Model Question Paper – 1

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*

(ii) *Use the graph sheet for the question on Linear programming in PART E.*

PART – A

Answer ALL the questions **$10 \times 1 = 10$**

1. Give an example of a relation which is symmetric and transitive but not reflexive.
2. Write the domain of $f(x) = sec^{-1}x$.
3. Define a diagonal matrix.
4. Find the values of x for which, $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.
5. Find the derivative of $cos(x^2)$ with respect to x.
6. Evaluate: $\int (1-x)\sqrt{x}\, dx$.
7. If the vectors $2\hat{i} + 3\hat{j} - 6\hat{k}$ and $4\hat{i} - m\hat{j} - 12\hat{k}$ are parallel find m.
8. Find the equation of the plane having intercept 3 on the y axis and parallel to ZOX plane.
9. Define optimal solution in linear programming problem.
10. An urn contains 5 red and 2 black balls. Two balls are randomly selected. Let X represents the number of black balls, what are the possible values of X?

PART B

Answer any TEN questions: **$10 \times 2 = 20$**

11. Define binary operation on a set. Verify whether the operation $*$ defined on Z, by $a * b = ab + 1$ is binary or not.
12. Find the simplest form of $tan^{-1}\sqrt{3} - sec^{-1}(-2)$.
13. Evaluate $sin\left\{\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right\}$.
14. Find the area of the triangle whose vertices are $(3,8)$, $(-4,2)$ and $(5,1)$ using determinants.

15 Check the continuity of the function f given by $f(x) = 2x + 3$ at $x = 1$.

16. Find the derivative of $(3x^2 - 7x + 3)^{5/2}$ with respect to x.

17. If the radius of a sphere is measured as $9\ cm$ with an error, $0.03\ cm$, then find the approximate error in calculating its volume.

18. Evaluate: $\int \frac{dx}{\sin^2 x \cos^2 x}$.

19. Evaluate: $\int \log x\ dx$

20. Find the order and degree of the differential equation,

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0.$$

21. If the position vectors of the points$A$ and B respectively are $\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{j} - \hat{k}$ find the direction cosines of $\overrightarrow{AB}$.

22. Find a vector of magnitude 8 units in the direction of the vector, $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$.

23. Find the distance of the point$(2,\ 3, -5)$ from the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} - 2\hat{k}) = 9$.

24. A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F is the event ' the number appearing is even', then find whether E and F are independent?

PART C

Answer any TEN questions: **$10 \times 3 = 30$**

25. Verify whether the function, $f : A \to B$, where $A = R - \{3\}$ and $B = R - \{1\}$, defined by $f(x) = \frac{x-2}{x-3}$ is one-one and on-to or not. Give reason.

26. Prove that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ when $xy < 1$.

27. Express $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

28. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$ prove that $\frac{dy}{dx} = \frac{1}{2(1+x^2)}$.

29. If $x = at^2$ and $y = 2at$ find $\frac{dy}{dx}$.

30. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is

(i) strictly increasing; (ii) strictly decreasing.

31. Find the antiderivative of f(x) given by $f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$.

32. Evaluate: $\int \frac{dx}{x + x\log x}$.

33. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 4$, $x = 9$ and the x-axis in the first quadrant.

34. Form the differential equation of the family of circles touching the y-axis at origin.

35. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

36. Find the area of the triangle ABC where position vectors of A, B and C are $\hat{\imath} - \hat{\jmath} + 2\hat{k}$, $2\hat{\jmath} + \hat{k}$ and $\hat{\jmath} + 3\hat{k}$ respectively.

37. Find the Cartesian and vector equation of the line that passes through the points (3, -2, -5) and (3, -2, 6).

38. Consider the experiment of tossing two fair coins simultaneously, find the probability that both are head given that at least one of them is a head.

PART D

Answer any SIX questions: **$6 \times 5 = 30$**

39. Prove that the function, $f : N \to Y$ defined by $f(x) = x^2$, where $Y = \{y : y = x^2, x \in N\}$ is invertible. Also write the inverse of $f(x)$.

40. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$. Calculate AB, AC and A(B+C). Verify that $AB + AC = A(B + C)$.

41. Solve the following system of equations by matrix method:

$x + y + z = 6$; $y + 3z = 11$ and $x - 2y + z = 0$.

42. If $y = 3\cos(\log x) + 4\sin(\log x)$ show that $x^2 y_2 + xy_1 + y = 0$.

43. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2 cm/minute. When $x = 10\ cm$ and $y = 6\ cm$, find the rates of change of (i) the perimeter and (ii) the area of the rectangle.

44. Find the integral of $\dfrac{1}{\sqrt{a^2 - x^2}}$ with respect to x and hence evaluate $\displaystyle\int \frac{dx}{\sqrt{7-6x-x^2}}$.

45. Using integration find the area of the region bounded by the triangle whose vertices are $(-1,0)$, $(1,3)$ and $(3,2)$.

46. Solve the differential equation $x\log x\dfrac{dy}{dx} + y = \dfrac{2}{x}\log x$.

47. Derive the equation of a plane in normal form(both in the vector and Cartesian form).

48. If a fair coin is tossed 8 times. Find the probability of (i) at least five heads and (ii) at most five heads.

PART E

Answer any ONE question: **1×10=10**

49. (a) Prove that $\displaystyle\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ and evaluate $\displaystyle\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

(b) Prove that $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

50.(a) A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1hour on machine B to produce a package of bolts. He earns a profit of Rs.17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit if he operates his machines for at most 12 hours a day?

(b) Determine the value of k, if $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{if } x \neq \dfrac{\pi}{2} \\ 3, & \text{if } x = \dfrac{\pi}{2} \end{cases}$

is continuous at $x = \dfrac{\pi}{2}$.

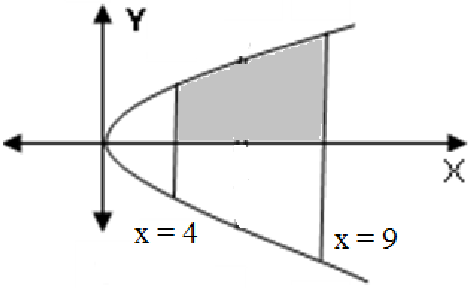
SCHEME OF VALUATION
Model Question Paper – 1

II P.U.C MATHEMATICS (35)

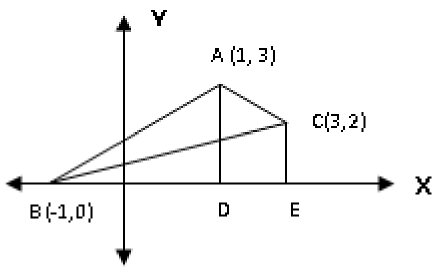
Q.no		Marks
1	Let $A = \{1,\ 2,\ 3\}$. Writing an example of the type $R = \{(1,\ 1),\ (1,\ 2),\ (2,\ 1),\ (2,\ 2)\}$.	1
2	Writing the domain $\lvert x\rvert \geq 1$ **OR** $x \geq 1$ *and* $x \leq -1$ **OR** $\{x : x \geq 1\ or\ \ x \leq -1\}$ **OR** $(-\infty, -1] \cup [1,\ \infty)$.	1
4	Getting: $x = \pm 2\sqrt{2}$	1
5	Getting: $-\sin(x^2).(2x)$ **OR** $-2x\sin\left(x^2\right)$	1
6	Getting: $\dfrac{2x^{3/2}}{3} - \dfrac{2x^{5/2}}{5} + c$	1
7	Getting: $m = -6$.	1
8	Getting: Equation of the plane is $y = 3$	1
9.	Writing the Definition.	1
10.	Possible values of X are 0, 1, and 2	1
11.	Writing the definition.	1
	Giving the reason, if a and b are any two integers then $ab + 1$ is also a unique integer.	1
12.	Writing $\tan^{-1}\sqrt{3} = \dfrac{\pi}{3}$ **OR** $\sec^{-1}(-2) = \dfrac{2\pi}{3}$	1
	Getting the answer $-\dfrac{\pi}{3}$	1
13.	Writing $sin\left\{\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right\} = sin\left\{\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right\}$.	1
	Getting the answer 1.	1
14.	Writing: Area $= \dfrac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$	1
	Getting: Area $= \dfrac{61}{2}$	1
15	Getting: $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1}(2x+3) = 2(1) + 3 = 5$.	1
	Getting : $\lim\limits_{x\to 1} f(x) = 5 = f(1)$	1

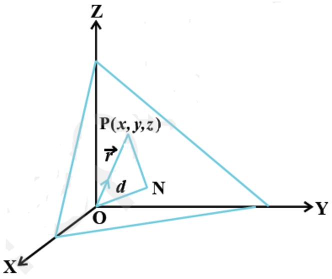
	and concluding f is continuous at $x = 1$.	
16	For writing: $\frac{dy}{dx} = \frac{5}{2}(3x^2 - 7x + 3)^{\frac{5}{2}-1} \times \frac{d}{dx}(3x^2 - 7x + 3)$.	1
	Getting: $\frac{5}{2}(3x^2 - 7x + 3)^{\frac{3}{2}}(6x - 7)$.	1
17.	Writing $dV = \frac{dV}{dr}.\Delta r$ **OR** writing $\frac{dV}{dr} = \frac{4}{3}\pi.3r^2$.	1
	Getting $dV = 4\pi \times 81 \times 0.03 = 9.72\pi\ cm^3$.	1
18.	Writing: $\int(sec^2 x + cosec^2 x)dx$.	1
	Getting: $tan\, x - cot\, x + c$.	1
19	Getting: $\log x \int 1\,.dx - \int \frac{d}{dx}.\log x\,.\int 1\, dx.dx$.	1
	Getting: $x \log x - x + c$.	1
20	Writing: Order = 2.	1
	Writing: Degree = 1.	1
21.	Getting: $\overrightarrow{AB} = -\hat{\imath} - \hat{\jmath} + 2\hat{k}$.	1
	Finding $\lvert\overrightarrow{AB}\rvert = \sqrt{6}$ and writing the direction cosines: $\frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$.	1
22.	Finding $\lvert\vec{a}\rvert = \sqrt{30}$ **OR** writing $8.\frac{\vec{a}}{\lvert\vec{a}\rvert}$.	1
	Getting: $8\hat{a} = \frac{8(5\vec{\imath} - \vec{\jmath} + 2\vec{k})}{\sqrt{30}}$ or $\frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$	1
23.	Writing equation of the plane $x + 2y - 2z - 9 = 0$ **OR** writing the formula $d = \frac{\lvert ax_1 + by_1 + cz_1 + d\rvert}{\sqrt{a^2 + b^2 + c^2}}$ **OR** writing $\left\lvert\frac{1(2) + 2(3) - 2(-5) - 9}{\sqrt{1 + 2^2 + 2^2}}\right\rvert$.	1
	Getting the answer $d = 3$.	1
24.	Writing Sample space $S = \{1,2,3,4,5,6\}$, $E = \{3,6\}$, $F = \{2,4,6\}$ and $E \cap F = \{6\}$ **OR** getting $P(E) = \frac{1}{3}$ **OR** getting $P(F) = \frac{1}{2}$	1
	Getting $P(E \cap F) = \frac{1}{6}$ and $P(E \cap F) = P(E) \cdot P(F)$, and concluding E and F are independent events.	1

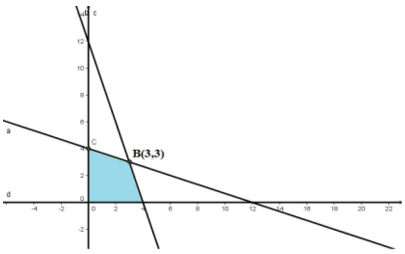
25	$f(x) = \frac{x-2}{x-3}$. Writing $\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$.	1
	Getting: $x_1 = x_2$.	1
	Proving the function is onto.	1
26	Letting $tan^{-1} x = A,\ tan^{-1} y = B$ and writing $tan\, A = x,\ tan\, B = y$.	1
	Getting $tan\,(A+B) = \frac{x+y}{1-xy}$.	1
	Getting $tan^{-1} x + tan^{-1} y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$ when $xy < 1$	1
27	Writing: $A = \frac{1}{2}(A+A') + \frac{1}{2}(A - A')$	1
	Getting: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 5 \\ 5 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	1+1
28	Taking $x = tan\,\theta$ and getting $y = tan^{-1}\left(\frac{\sqrt{1+tan^2\theta}-1}{tan\,\theta}\right) = tan^{-1}\left(\frac{sec\,\theta - 1}{tan\,\theta}\right)$	1
	Getting $y = tan^{-1}\left(tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2} \cdot tan^{-1} x$	1
	Proving $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{1+x^2} = \frac{1}{2(1+x^2)}$.	1
29	Getting $\frac{dx}{dt} = 2a\,t$	1
	Getting $\frac{dy}{dt} = 2a$	1
	Getting $\frac{dy}{dx} = \frac{1}{t}$.	1
30	Getting $f'(x) = 12x^2 - 12x - 72$.	1
	Getting the set of values for strictly increasing, $(-\infty, -2) \cup (3, \infty)$	1
	Getting the set of values for strictly decreasing,$(-2, 3)$	1
31	Getting $f(x) = x^4 + \frac{1}{x^3} + c$	1
	Using $f(2) = 0$ and getting $c = -\frac{129}{8}$.	1
	Writing $f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$.	1
32	Writing $I = \int \frac{\frac{1}{x}}{1+\log x} dx$.	1
	Taking $log\, x = t$ and writing $\frac{1}{x} dx = dt$.	1
	$\therefore I = \int \frac{1}{1+t} dt = log(1+t) + c = log(1 + log\, x) + c$	1

33	 Drawing the figure	1
	Writing Area = $\int_4^9 y\,dx$	1
	Getting: Area = $\frac{38}{3}$ $sq.units$	1
34	Writing the equation $x^2 + y^2 - 2ax = 0$ **OR** $(x - a)^2 + y^2 = a^2$.	1
	Getting: $2x + 2y\frac{dy}{dx} = 2a$ **OR** $2(x-a) + 2y\frac{dy}{dx} = 0$.	1
	Getting the answer $y^2 - x^2 - 2xy\frac{dy}{dx} = 0$.	1
35	Writing $\overrightarrow{(a + \lambda b)}.\vec{c} = \vec{0}$.	1
	Getting $3(2 - \lambda) + 1(2 + 2\lambda) + 0(3 + \lambda) = 0$	1
	Getting $\lambda = 8$.	1
36	Getting $\overrightarrow{AB} = -\hat{\imath} + 3\hat{\jmath} - \hat{k}$ **OR** $\overrightarrow{AC} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$	1
	Getting $\overrightarrow{AB} \times \overrightarrow{AC} = 5\hat{\imath} + 2\hat{\jmath} + \hat{k}$.	1
	Getting $\lvert\overrightarrow{AB} \times \overrightarrow{AC}\rvert = \sqrt{30}$ and area = $\frac{\sqrt{30}}{2}$ sq. units	1
37	Taking $\vec{a}$ and $\vec{b}$ as position vectors of given points and finding $\vec{b} - \vec{a} = 11\hat{k}$. **OR** Writing formula for vector equation of the line $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.	1
	Getting vector equation of line $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} - 5\hat{k} + \lambda(11\hat{k})$ **OR** $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} - 5\hat{k} + \lambda(-11\hat{k})$ **OR** $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k} + m(11\hat{k})$ **OR** $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k} + m(-11\hat{k})$.	1
	Writing Cartesian equation of the line $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}$ **OR** $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{-11}$ **OR** $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z-6}{11}$. **OR** $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z-6}{-11}$.	1
38	Writing , Sample space S = {HH, HT, TH, TT} and events A = { HH}; B = { HH, HT, TH}	1
	Writing $A \cap B = \{HH\}$, $P(A \cap B) = \frac{1}{4}$, $P(B) = \frac{3}{4}$	1

	Getting $P(E)=\frac{1}{3}$, $P(F)=\frac{1}{2}$ and $P(E\cap F)=\frac{1}{6}$ and writing $P(E\cap F)=P(E)\cdot P(F)$ $\therefore$ E and F are independent events.	1
39	Defining $g: Y \to N,\ g(y) = \sqrt{y},\ y \in Y$ **OR** Defining $g: Y \to N,\ g(x) = \sqrt{x},\ x \in Y$ **OR** Writing $x^2 = y \Rightarrow x = \sqrt{y}$.	1
	Getting $gof(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = x$.	1
	Stating $gof = I_N$.	1
	Getting $fog(y) = f(\sqrt{y}) = \sqrt{y}^2 = y,\ y \in Y$ **OR** $fog(x) = f(\sqrt{x}) = \sqrt{x}^2 = x, x \in Y$ and stating $fog = I_Y$.	1
	Writing $f^{-1}(x) = \sqrt{x}$ OR $f^{-1} = \sqrt{x}$.	1
40	Finding: AB	1
	Finding: AC	1
	Finding: B + C	1
	Finding: A(B + C)	1
	Conclusion	1
41	Let $A=\begin{bmatrix}1 & 1 & 1\\0 & 1 & 3\\1 & -2 & 1\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}6\\11\\0\end{bmatrix}$ Getting $\lvert A\rvert=9$.	1
	Getting $\text{adj}A=\begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}$ (any 4 cofactors correct award 1 mark)	2
	Writing $X=A^{-1}B=\frac{1}{\lvert A\rvert}(\text{adj}A)B$ **OR** $X = \frac{1}{9}\begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}\begin{bmatrix}6\\11\\0\end{bmatrix}$	1
	Getting $x = 1,\ y = 2,\ z = 3$.	1
42	Getting $y_1 = 3[-\sin(logx)] \times \frac{1}{x} + 4[\cos(\log x)] \times \frac{1}{x}$	1
	Getting $x\,y_1 = -3\ \sin(logx) + 4\cos(\log x)$	1
	Getting $x\,y_2 + y_1.1 = -3[\cos(logx)]\frac{1}{x} + 4[-\sin(\log x)]\frac{1}{x}$	1
	Getting $x^2y_2 + xy_1 = -[3\cos(\log x) + 4\sin(logx)]$	1

	Getting $x^2y_2 + xy_1 + y = 0$	1
43	Writing $\frac{dx}{dt} = -3$ and $\frac{dy}{dt} = 2.$	1
	Writing perimeter $P = 2(x+y)$ and area $a = xy$.	1
	Getting $\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$ **OR** $\frac{dA}{dt} = x.\frac{dy}{dt} + y.\frac{dx}{dt}$	1
	Getting $\frac{dP}{dt} = -2\ cm/minute$	1
	Getting $\frac{dA}{dt} = 2\ cm^2/minute$	1
44	Taking $x = a\sin\theta$ and writing $dx = a\cos\theta\ d\theta$	1
	Writing $\int \frac{dx}{\sqrt{a^2 - x^2}} = \int \frac{a\cos\theta\ d\theta}{\sqrt{a^2 - a^2\sin^2\theta}}$	1
	Getting the answer $sin^{-1}\left(\frac{x}{a}\right) + c$	1
	Getting $\int \frac{1}{\sqrt{7 - 6x - x^2}}\,dx = \int \frac{1}{\sqrt{16 - (x+3)^2}}\,dx$	1
	Getting $sin^{-1}\left(\frac{x+3}{4}\right) + c$	1
45		1
	Getting the equation of the sides AB, AC and BC, $y = \frac{3x+3}{2}$, $y = \frac{-x+7}{2}$, $y = \frac{x+1}{2}$ (any one equation correct award one mark)	2
	Writing area of triangle ABC= $\int_{-1}^{1} \frac{3x+3}{2}dx + \int_{1}^{3} \frac{-x+7}{2}dx - \int_{-1}^{3} \frac{x+1}{2}dx$	1
	Area = 4 sq. units.	1
46	Writing: $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x^2}$	1
	Comparing with standard form and writing P and Q $P = \frac{1}{x\log x}$ and $Q = \frac{2}{x^2}$	1
	Finding I.F : $I.F = e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x.$	1
	Writing solution in the standard form: $y.\log x = \int \frac{2}{x^2}\log x\ dx + C$	1
	Getting: $y.\log x = -\frac{2}{x}(1 + log\lvert x\rvert) + C$	1

47		1
	Getting $\overrightarrow{ON} = d\,\hat{n}$.	1
	Let P(x, y, z) be a point on the plane having p.v. vector. Stating $\overrightarrow{NP} \perp \overrightarrow{ON}$ and getting $(\vec{r} - d\,\hat{n}).\hat{n} = 0$, $\vec{r}.\hat{n} = d$	1
	Let l, m, n be the direction cosines of $\hat{n}$. Writing $\hat{n} = l\hat{i} + m\hat{j} + n\hat{k}$	1
	Getting $lx + my + nz = d$	1
48	Writing $P(X = x) = {}^{n}C_x q^{n-x} p^x$, x = 0, 1, … , n **OR** $n = 8$ $p = 1/2$, $q = 1/2$.	1
	Getting $P(X = x) = {}^{8}C_x \left(\frac{1}{2}\right)^{8-x} . \left(\frac{1}{2}\right)^{x} = {}^{8}C_x \left(\frac{1}{2}\right)^{8}$.	1
	Stating P (at least five heads) $= P(x = 5) + P(x = 6) + P(x = 7) + P(x = 8)$	1
	Getting $= \frac{37}{256}$	1
	Stating $P(at\ most\ five\ heads) = P(x \leq 5)$ $= P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5)$ and getting $\frac{109}{128}$	1
49 (a)	Taking $t = a + b - x$ **OR** $x = a + b - t$ and $dt = -\,dx$;	1
	Proving $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$	1
	Getting $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$	1
	Getting $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)} + \sqrt{\sin\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right)}} dx$	1
	Getting $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\, dx = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	1
	Getting $I = \frac{\pi}{12}$	1

49 (b)	Operating $C_1 \to C_1 + C_2 + C_3$ $\text{LHS} = \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$	1
	Taking $2(x+y+z)$ from first column $\text{LHS} = 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$	1
	Operate $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ $\text{LHS} = 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$	1
	Getting: LHS = RHS	1
50 (a)	Formulating and writing the constraints $x + 3y \leq 12$; $3x + y \leq 1$; $x \geq 0,\ y \geq 0$	1 + 1
		1
	Getting corner points	1
	Writing Maximize $Z = 17.5x + 7y$ and Evaluating objective function Z at each Corner points.	1
	Writing maximum value $Z = 73.5$ at B(3,3)	1
50 (b)	Stating $f\left(\frac{\pi}{2}\right) = \lim_{x \to \frac{\pi}{2}} \left(\frac{k \cos x}{\pi - 2x}\right)$	1
	Taking $x - \frac{\pi}{2} = h$ and stating $h \to 0$	1
	Getting $\lim_{h \to 0} \frac{k(-\sin h)}{-2h}$	1
	Obtaining $k = 6$	1

Model Question Paper – 2

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

(ii) Use the graph sheet for the question on Linear programming in PART E.

PART – A

Answer ALL the questions **$10 \times 1 = 10$**

1. Define bijective function.
2. Find the principal value of $cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.
3. Construct a 2×3 matrix whose elements are given by $a_{ij} = |i-j|$.
4. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, find $|2A|$.
5. If $y = tan\,(2x+3)$ find $\frac{dy}{dx}$.
6. Write the integral of $\frac{1}{x\sqrt{x^2-1}}$, $x > 1$ with respect to x.
7. Write the vector joining the points $A(2,\ 3,\ 0)$ and $B(-1,-2,-4)$.
8. Find the equation of the plane which makes intercepts $1,\ -1\ and\ 2$ on the x, y and z axes respectively.
9. Define feasible region.
10. If P(B) = 0.5 and P $(A \cap B)$=0.32, find P(A/B).

PART B

Answer any TEN questions **$10 \times 2 = 20$**

11. A relation R is defined on the set $A = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$ by $R = \{(x,\ y): y\ \ is\ \ divisible\ \ by\ \ x\}$. Verify whether R is symmetric and reflexive or not. Give reason.

12. Write the simplest form of $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \frac{\pi}{2}$.

13. If $\sin\left\{\sin^{-1}\frac{1}{5} + \cos^{-1} x\right\} = 1$, find x.

14. If each element of a row is expressed as sum of two elements then verify for a third order determinant that the determinant can be expressed as sum of two determinants.

15. If $\sqrt{x}+\sqrt{y}=\sqrt{a}$, prove that $\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$.

16. If $y=(\sin^{-1}x)^x$ find $\frac{dy}{dx}$.

17. Find the local maximum value of the function $g(x)= x^3-3x$.

18. Evaluate $\int log(\sin x).(\cot x)dx$.

19. Find $\int_0^{\pi/2}\cos 2x\ dx$.

20. Form the differential equation of the family of curves $\frac{x}{a}+\frac{y}{b}=1$ by eliminating the constants a and b.

21. If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=8$ find $|\vec{x}|$.

22. Show that the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ is equally inclined to the positive direction of the axes.

23. Find the angle between the pair of lines $\vec{r}=3\hat{i}+5\hat{j}-\hat{k}+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{r}=7\hat{i}+4\hat{k}+\mu\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$.

24. Probability distribution of x is

x	0	1	2	3	4
$P(x_1)$	0.1	k	2k	2k	k

Find k.

PART C

Answer any TEN questions **$10\times3=30$**

25. If $*$ is a binary operation defined on $A=N\times N$, by $(a,b)*(c,d)=(a+c,b+d)$, prove that $*$ is both commutative and associative. Find the identity if it exists.

26. Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$.

27. By using elementary transformations, find the inverse of the matrix $A=\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}$.

28. If $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$ prove that $\frac{dy}{dx}=\tan\left(\frac{\theta}{2}\right)$.

29. If a function $f(x)$ is differentiable at $x=c$ prove that it is continuous at $x=c$.

30. Prove that the curves $x=y^2$ and $xy=k$ cut at right angles if $8k^2=1$.

31. Evaluate: $\int\sin(ax+b)\cos(ax+b)dx$.

32. Evaluate: $\int\tan^{-1}x\,dx$.

33. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 2$.

34. Find the equation of the curve passing through the point $(-2, 3)$, given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

35. For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ prove that
$\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = 2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$.

36. Find a unit vector perpendicular to the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

37. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$, both in vector form and Cartesian form.

38. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. Find the probability that it is actually head.

PART D

Answer any SIX questions **$6 \times 5 = 30$**

39. Let $f : N \to R$ be defined by $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of the function, is invertible. Also find the inverse of f.

40. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$. Calculate AC, BC and $(A+B)C$. Also, verify that $(A+B)C = AC + BC$.

41. Solve the following system of equations by matrix method,

$3x - 2y + 3z = 8$; $2x + y - z = 1$ and $4x - 3y + 2z = 4$.

42. If $y = Ae^{mx} + Be^{nx}$, prove that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

43. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 centimeter?

44. Find the integral of $\sqrt{x^2+a^2}$ with respect to x and evaluate $\int\sqrt{4x^2+9}\,dx$.

45. Solve the differential equation $ydx-\left(x+2y^2\right)dy=0$.

46. Find the area of the circle $4x^2+4y^2=9$ which is interior to the parabola $x^2=4y$.

47. Derive the condition for the coplanarity of two lines in space both in the vector form and Cartesian form.

48. Find the probability of getting at most two sixes in six throws of a single die.

PART E

Answer any ONE question **$1\times$10=10**

49. (a) Minimize and Maximize $z = 3x + 9y$ subject to the constraints

$x + 3y \leq 60$
$x + y \geq 10$
$x \leq y$
$x \geq 0, y \geq 0$, by the graphical method.

(b) Prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

50.(a) Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if f(x) is even} \\ 0, & \text{if f(x) is odd} \end{cases}$

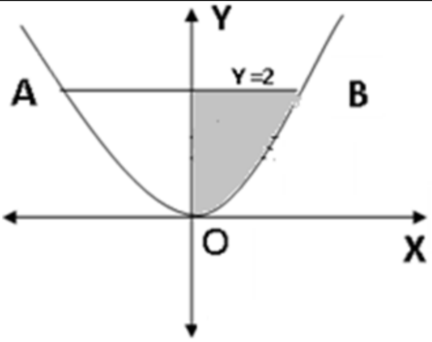
and evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\,dx$

(b) Define a continuity of a function at a point. Find all the points of discontinuity of f defined by $f(x)=|x|-|x+1|$.

SCHEME OF VALUATION
Model Question Paper – 2
II P.U.C MATHEMATICS (35)

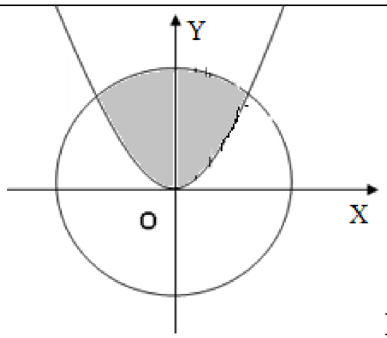
Q.no		Marks
1	Writing the definition.	1
2	Getting $cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.	1
3	Getting: $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$	1
4	Getting $\|2A\| = -24$	1
5	Getting $sec^2(2x+3).2$ **OR** $2.sec^2(2x+3)$	1
6	Writing $-\text{cosec}^{-1} x + c$ **OR** $\sec^{-1} x + c$	1
7	Getting $\overrightarrow{AB} = -3\hat{i} - 5\hat{j} - 4\hat{k}$	1
8	Writing $\frac{x}{1} + \frac{y}{-1} + \frac{z}{2} = 1$	1
9	Writing the definition	1
10	Getting: $P(A/B) = \frac{P(A\cap B)}{P(B)} = 0.64$	1
11	Stating the reason if y *is divisible by* x then it is not necessary that x *is divisible by* y.	1
	Stating the reason x *is divisible by* x, $\forall\ x \in A$.	1
12	Dividing numerator and denominator by $cosx$ and getting $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$.	1
	Getting the answer $\frac{\pi}{4} - x$.	1
13	Writing $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\} = \sin^{-1}\left\{\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right\}$.	1
	Getting the answer 1.	1
14	Writing $\Delta = \begin{vmatrix} a_1 + x & a_2 + y & a_3 + z \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ and expanding by definition	1
	Getting $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$	1
15	Getting $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}.\frac{dy}{dx} = 0$	1

	Getting $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$.	1
16	Let $v = (sin^{-1}x)^x$. Getting $logv = x.log\,(sin^{-1}x)$	1
	Getting $\frac{dv}{dx} = v\left[x.\frac{1}{sin^{-1}x}.\frac{1}{\sqrt{1-x^2}} + log\,(sin^{-1}x)\right]$	1
17	Getting $g'(x) = 3x^2 - 3$.	1
	Getting the local maximum value $= 2$.	1
18	Substituting $\log\sin x = t$ and writing $\cot x\,.\,dx = dt$	1
	Getting $\frac{(\log\sin x)^2}{2} + c$	1
19	Getting $\left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$	1
	Getting $\frac{1}{2}[\sin\pi - 0] = 0$	1
20	Getting $y = -\frac{b}{a}x + b$ **OR** finding $\frac{dy}{dx}$.	1
	Getting $\frac{d^2y}{dx^2} = 0$	1
21	Writing $\vec{x}.\vec{x} - \vec{a}.\vec{x} + \vec{x}.\vec{a} - \vec{a}.\vec{a} = 8$ **OR** $\vec{x}^2 - \vec{a}^2 = 8$ **OR** $\lvert\vec{x}\rvert^2 - \lvert\vec{a}\rvert^2 = 8$	1
	Getting $\lvert\vec{x}\rvert = 3$	1
22	Getting magnitude $= \sqrt{3}$	1
	Concluding that the direction cosines are equal	1
23	Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} + 2\hat{k}$ Getting $\vec{a}\bullet\vec{b} = 6$ **OR** $\lvert\vec{a}\rvert = \sqrt{3}$ **OR** $\lvert\vec{b}\rvert = 2\sqrt{3}$	1
	Getting angle between the vectors $= 0$	1
24	Writing :$\sum P(x_i) = 1$	1
	Getting: $K = 0.15$.	1
25	Proving commutative.	1
	Proving associative.	1
	Proving identity does not exist.	1
26	Writing $2tan^{-1}x = tan^{-1}\frac{2x}{1-x^2}$ **OR** $2tan^{-1}\frac{1}{2} = tan^{-1}\frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}$.	1
	Getting $2tan^{-1}\frac{1}{2} = tan^{-1}\frac{4}{3}$.	1
	Proving $tan^{-1}\frac{4}{3} + tan^{-1}\frac{1}{7} = tan^{-1}\frac{31}{17}$.	1
27	Writing $A = IA$	1

	Getting any one non diagonal element is zero	1
	Getting the inverse $\frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$	1
28	Getting $\frac{dx}{d\theta} = a[1+\cos\theta]$ **OR** $\frac{dy}{d\theta} = a\sin\theta$	1
	Getting $\frac{dy}{dx} = \frac{a\sin\theta}{a[1+\cos\theta]}$	1
	Getting $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$.	1
29	Stating $\lim_{x\to c}\frac{f(x)-f(c)}{x-c} = f'(c)$	1
	Writing $f(x)-f(c) = \frac{f(x)-f(c)}{x-c}.(x-c)$	1
	Getting $\lim_{x\to c} f(x) = f(c)$	1
30	Finding the point of intersection $\left(k^{2/3}, k^{1/3}\right)$	1
	Finding the slope of the tangent to the first curve at point of intersection $m_1 = \frac{1}{2k^{1/3}}$ **OR** similarly to find $m_2 = -\frac{1}{k^{1/3}}$ **OR** writing the orthogonality condition $m_1 m_2 = -1$	1
	To showing the required condition.	1
31	Writing $\frac{1}{2}\int 2.\sin(ax+b).\cos(ax+b)dx$	1
	Writing $\frac{1}{2}\int \sin 2(ax+b)dx$	1
	Getting $\frac{1}{2}\left[-\frac{\cos 2(ax+b)}{2a}\right] + c$	1
32	Writing $\int \tan^{-1} x\, dx$ $= \tan^{-1}x.\int 1\,dx - \int \frac{d}{dx}\tan^{-1}x.\int 1\,dx.dx.$	1
	Getting $x\tan^{-1}x - \int \frac{x}{1+x^2}dx$	1
	Getting $x\tan^{-1}x - \frac{1}{2}\log\lvert 1+x^2\rvert + c$	1
33	 Drawing the figure and explaining it	1
	Stating required area $= 2\int_0^2 \sqrt{y}.dy$	1
	Getting area $= \frac{8\sqrt{2}}{3}$ sq.units	1

34	Writing: $\frac{dy}{dx}=\frac{2x}{y^2}$	1
	Stating $\int y^2 dy = \int 2x\, dx$	1
	Getting $\frac{y^3}{3}=x^2+c$ and getting $c = 5$	1
35	Writing $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]$ $=(\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\}$	1
	For expanding : $(\vec{a}+\vec{b}).\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\}$	1
	Getting $2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$	1
36	Getting $\vec{a}+\vec{b}=2\hat{\imath}+3\hat{\jmath}+4\hat{k}$ **OR** $\vec{a}-\vec{b}=-\hat{\jmath}-2\hat{k}$ **OR** writing the formula $\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{\lvert(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\rvert}$	1
	Getting $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=-2\hat{\imath}+4\hat{\jmath}-2\hat{k}$ **OR** $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=2\hat{\imath}-4\hat{\jmath}+2\hat{k}$	1
	Getting the answer $\frac{-2\hat{\imath}+4\hat{\jmath}-2\hat{k}}{\sqrt{24}}$ **OR** $\frac{2\hat{\imath}-4\hat{\jmath}+2\hat{k}}{\sqrt{24}}$	1
37	Writing: $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}-2\hat{k}$ **OR** writing the formula $\vec{r}=\vec{a}+\lambda\vec{b}$	1
	Writing: the equation of the line is $=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$	1
	Getting the equation $\frac{x-1}{3}=\frac{y-2}{2}=\frac{z-3}{-2}$	1
38	Writing : $P(E_1)=\frac{1}{2}$ and $P(E_2)=\frac{1}{2}$	1
	Writing: $P(A\mid E_1)=\frac{4}{5}$ and $P(A\mid E_2)=\frac{1}{5}$	1
	Getting: $P(E_1\mid A)=\frac{P(E_1)P(A\mid E_1)}{P(E_1)P(A\mid E_1)+P(E_2)P(A\mid E_2)}=\frac{4}{5}$	1
39	Getting $y=4x^2+12x+15 \Rightarrow x=\frac{\sqrt{y-6}-3}{2}$.	1
	Stating $g(y)=\frac{\sqrt{y-6}-3}{2},\ y\in S$ **OR** $g(x)=\frac{\sqrt{x-6}-3}{2},\ x\in S$.	1
	Proving $gof(x)=g(4x^2+12x+15)=x$ and writing $gof=I_N$.	1
	Proving $fog(y)=f\left(\frac{\sqrt{y-6}-3}{2}\right)=y,\ y\in S$ **OR** $fog(x)=f\left(\frac{\sqrt{x-6}-3}{2}\right)=x,\ x\in S$	1

	and writing $fog = I_S$.	
	Writing $f^{-1}(x) = \frac{\sqrt{x-6}-3}{2}$ **OR** $f^{-1} = \frac{\sqrt{x-6}-3}{2}$.	1
40	Finding : A+B	1
	Finding : (A+B)C	1
	Finding : AC	1
	Finding : BC	1
	Verifying $(A+B)C = AC+BC$	1
41	Let $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ Getting $\lvert A \rvert = -17$.	1
	Getting $\text{adj}A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$ (any 4 cofactors correct award 1 mark)	2
	Writing $X = A^{-1}B = \frac{1}{\lvert A \rvert}(\text{adj}A)B$ **OR** $X = \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	1
	Getting x =1, y = 2 and z = 3	1
42	Getting $\frac{dy}{dx} = Am.e^{mx} + Bn.e^{nx}$	1
	Getting $\frac{d^2y}{dx^2} = Am.me^{mx} + Bn.ne^{nx}$	1
	Writing LHS = $(Am^2e^{mx} + Bn^2e^{nx})$ $-(m+n)(Am.e^{mx} + Bn.e^{nx}) + mny$	1
	Evaluating the brackets	1
	Getting the answer zero	1
43	Writing $\frac{dV}{dt} = 9$ **OR** $V = x^3$	1
	Getting $\frac{dV}{dt} = 3x^2.\frac{dx}{dt}$	1
	Getting $\frac{dx}{dt} = \frac{1}{300} cm/sec$	1
	Writing $S = 6x^2$ and $\frac{dS}{dt} = 12x.\frac{dx}{dt}$	1
	Getting $\frac{dS}{dt} = \frac{2}{5} cm^2/sec$	1
44	Writing $I = \int \sqrt{x^2+a^2}\,dx$ $= \sqrt{x^2+a^2}\int 1\,dx - \int \frac{d}{dx}\sqrt{x^2+a^2}.\int 1\,dx.dx$	1
	Getting $x\sqrt{x^2+a^2} - \frac{1}{2}\int \frac{2x^2}{\sqrt{x^2+a^2}}dx$	1
	Getting $x\sqrt{x^2+a^2} - \int \frac{x^2+a^2-a^2}{\sqrt{x^2+a^2}}dx$	1

	Getting $I = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left\|x+\sqrt{x^2+a^2}\right\| + c$	1
	Getting $\int\sqrt{4x^2 + 9}dx$ $= \dfrac{x}{2}\sqrt{4x^2+a^2} + \dfrac{a^2}{4}\log\left\|2x+\sqrt{4x^2+a^2}\right\| + c$	1
45	Writing $y\frac{dx}{dy} = x + 2y^2$	1
	Getting $\frac{dx}{dy} - \frac{x}{y} = 2y$ and writing $P = -\frac{1}{y},\ Q = 2y$	1
	Getting I.F $= e^{\int -\frac{1}{y}dy} = \frac{1}{y}$	1
	Writing the solution $x\frac{1}{y} = \int (2y)\left(\frac{1}{y}\right)dy + C$	1
	Getting the answer $x = 2y^2 + Cy$	1
46	 Drawing figure	1
	Finding the points of intersection $x = \pm\sqrt{2}$	1
	Writing area of the region $= 2\left\{\int_0^{\sqrt{2}}\frac{1}{2}\sqrt{9-4x^2}\,dx - \int_0^{\sqrt{2}}\frac{x^2}{4}\,dx\right\}$ **OR** area of the region $= \displaystyle\int_{-\sqrt{2}}^{\sqrt{2}}\left(\frac{1}{2}\sqrt{9-4x^2}\right)dx - \int_{-\sqrt{2}}^{\sqrt{2}}\left(\frac{x^2}{4}\right)dx$	1
	Getting the answer $= \left\{\frac{\sqrt{2}}{2} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)\right\} - \frac{\sqrt{2}}{3}$	1 + 1
47	Writing the equations $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$	1
	Stating $\vec{AB} = \vec{a_1} - \vec{a_2}$ is perpendicular to $\vec{b_1}\times\vec{b_2}$	1
	Getting $(\vec{a_1}-\vec{a_2}).(\vec{b_1}\times\vec{b_2}) = 0$	1
	Writing $\vec{AB} = (x_1-x_2)\hat{\imath} + (y_1-y_2)\hat{\jmath} + (z_1-z_2)\hat{k}$	1
	Getting $\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0$	1
48	Writing $p = \frac{1}{6},\ q = \frac{5}{6}$ and $n = 6$	1
	Writing P(at most 2 successes) $= P(x=0) + P(x=1) + P(x=2)$	1
	Getting $P(X=0) = \left(\frac{5}{6}\right)^6,\ P(X=1) = \left(\frac{5}{6}\right)^5,$ $P(X=2) = 15\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$	2

	Getting the answer $\frac{35}{18}\left(\frac{5}{6}\right)^4$	1
49 (a)	Drawing graph of the system of linear inequalities	2
	Showing feasible region ABCD and getting corner P	1
	Getting corresponding value of Z at each corner point	1
	Obtaining minimum value Z=60 at $x = 5, y = 5$	1
	Obtaining maximum value Z=180, at $x = 15,\ y = 15$ and $x = 0, y = 20$	1
49(b)	Getting $\begin{vmatrix} x & x^2 & yz \\ y-x & y^2-x^2 & zx-yz \\ z-x & z^2-x^2 & xy-yz \end{vmatrix}$ (any one row correct award the mark)	1
	Getting $(y-x)(z-x)\begin{vmatrix} x & x^2 & yz \\ 1 & y+x & -z \\ 1 & z+x & -y \end{vmatrix}$	1
	Getting $(y-x)(z-x)\begin{vmatrix} x & x^2 & yz \\ 1 & y+x & -z \\ 0 & z-y & -y+z \end{vmatrix}$	1
	Getting $(x-y)(y-z)(z-x)(xy+yz+zx)$	1
50 (a)	Writing $\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx$	1
	Taking $t = -x$ and $dt = -dx$	1
	Getting $\int_{-a}^{a} f(x)dx = \int_{0}^{a} f(-x)dx + \int_{0}^{a} f(x)dx$	1
	Getting $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ when $f(x)$ is even	1
	Getting $\int_{-a}^{a} f(x)dx = 0$ when $f(x)$ is odd	1
	Writing $\int_{-\pi/2}^{\pi/2} sin^7 x\, dx = 0$ with reason.	1
50 (b)	Definition	1
	Let $g(x) = \lvert x \rvert$ and $h(x) = \lvert x+1 \rvert$. As modulus functions are continuous, therefore g and h are continuous.	1
	As difference of two continuous functions is again continuous function, therefore f is continuous.	1
	There is no point of discontinuity.	1

Model Question Paper – 3

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

(ii) ***Use the graph sheet for the question on Linear programming in PART E.***

PART – A

Answer ALL the questions **$10 \times 1 = 10$**

1. Let $*$ be a binary operation defined on set of rational numbers, by $a * b = \frac{ab}{4}$. Find the identity element.
2. Write the set of values of x for which $2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}$ holds.
3. What is the number of the possible square matrices of order 3 with each entry 0 or 1?
4. If A is a square matrix with $|A| = 6$, find the values of $|AA'|$.
5. The function $f(x) = \frac{1}{x-5}$ is not continuous at $x = 5$. Justify the statement.
6. Write the antiderivative of e^{2x} with respect to x.
7. Define collinear vectors.
8. Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin.
9. Define Optimal Solution.
10. A fair die is rolled. Consider events $E = \{2, 4, 6\}$ and $F = \{1, 2\}$. Find $P(E|F)$.

PART B

Answer any TEN questions **$10 \times 2 = 20$**

11. Prove that the greatest integer function, $f : R \to R$, defined by $f(x) = [x]$, where $[x]$ indicates the greatest integer not greater than x, is neither one-one nor onto.
12. Prove that $2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$.
13. Find $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.
14. Find the equation of the line passing through (1, 2) and (3, 6) using the determinants.

15. If $y=\sin(\log_e x)$, prove that $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{x}$.

16. Find the derivative of $x^x-2^{\sin x}$ with respect to x.

17. Find a point on the curve $y=x^3-11x+5$ at which the tangent is $y=x-11$.

18. Find $\int e^x \sec x(1+\tan x)dx$.

19. Evaluate $\int \log x\, dx$.

20. Prove that the differential equation $x^2\frac{dy}{dx}=x^2-2y^2+xy$ is a homogeneous differential equation of degree 0.

21. Find k if the vectors $\hat{\imath}+3\hat{\jmath}+\hat{k}$, $2\hat{\imath}-\hat{\jmath}-\hat{k}$ and $k\hat{\imath}+7\hat{\jmath}+3\hat{k}$ are coplanar.

22. Find the area of the parallelogram whose adjacent sides are the vectors $3\hat{\imath}+\hat{\jmath}+4\hat{k}$ and $\hat{\imath}-\hat{\jmath}+\hat{k}$.

23. Find equation of the plane passing through the line of intersection of the planes $x+y+z=6$ and $2x+3y+4z-5=0$ and the point, $(1, 1, 1)$.

24. Two cards drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

PART C

Answer any TEN questions **$10\times3 = 30$**

25. Show that the relation R in the set of all integers, Z defined by $R=\{(a, b) : 2 \textit{ divides } a-b\}$ is an equivalence relation.

26. If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, find x.

27. If A and B are square matrices of the same order, then show that $(AB)^{-1}=B^{-1}A^{-1}$.

28. Verify the mean value theorem for $f(x)=x^2-4x-3$ in the interval $[a, b]$, where $a=1$ and $b=4$.

29. If $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $-\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$ find $\frac{dy}{dx}$.

30. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is maximum?

31. Evaluate $\int_0^2 e^x\,dx$ as the limit of the sum.

32. Find $\int \frac{1}{1+\tan x}dx$.

33. Find the area bounded by the parabola $y^2 = 5x$ and the line $y = x$.

34. In a bank, principal p increases continuously at the rate of 5% per year. Find the principal in terms of time t.

35. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

36. Show that the position vector of the point$P$ which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m : n$ is $\frac{m\vec{b}+n\vec{a}}{m+n}$.

37. Find the distance between the parallel lines
$\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + m(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and
$\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + n(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$.

38. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

PART D

Answer any SIX questions **$6 \times 5 = 30$**

39. Verify whether the function, $f : N \to Y$ defined by $f(x) = 4x + 3$, where $Y = \{y : y = 4x + 3,\ x \in N\}$ is invertible or not. Write the inverse of$f(x)$ if exists.

40. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$, verify that $(AB)' = B'A'$.

41. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations $2x - 3y + 5z = 11$; $3x + 2y - 4z = -5$ and $x + y - 2z = -3$.

42. If $y = \left(\tan^{-1} x\right)^2$ then show that $\left(x^2+1\right)^2 \frac{d^2y}{dx^2} + 2x\left(x^2+1\right)\frac{dy}{dx} = 2$.

43. A particle moves along the curve, $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

44. Find the integral of $\frac{1}{\sqrt{x^2+a^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{x^2+2x+2}}dx$

45. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a > b)$ by the method of integration and hence find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

46. Find the particular solution of the differential equation $\frac{dy}{dx}+y\cot x=4x\cdot \operatorname{cosec} x$, $x \neq 0$, given that y = 0 when $x = \frac{\pi}{2}$.

47. Derive the equation of the line in space passing through a point and parallel to a vector both in the vector and Cartesian form.

48. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize at least once and exactly once.

PART E

Answer any ONE question **$1 \times 10 = 10$**

49. (a) Prove that $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$ when $f(2a-x) = f(x)$ and hence evaluate $\int_0^{\pi}|\cos x|\, dx$.

(b) Find the values of a and b such that the function defined by
$$f(x)=\begin{cases}5, & \text{if } x\le 2\\ ax+b, & \text{if } 2<x<10\\ 21, & \text{if } x\ge 10\end{cases}$$, is continuous function

50. (a) A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F1 and F2 are available. Food F1 costs Rs 4 per unit food and F2 costs Rs 6 per unit. One unit of food F1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

(b) Prove that $\begin{vmatrix}1 & x & x^2\\ x^2 & 1 & x\\ x & x^2 & 1\end{vmatrix}=(1-x^3)^2$.

SCHEME OF VALUATION

Model Question Paper – 3

II P.U.C MATHEMATICS (35)

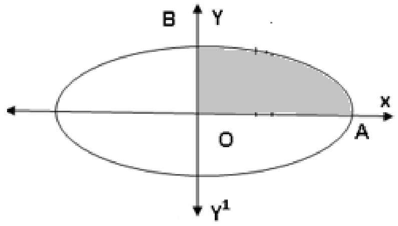
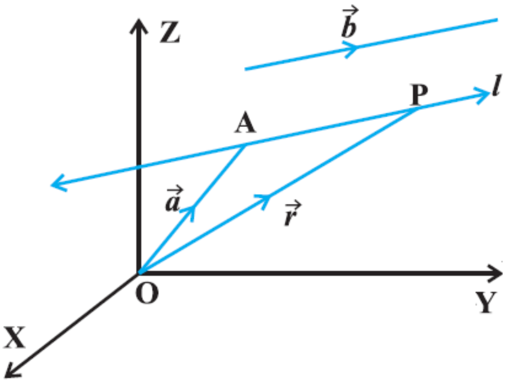
Q.no		Marks
1	Proving identity $e = 4$.	1
2	Writing $-1 < x < 1$ **OR** $\lvert x \rvert < 1$.	1
3	Getting 2^9.	1
4	Getting answer= 36.	1
5	Giving reason: function is not defined at $x = 5$	1
6	Getting $\frac{e^{2x}}{2} + c$.	1
7	Writing the definition.	1
8	Getting: the distance of the plane from the origin $= \frac{6}{\sqrt{29}}$.	1
9	Writing the definition.	1
10	Getting: $P(E\mid F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{2}$	1
11	Giving counter example of the type $f(2.6) = f(2.7) = 2$, but $2.6 \neq 2.7$.	1
	Giving the reason, non integral cannot be an image.	1
12	Letting $x = sin\theta, \ -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ **OR** using $sin\,2\theta = 2.sin\theta.cos\theta$.	1
	Obtaining LHS = RHS.	1
13	Getting $cos\,\frac{7\pi}{6} = -\,cos\,\frac{\pi}{6}$ **OR** $cos\,\frac{7\pi}{6} = cos\left(\pi + \frac{\pi}{6}\right) = cos\left(\pi - \frac{\pi}{6}\right)$.	1
	Getting $cos^{-1}\left(cos\frac{7\pi}{6}\right) = \frac{5\pi}{6}$.	1
14	Writing $\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$.	1
	Getting $4x - 2y = 0$.	1
15	Writing $sin^{-1}y = \log x$.	1
	Getting $\frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{x}$.	1
16	Writing $\frac{d}{dx}x^x = x^x(1 + logx)$ **OR** $\frac{d}{dx}2^{sinx} = 2^{sinx}.(log\,2).cosx$.	1

	Getting the answer $x^x(1+logx) - 2^{sinx}.cosx$.	1
17	Getting $\frac{dy}{dx} = 3x^2 - 11$ **OR** writing slope= 1.	1
	Getting the point $(2, -9)$.	1
18	Writing $\int e^x(\sec x + \sec x \cdot \tan x)dx$ and $\frac{d}{dx}\sec x = \sec x . \tan x$.	1
	Getting $e^x \sec x + c$.	1
19	Writing $\log x \cdot \int 1\, dx - \int 1.\frac{d}{dx}\log x\, dx$.	1
	Getting $= x\log x - x + c$.	1
20	Writing $\frac{dy}{dx} = \frac{x^2-2y^2+xy}{x^2}$ **OR** $F(x,\ y) = \frac{x^2-2y^2+xy}{x^2}$	1
	Using $F(\lambda x,\ \lambda y) = \frac{(\lambda x)^2 - 2(\lambda y)^2 + \lambda x \lambda y}{(\lambda x)^2}$ and getting $F(\lambda x,\ \lambda y) = \lambda^0 F(x,y)$.	1
21	Writing $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ k & 7 & 3 \end{vmatrix} = 0$ **OR** $\begin{vmatrix} k & 7 & 3 \\ 2 & -1 & -1 \\ 1 & 3 & 1 \end{vmatrix} = 0$.	1
	Getting $k = 0$.	1
22	Taking $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and writing $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & j & k \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$ **OR** writing the formula: area of the parallelogram $= \lvert \vec{a} \times \vec{b} \rvert$.	1
	Getting the answer $= \sqrt{42}$ sq. units.	1
23	Writing $(x + y + z - 6) + m(2x + 3y + 4z - 5) = 0$,	1
	Getting $m = 3/4$ and getting the equation of the plane as $10x + 13y + 16z - 39 = 0$.	1
24	Writing: $n(s) = 52$, $n(A) = \frac{26}{52}$ and $n(B) = \frac{25}{51}$	1
	Getting : $P(A \cap B) = P(A) \cdot P(B) = \frac{25}{102}$	1
25	Proving reflexive.	1
	Proving symmetric.	1
	Proving transitive.	1
26	Writing : $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right)$ **OR** Writing $tan^{-1}\frac{x-1}{x-2} = tan^{-1}1 - tan^{-1}\frac{x+1}{x+2}$.	1

	Getting $\tan^{-1}\left(\frac{2x^2-4}{-3}\right)=\frac{\pi}{4}$ **OR** $tan^{-1}\frac{x-1}{x-2} = tan^{-1}\frac{1}{2x+3}$	1
	Getting $x = \pm\frac{1}{\sqrt{2}}$.	1
27	Stating: $(AB)(AB)^{-1} = I$	1
	Pre multiplying by A^{-1} and getting $B(AB)^{-1} = A^{-1}$	1
	Getting $(AB)^{-1} = B^{-1}A^{-1}$.	1
28	Stating $f(x)$ is continuous in [1, 4] **OR** stating differentiable in (1, 4) **OR** Getting $f(1) = -6$ **OR** $f(4) = -3$.	1
	Getting $\frac{f(b)-f(a)}{b-a} = 1$	1
	Getting $c = \frac{5}{2}$	1
29	Taking $x = tant$	1
	Getting $y = tan^{-1}(tan\,3t)$	1
	Getting $\frac{dy}{dx} = \frac{3}{1+x^2}$	1
30	Let x be the height of the box and V the volume of the box. Writing $V = x(18-2x)^2$.	1
	Getting $\frac{dV}{dt}=(18-2x)^2-2x(18-x)$	1
	Getting $x = 4.5$	1
31	Writing $h = \frac{2-0}{n} = \frac{2}{n}$ **OR** writing the formula $\int_a^b f(x)\,dx$ $= (b-a).\lim_{n\to\infty}\frac{1}{n}\{f(a) + f(a+h) + ... + f(a+(n-1)h\}$ **OR** writing $\int_0^2 e^x\,dx$ $= (2-0).\lim_{n\to\infty}\frac{1}{n}\{e^0 + e^{2/n} + e^{4/n} + \cdots + e^{(2n-2)/n}\}$	1
	Getting $\int_0^2 e^x\,dx = (2-0).\lim_{n\to\infty}\frac{1}{n}\left\{\frac{e^{2n/n}-1}{e^{2/n}-1}\right\}$	1
	Getting the answer $e^2 - 1$	1
32	Getting $\frac{1}{2}\int\frac{\cos x+\sin x+\cos x-\sin x}{\cos x+\sin x}dx$	1
	Getting $\frac{1}{2}x + \frac{1}{2}\log\lvert\cos x + \sin x\rvert + c$.	1+1
33	Finding points of intersection (0,0) and (5,5)	1
	Writing area= $\int_0^5\sqrt{5x}\,.dx - \int_0^5 x.dx$	1
	Getting the answer $= \frac{50}{3} - \frac{25}{2} = \frac{25}{6}\ sq.units$	1
34	Writing $\frac{dp}{dt}=\frac{5}{100}p$	1

	Writing $\int \frac{1}{p}\, dp = \frac{1}{20}\int dt$	1
	Getting $p = c.e^{t/20}$.	1
35	Knowing $\lvert\vec{a}+\vec{b}+\vec{c}\rvert^2 = \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + \lvert\vec{c}\rvert^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a}$ **OR** writing $\lvert\vec{a}+\vec{b}+\vec{c}\rvert = 0,\ \lvert\vec{a}\rvert = 1,\ \lvert\vec{b}\rvert = 1,\ \lvert\vec{c}\rvert = 1$.	1
	Writing $0 = 1+1+1+2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$.	1
	Getting $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{3}{2}$.	1
36	Writing :Let Pdivide the line joining the points A and B having the position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m : n$. (**OR** drawing the figure) Writing $\overrightarrow{AP} = \frac{m}{n}\overrightarrow{PB}$ **OR** $n\overrightarrow{AP} = m\overrightarrow{PB}$.	1
	Getting $n(\overrightarrow{OP} - \vec{a}) = m(\vec{b} - \overrightarrow{OP})$.	1
	Getting $\overrightarrow{OP} = \frac{m\vec{b} + n\vec{a}}{m+n}$.	1
37	Writing $\vec{a}_1 = \hat{\imath} + 2\hat{\jmath} - 4\hat{k},\ \vec{a}_2 = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ **OR** Writing the formula to find the distance $= \left\lvert\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}}{\lvert\vec{b}\rvert}\right\rvert$. **OR** getting $\vec{a}_2 - \vec{a}_1 = 2\hat{\imath} + \hat{\jmath} - \hat{k}$	1
	Finding $(\vec{a}_2 - \vec{a}_1)\times\vec{b} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 1 & -1\\ 2 & 3 & 6\end{vmatrix} = 9\hat{\imath} - 14\hat{\jmath} + 4\hat{k}$.	1
	Getting the distance $= \frac{\sqrt{293}}{7}$ units.	1
38	Writing: $\mathrm{P(E_1)} = \frac{1}{2}, \mathrm{P(E_2)} = \frac{1}{2}$	1
	Writing: $\mathrm{P}(\mathrm{A}\mid\mathrm{E_1}) = \frac{1}{2}$ and $\mathrm{P}(\mathrm{A}\mid\mathrm{E_2}) = \frac{1}{4}$	1
	Getting: $\mathrm{P}(\mathrm{E_1}\mid\mathrm{A}) = \frac{\mathrm{P(E_1)P(A\mid E_1)}}{\mathrm{P(E_1)P(A\mid E_1)+P(E_2)P(A\mid E_2)}} = \frac{2}{3}$	1
39	Defining $g : Y \to N,\ g(y) = \frac{y-3}{4} y \in Y$ **OR** defining $g : Y \to N,\ g(x) = \frac{x-3}{4},\ x \in Y$ **OR** writing $y = 4x + 3 \Rightarrow x = \frac{y-3}{4}$.	1
	Getting $gof(x) = g(4x+3) = x$.	1
	Stating $gof = I_N$.	1
	Getting $fog(y) = f\left(\frac{y-3}{4}\right) = y,\ y \in Y$ **OR** $fog(x) = f\left(\frac{x-3}{4}\right) = x,\ x \in Y$ and stating $fog = I_Y$.	1

	Writing $f^{-1}(x) = \frac{x-3}{4}$ **OR** $f^{-1} = \frac{x-3}{4}$.	1
40	Finding : AB	1
	Finding : $(AB)'$	1
	Finding : A' and B'	1
	Finding : $B'A'$	1
	Conclusion.	1
41	Finding: $\|A\| = -1$	1
	Finding: $adjA = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ Any four cofactors are correct award 1 mark.	2
	Getting : $A^{-1} = \frac{adjA}{\|A\|} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$	1
	Finding : $X = A^{-1}B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ Therefore x = 1, y = 2 and z =3.	1
42	Getting $\frac{dy}{dx} = 2\tan^{-1}x \frac{d}{dx}\tan^{-1}x$.	1
	Getting $\frac{dy}{dx} = 2\tan^{-1}x \frac{1}{1+x^2}$.	1
	Writing $(1+x^2)\frac{dy}{dx} = 2\tan^{-1}x$	1
	Differentiating $(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}2x = 2\frac{1}{1+x^2}$	1
	Getting $(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$.	1
43	Writing $\frac{dy}{dt} = 8.\frac{dx}{dt}$	1
	Getting $6.\frac{dy}{dt} = 3x^2.\frac{dx}{dt}$	1
	Getting $x = \pm 4$.	1
	Finding $y = 11$ and $y = \frac{-31}{3}$	1
	Writing the points $(4, 11)$, $\left(-4, \frac{-31}{3}\right)$	1
44	Substituting $x = a\tan\theta$ and writing $dx = a\sec^2\theta\, d\theta$	1
	Getting $\int \sec\theta\, d\theta$	1
	Writing integral $= \log\|\sec\theta + \tan\theta\| + c$	1
	Getting $\log\left\|x + \sqrt{x^2+a^2}\right\| + c$	1
	Getting $\int \frac{1}{\sqrt{x^2+2x+2}}dx = \log\left\|x+1+\sqrt{x^2+2x+2}\right\| + c$	1

45	 Drawing the figure **OR** stating: Ellipse is a symmetrical closed curve centered at the origin. Hence area of the ellipse is 4 times the area of the region in the first quadrant.	1
	Writing area of the ellipse $= 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx$	1
	Knowing $\int \sqrt{a^2 - x^2}\,.dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$ **OR** putting $x = a.\sin t$ and $dx = a.\cos t.dt$	1
	Getting area $= \pi ab$ sq. units	1
	Getting area of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1 = 12\pi$ sq. units.	1
46	Stating: The given differential equation is a linear differential equation **OR** $P = \cot x$ and $Q = 4x.cosecx$	1
	Getting I. F. $= e^{\int \cot x dx} = sinx$	1
	Getting $y.sinx = \int 4x \operatorname{cosec} x.sinx.dx + C$	1
	Getting $y.sinx = 2x^2 + C$	1
	Taking $x = \frac{\pi}{2}$, $y = 0$ and getting $C = -\frac{\pi^2}{2}$	1
47	 Drawing figure with explanation	1
	Concluding $\overrightarrow{AP} = \vec{r} - \vec{a}$.	1
	Getting $\vec{r} = \vec{a} + \lambda\vec{b}$.	1
	Writing $P \equiv (x, y, z)$, $A \equiv (x_1, y_1, z_1)$ and $\vec{b} = (a, b, c)$, $\vec{r} - \vec{a} = \lambda\vec{b} \Rightarrow (x - x_1, y - y_1, z - z_1) = \lambda(a, b, c)$.	1
	Getting $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \lambda$.	1
48	Writing: $n = 50$, $p = \frac{1}{100}$, $q = \frac{99}{100}$	1
	Writing: $P(x = x) = {}^{n}C_x q^{n-x} p^x$, $n = 0,1,2,\cdots,50$.	1

	Writing: $P(x \geq 1) = 1 - P(x = 0)$	1
	Getting: $P(x \geq 1) = 1 - \left(\frac{99}{100}\right)^{50}$	1
	Getting: $P(x = 1) = \frac{1}{2}\left(\frac{99}{100}\right)^{49}$	1
49(a)	Writing $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_a^{2a} f(x)dx.$	1
	Substituting $x = 2a - t,\ dx = -dt.$	1
	Getting $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a - t)dt.$	1
	Getting $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx.$	1
	Proving $\int_0^{\pi} \lvert\cos x\rvert\, dx = 2\int_0^{\pi/2} \lvert\cos x\rvert\, dx.$	1
	Getting the answer = 2 .	1
49(b)	Stating LHL = RHL at x = 2 and x = 10.	1
	Getting: $5 = 2a + b$	1
	Getting $21 = 10a + b$	1
	Solving to get a = 2 and b = 1.	1
50(a)	Writing: To minimize $z = 4x + 6y$	1
	Writing: constraints $3x + 6y \geq 80$ $4x + 3y \geq 100$ $x \geq 0, y \geq 0$	1
	Drawing graph and identifying the feasible region	2
	Writing: corner point Getting corresponding value of z at each corner point	1
	Getting minimum value of $Z = 104$ at $x = 24,\ y = \frac{4}{3}$	1
50(b)	Getting $LHS = \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix}$	1
	Getting $= (1 + x + x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$	1
	Getting $= (1 + x + x^2)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x & x-x^2 \\ 0 & x^2-1 & 1-x \end{vmatrix}$	1
	Getting $(1 - x^3)^2$	1