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Question Paper Code : 96351

M.C.A. DEGREE EXAMINATION, FEBRUARY/MARCH 2014.

First Semester

DMC 7101 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulations 2013)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Write the predicate "x is the father of the mother of y".

2. Find the truth table for $P \to Q$

3. There are n stations on a Railway. Prove that a train can stop at three of these in $\frac{1}{6}(n-2)(n-3)(n-4)$ ways, no two of the stopping stations being consecutive.

4. Form the recurrence relation, given $f_n = 3.5^n, n \geq 0$.

5. Let $(G,*)$ be a group. Then show that for any $a \in G$, the inverse of a is unique.

6. Define normal subgroup.

7. Draw the Harse diagram of a set of all divisors of 24.

8. Write the dual of the following statements.

   (a) $(0.a)+(b.1)=b$.

   (b) $(a+0)+(1+a')=1$.

9. Obtain the grammar for Language, $L=\{a^n b^n / n \geq 0\}$.

10. Construct on NFA with 3 states that accepts the language $0^*1^*0^*0^*$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Construct the truth table for $(p \vee q) \vee (p \wedge s)$. (8)

(ii) Obtain the principal conjunctive normal form for $(p \wedge q) \vee (\sim p \wedge q \wedge r)$. (8)

Or

(b) (i) Is the conclusion $(\exists z)\, p(z)$ valid from the premises $(\forall x)[p(x) \rightarrow \phi(x)]$, $(\exists y)\, p(y)$. (8)

(ii) Show that $\sim (p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$. (8)

12. (a) (i) Using generating function, solve the differences equation $y_{n+2} - 6y_{n+1} + 8y_n = 0$, $y_0 = 1, y_1 = 4$. (8)

(ii) Use mathematical induction to show that 3 divides $n^3 + 2n$ whenever n is a non-negative integer. (8)

Or

(b) (i) Find the general solution of $f(n) - 3f(n-1) - 4f(n-2) = 4^n$. (8)

(ii) In a certain village, there are 3 sports clubs, the rugby club, the soccer club and the cricket club. Every one who belongs to the cricket club also belongs to the soccer club or rugby club (or both). The following additional information is known.

42 people belong to the soccer club;

45 people belong to the rugby club;

7 people belong to the both the soccer and rugby clubs;

11 people belong to the both the soccer and cricket clubs;

28 people belong to both rugby and cricket clubs; twice as many people belong only to the soccer club as belong only to the rugby club. Find the number of people in the village who belong to

(1) All three clubs

(2) The cricket club

(3) Only the soccer club. (8)

13. (a) (i) List all the elements of the permutation group $(D_3, 0)$ and form the composition table. (8)

(ii) If $f : (G \rightarrow G_1)$ is a group homomorphism, prove that f is one-to-one if and only if ker $f = \{e\}$. (8)

Or

(b) (i) State and prove Lagrange's theorem. (12)

(ii) Define ring. (4)

14. (a) (i) Let L be a set of positive integers. Show that the "divide" relation is a partial ordered relation. (6)

(ii) In a Lattice, show that

$(a * b) \oplus (b * c) \oplus (c * a) \leq (a \oplus b) * (b \oplus c) * (c \oplus a)$. (10)

Or

(b) (i) Show that De Morgan's law $(a \wedge b)' = a' \vee b'$ holds in a complemented distributive lattice. (6)

(ii) Describe the truth table for the given K–Map. (10)

CD \ AB	00	01	11	10
00	0	4	12 1	8
01	1	5 1	13	9 1
11	3 1	7	15 1	11
10	2	6	14	10

15. (a) (i) Show that the set $L = \{a^{i^2} / i \geq 1\}$ is not regular. (8)

(ii) What are the applications of finite state machines? Construct a DFA with reduced states equivalent to the regular expression $(ab + (b + aa))b * a$. (8)

Or

(b) (i) Prove the theorem : For every context-free grammar G_1, there is an equivalent grammar G_2 in chomsky normal form. Find a grammar in Chomsky Normal form equivalent to $S \to cCdD,\ C \to cC / c,\ D \to dD/d$. (8)

(ii) Construct a DFA equivalent to an NFA whose transition table is defined by (8)

state	a	b
(initial) q_0	$\{q_1, q_3\}$	$\{q_2, q_3\}$
q_1	q_1	q_3
q_2	q_3	q_2
(final) q_3	ϕ	ϕ