue measurable if

(a.) f is monotone

(b.) $\{x\in R: f(x)\geq\alpha\}$ is measurable for each $\alpha\in R$

(c.) $\{x\in R: f(x)=\alpha\}$ is measurable for each $\alpha\in R$

(d.) For each open set G in R, $f^{-1}(G)$ is measurable

12. Let $\{e_n\}_{n=1}^{\infty}$ be an orthonormal sequence in a Hilbert space H and let $x(\neq 0)\in H$. Then

(a.) $\lim_{n\to\infty}\langle x,e_n\rangle$ does not exist

(b.) $\lim_{n\to\infty}\langle x,e_n\rangle=\|x\|$

(c.) $\lim_{n\to\infty}\langle x,e_n\rangle=1$

(d.) $\lim_{n\to\infty}\langle x,e_n\rangle=0$

13. The subspace $Q\times[0,1]$ of R^2 (with the usual topology) is

(a.) dense is R^2

(b.) connected

(c.) separable

(d.) compact

14. $Z_2[x]/\langle x^3+x^2+1\rangle$ is

(a.) a field having 8 elements

(b.) a field having 9 elements

(c.) an infinite field

(d.) NOT a field

15. The number of element of a principal ideal domain can be

(a.) 15

(b.) 25

(c.) 35

(d.) 36

16. Let, F, G and H be pair wise independent events such that $P(F)=P(G)=P(H)=\frac{1}{3}$ and $(F\cap G\cap H)=\frac{1}{4}$ Then the probability that at least one event among F, G and H occurs is

(a.) $\frac{11}{12}$

(b.) $\frac{7}{12}$

(c.) $\frac{5}{12}$

(d.) $\frac{3}{4}$

17. Let X be a random variable such that $E(X^2)=E(X)=1$. Then $E(X^{100})=$

(a.) 0

(b.) 1

(c.) 2^{100}

(d.) $2^{100}+1$

18. For which of the following distributions, the weak law of large numbers does NOT hold?

(a.) Normal

(b.) Gamma

(c.) Beta

(d.) Cauchy

19. If $D\equiv\frac{d}{dx}$ then the value of $\frac{1}{(xD+1)}\left(x^{-1}\right)$ is

(a.) $\log x$

(b.) $\frac{\log x}{x}$

(c.) $\frac{\log x}{x^2}$

(d.) $\frac{\log x}{x^3}$

20. The equation

$\left(\alpha xy^3+y\cos x\right)dx+\left(x^2y^2+\beta\sin x\right)dy=0$

is exact for

(a.) $\alpha = \frac{3}{2}, \beta = 1$

(b.) $\alpha = 1, \beta = \frac{3}{2}$

(c.) $\alpha = \frac{2}{3}, \beta = 1$

(d.) $\alpha = 1, \beta = \frac{2}{3}$

TWO MARKS QUESTIONS (21-60)

21. If $A = \begin{pmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2} \end{pmatrix}$, then the trace of A^{102} is

(a.) 0
(b.) 1
(c.) 2
(d.) 3

22. Which of the following matrices is NOT diagonalizable?

(a.) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

(b.) $\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$

(c.) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(d.) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

23. Let V be the column space of the matrix $A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -1 \end{pmatrix}$. Then the orthogonal projection of $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ on V is

(a.) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

(b.) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(c.) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

(d.) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

24. Let $\sum_{n=-\infty}^{\infty} a_n (z+1)^n$ be the Laurent series expansion of $f(z) = \sin\left(\frac{z}{z+1}\right)$. Then $a_{-2} =$

(a.) 1

(b.) 0

(c.) $\cos(1)$

(d.) $\frac{-1}{2}\sin(1)$

25. Let $u(x, y)$ be the real part of an entire function $f(z) = u(x, y) + iv(x, y)$ for $z = x + iy \in C$. If C is the positively oriented boundary of a rectangular region R in R^2, then $\oint_C \left[\frac{\partial u}{\partial y} dx - \frac{\partial u}{\partial x} dy\right] =$

(a.) 1

(b.) 0

(c.) 2π

(d.) π

26. Let $\phi : [0,1] \to R$ be three times continuously differentiable, Suppose that the iterates defined by $x_{n+1} = \phi(x_n), n \geq 0$ converge to the fixed point ξ of ϕ. If the order of convergence is three then

(a.) $\phi'(\xi) = 0, \phi''(\xi) = 0$

(b.) $\phi'(\xi) \neq 0, \phi''(\xi) = 0$

(c.) $\phi'(\xi) = 0, \phi''(\xi) \neq 0$

(d.) $\phi'(\xi) \neq 0, \phi''(\xi) \neq 0$

27. Let $f : [0,2] \to R$ be a twice continuously differentiable function. If $\int_0^2 f(x)dx \approx 2f(1)$, then the error in the approximation is

(a.) $\frac{f'(\xi)}{12}$ for some $\xi \in (0, 2)$

(b.) $\frac{f'(\xi)}{2}$ for some $\xi \in (0,2)$

(c.) $\frac{f''(\xi)}{3}$ for some $\xi \in (0,2)$

(d.) $\frac{f''(\xi)}{6}$ for some $\xi \in (0,2)$

28. For a fixed $t \in R$, consider the linear programming problem:

Maximize $z = 3x + 4y$

Subject to $x + y \leq 100$

$x + 3y \leq t$

and $x \geq 0, y \geq 0$

The maximum value of z is 400 for t =

(a.) 50

(b.) 100

(c.) 200

(d.) 300

29. The minimum value of

$z = 2x_1 - x_2 + x_3 - 5x_4 + 22x_5$ subject to

$x_1 - 2x_4 + x_5 = 6$

$x_2 + x_4 - 4x_5 = 3$

$x_3 + 3x_4 + 2x_5 = 10$

$x_j \geq 0, j = 1, 2, ..., 5$

is

(a.) 28

(b.) 19

(c.) 10

(d.) 9

30. Using the Hungarian method, the optimal value of the assignment problem whose cost matrix is given by

5	23	14	8
10	25	1	23
35	16	15	12
16	23	11	7

is

(a.) 29

(b.) 52

(c.) 26

(d.) 44

31. Which of the following sequence $\{f_n\}_{n=1}^{\infty}$ of functions does NOT converge uniformly on [0, 1]?

(a.) $f_n(x) = \frac{e^{-x}}{n}$

(b.) $f_n(x) = (1-x)^n$

(c.) $f_n(x) = \frac{x^2 + nx}{n}$

(d.) $f_n(x) = \frac{\sin(nx+n)}{n}$

32. Let $E = \{(x,y) \in R^2 : 0 < x < y\}$. Then $\iint_E ye^{-(x+y)}dxdy =$

(a.) $\frac{1}{4}$

(b.) $\frac{3}{2}$

(c.) $\frac{4}{3}$

(d.) $\frac{3}{4}$

33. Let $f_n(x) = \frac{1}{n}\sum_{k=0}^{n}\sqrt{k(n-k)}\binom{n}{k}x^k(1-x)^{n-k}$ for $x \in [0,1], n = 1, 2, \ldots.$ If $\lim_{n\to\infty} f_n(x) = f(x)$ for $x \in [0,1]$, then the maximum value of $f(x)$ on [0,1] is

(a.) 1

(b.) $\frac{1}{2}$

(c.) $\frac{1}{3}$

(d.) $\frac{1}{4}$

34. Let $f : (c_{00}, \|\cdot\|_1) \to C$ be a non zero continuous linear functional. The number of Hahn-Banach extensions of f to $(l^1, \|\cdot\|_1)$ is

(a.) One

(b.) Two

(c.) Three

(d.) infinite

35. If $I:\left(l^1,\|.\|_2\right)\to\left(l^1,\|.\|_1\right)$ is the identity map, then

(a.) Both I and I^{-1} are continuous

(b.) I is continuous but I^{-1} is NOT continuous

(c.) I^{-1} is continuous but I is NOT continuous

(d.) Neither I and I^{-1} is continuous

36. Consider the topology $\tau=\{G\subseteq R: R\setminus G \text{ is compact in } (R,\tau_u)\}\cup\{\phi,R\}$ on R, where τ_u is the usual topology on R and ϕ is the empty set. Then (R,τ) is

(a.) a connected Hausdorff space

(b.) connected but NOT Hausdorff

(c.) hausdorff but NOT connected

(d.) neither connected nor Hausdorff

37. Let

$\tau_1=\{G\subseteq R: G \text{ is finite or } R\setminus G \text{ is finite}\}$

and

$\tau_2=\{G\subseteq R: G \text{ is contable or } R\setminus G \text{ is contable}\}$

Then

(a.) neither τ_1 nor τ_2 is a topology on R

(b.) τ_1 is a topology on R but τ_2 is NOT a topology on R

(c.) τ_2 is a topology on R but τ_1 is NOT a topology on R

(d.) both τ_1 and τ_2 are topologies on R

38. Which one of the following ideals of the ring $Z[i]$ of Gaussian integers is NOT maximal?

(a.) $\langle 1+i\rangle$

(b.) $\langle 1-i\rangle$

(c.) $\langle 2+i\rangle$

(d.) $\langle 3+i\rangle$

39. If Z(G) denotes the centre of a group G, then the order of the quotient group G/Z(G) cannot be

(a.) 4

(b.) 6

(c.) 15

(d.) 25

40. Let Aut(G) denote the group of automorphism of a group G. Which one of the following is NOT a cyclic group?

(a.) $Aut(Z_4)$

(b.) $Aut(Z_6)$

(c.) $Aut(Z_8)$

(d.) $Aut(Z_{10})$

41. Let X be a non-negative integer valued random variable with $E(X^2)=3$ and $E(X)=1$. Then $\sum_{i=1}^{\infty} iP(X \geq i) =$

(a.) 1

(b.) 2

(c.) 3

(d.) 4

42. Let X be a random variable with probability density function $f \in \{f_0, f_1\}$, where

$$f_0(x)=\begin{cases} 2x, & if\ 0<x<1 \\ 0, & otherwise \end{cases} \text{ and } f_1(x)=\begin{cases} 3x^2, & if\ 0<x<1 \\ 0, & otherwise \end{cases}$$

For testing the null hypothesis $H_0 : f \equiv f_0$ against the alternative hypothesis $H_1 : f \equiv f_1$ at level of significance α = 0.19, the power of the most powerful test is

(a.) 0.729

(b.) 0.271

(c.) 0.615

(d.) 0.385

43. Let X and Y be independent and identically distributed U(0, 1) random variables. Then $P\left(Y < \left(X - \frac{1}{2}\right)^2\right) =$

(a.) $\frac{1}{12}$

(b.) $\frac{1}{4}$

(c.) $\frac{1}{3}$

(d.) $\frac{2}{3}$

44. Let X and Y be Banach spaces and let $T : X \to Y$ be a linear map. Consider the statements:

P: If $x_n \to x$ in X then $Tx_n \to Tx$ in Y.

Q: If $x_n \to x$ in X and $Tx_n \to y$ in Y then $Tx = y$.

Then

(a.) P implies Q and Q implies P

(b.) P implies Q but Q does not imply P

(c.) Q implies P but P does not imply Q

(d.) Neither P implies Q nor Q implies P

45. If $y(x) = x$ is a solution of the differential equation $y'' - \left(\frac{2}{x^2} + \frac{1}{x}\right)(xy' - y) = 0, 0 < x < \infty$, then its general solution is

(a.) $\left(\alpha + \beta e^{-2x}\right)x$

(b.) $\left(\alpha + \beta e^{2x}\right)x$

(c.) $\alpha x + \beta e^{x}$

(d.) $\left(\alpha e^{x} + \beta\right)x$

46. Let $P_n(x)$ be the Legendra polynomial of degree n such that $P_n(1) = 1, n = 1, 2, ...$ If $\int_{-1}^{1}\left(\sum_{j=1}^{n}\sqrt{j(2j+1)}P_j(x)\right)^2 dx = 20$, then n =

(a.) 2

(b.) 3

(c.) 4

(d.) 5

47. The integral surface satisfying the equation $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x^2 + y^2$ and passing through the curve $x = 1 - 1, y = 1 + t, z = 1 + t^2$ is

(a.) $z = xy + \frac{1}{2}\left(x^2 - y^2\right)^2$

(b.) $z = xy + \frac{1}{4}\left(x^2 - y^2\right)^2$

(c.) $z = xy + \frac{1}{8}\left(x^2 - y^2\right)^2$

(d.) $z = xy + \frac{1}{16}\left(x^2 - y^2\right)^2$

48. For the diffusion problem $u_{xx} = u_t\,(0 < x < \pi, t > 0)$, $u(0,t) = 0$, $u(\pi,t) = 0$ and $u(x,0) = 3\sin 2x$, the solution is given by

(a.) $3e^{-t}\sin 2x$

(b.) $3e^{-4t}\sin 2x$

(c.) $3e^{-9t}\sin 2x$

(d.) $3e^{-2t}\sin 2x$

49. A simple pendulum, consisting of a bob of mass m connected with a string of length a, is oscillating in a vertical plane. If the string is making an angle θ with the vertical, then the expression for the Lagrangian is given as

(a.) $ma^2\left(\theta^2 - \frac{2g}{a}\sin^2\left(\frac{\theta}{2}\right)\right)$

(b.) $2mga\sin^2\left(\frac{\theta}{2}\right)$

(c.) $ma^2\left(\frac{\theta^2}{2} - \frac{2g}{a}\sin^2\left(\frac{\theta}{2}\right)\right)$

(d.) $\frac{ma}{2}\left(\theta^2 - \frac{2g}{a}\cos\theta\right)$

50. The extremal of the functional

$\int_0^1\left(y + x^2 + \frac{y'^2}{4}\right)dx, y(0)=0, y(1)=0$ is

(a.) $4\left(x^2 - x\right)$

(b.) $3\left(x^2 - x\right)$

(c.) $2\left(x^2 - x\right)$

(d.) $x^2 - x$

Common Data for Questions (51 & 52)

Let $T: R^3 \to R^3$ be the linear transformation defined by

$T(x_1, x_2, x_3) = (x_1 + 3x_2 + 2x_3 + 3x_1 + 4x_2 + x_3, 2x_1 + x_2 - x_3)$

51. The dimension of the range space of T^2 is

(a.) 0

(b.) 1

(c.) 2

(d.) 3

52. The dimension of the null space of T^3 is

(a.) 0

(b.) 1

(c.) 2

(d.) 3

Common Data for Questions (53 & 54)

Let $y_1(x)=1+x$ and $y_2(x)=e^x$ be two solutions of $y''(x)+P(x)y'(x)+Q(x)y(x)=0$.

53. $P(x)=$

(a.) $1+x$

(b.) $-1-x$

(c.) $\frac{1+x}{x}$

(d.) $\frac{-1-x}{x}$

54. The set of initial conditions for which the above differential equation has NO solution is

(a.) $y(0)=2, y'(0)=1$

(b.) $y(1)=0, y'(1)=1$

(c.) $y(1)=1, y'(1)=0$

(d.) $y(2)=1, y'(2)=2$

Common Data for Questions (55 & 56)

Let X and Y be random variables having the joing probability density function

$$f(x,y)=\begin{cases}\frac{1}{\sqrt{2\pi y}}e^{\frac{-1}{2y}(x-y)^2}, & if\ -\infty<x<\infty, 0<y<1\\ 0, & otherwise\end{cases}$$

55. The variance of the random variable X is

(a.) $\frac{1}{12}$

(b.) $\frac{1}{4}$

(c.) $\frac{7}{12}$

(d.) $\frac{5}{12}$

56. The covariance between the random variables X and Y is

(a.) $\frac{1}{3}$

(b.) $\frac{1}{4}$

(c.) $\frac{1}{6}$

(d.) $\frac{1}{12}$

Statement for Linked Answer Question (57 and 58)

Consider the function $f(z)=\frac{e^{iz}}{z(z^2+1)}$.

57. The residue of f at the isolated singular point in the upper half plane $\{z=x+iy\in C : y>0\}$ is

(a.) $\frac{-1}{2e}$

(b.) $\frac{-1}{e}$

(c.) $\frac{e}{2}$

(d.) 2

58. The Cauchy Principal Value of the integral $\int_{-\infty}^{\infty}\frac{\sin x\,dx}{x(x^2+1)}$ is

(a.) $-2\pi(1+2e^{-1})$

(b.) $\pi(1+e^{-1})$

(c.) $2\pi(1+e)$

(d.) $-\pi(1+e^{-1})$

Statement for Linked Answer Question (59 and 60)

Let $f(x,y)=kxy-x^3y-xy^3$ for $(x,y)\in R^2$, where k is a real constant. The directional derivative of f at the point (1, 2) in the direction of the unit vector $u=\left(\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ is $\frac{15}{\sqrt{2}}$.

59. The value of k is

(a.) 2

(b.) 4

(c.) 1

(d.)-2

60. The value of f at a local minimum in the rectangular region

$R=\left\{(x,y)\in R^2 : |x|<\frac{3}{2}, |y|<\frac{3}{2}\right\}$ is

(a.) - 2

(b.) - 3

(c.) $\frac{-7}{8}$

(d.) 0