Code-μ

Aakash
Medical | IIT-JEE | Foundations
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ANSWERS & HINTS
for
WBJEEM - 2014
SUB : MATHEMATICS

CATEGORY - I

Q.1 to Q.60 carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of 1/3 mark.

1. Let the equation of an ellipse be $\frac{x^2}{144}+\frac{y^2}{25}=1$. Then the radius of the circle with centre $\left(0,\sqrt{2}\right)$ and passing through the foci of the ellipse is

(A) 9 (B) 7 (C) 11 (D) 5

Ans : (C)

Hints : $a^2 = 144$, $b^2 = 25$ $P\left(0,\sqrt{2}\right)$, S(ae, 0)

Radius = PS, $S\left(\sqrt{119},0\right)$ PS = 11

2. If y = 4x + 3 is parallel to a tangent to the parabola $y^2 = 12x$, then its distance from the normal parallel to the given line is

(A) $\frac{213}{\sqrt{17}}$ (B) $\frac{219}{\sqrt{17}}$ (C) $\frac{211}{\sqrt{17}}$ (D) $\frac{210}{\sqrt{17}}$

Ans : (B)

Hints : m = slope of line = 4 ; a = 3

$y = mx - 2am - am^3$ (Normal)

y = 4x – 216.. Distance = $\frac{219}{\sqrt{17}}$

3. In a ΔABC, tan*A* and tan*B* are the roots of $pq(x^2 + 1) = r^2x$. Then ΔABC is

(A) a right angled triangle (B) an acute angled triangle

(C) an obtuse angled triangle (D) an equilateral triangle

Ans : (A)

Hints : $pqx^2 - r^2x + pq = 0$

tanA tanB = 1 so tan(A+B) is undefined $\therefore \angle C = \pi / 2$

4. Let the number of elements of the sets *A* and *B* be p and q respectively. Then the number of relations from the set *A* to the set *B* is

(A) 2^{p+q} (B) 2^{pq} (C) p + q (D) pq

Ans : (B)

Hints : 0(A) = p 0(B) = q ; 0(A × B) = pq

5. The function $f(x)=\dfrac{\tan\left\{\pi[x-\frac{\pi}{2}]\right\}}{2+[x]^2}$, where [x] denotes the greatest integer $\leq$ x, is

(A) continuous for all values of x
(B) discontinuous at $x=\dfrac{\pi}{2}$
(C) not differentiable for some values of x
(D) discontinuous at x = –2

Ans : (A)

Hints : $f(x) = 0\ \forall x \in R$

6. Let z_1, z_2 be two fixed complex numbers in the Argand plane and z be an arbitrary point satisfying $|z - z_1| + |z - z_2| = 2|z_1 - z_2|$. Then the locus of z will be

(A) an ellipse
(B) a straight line joining z_1 and z_2
(C) a parabola
(D) a bisector of the line segment joining z_1 and z_2

Ans : (A)

Hints : Possibility of ellipse P(z), $S_1(z_1)$, $S_2(z_2)$

$PS_1 + PS_2 = 2a = 2S_1S_2 = 4ae$

$\therefore$ So $e = \dfrac{1}{2}$ it is an ellipse

7. Let $S=\dfrac{2}{1}\,{}^nC_0+\dfrac{2^2}{2}\,{}^nC_1+\dfrac{2^3}{3}\,{}^nC_2+\ldots\ldots+\dfrac{2^{n+1}}{n+1}\,{}^nC_n$. Then *S* equals

(A) $\dfrac{2^{n+1}-1}{n+1}$
(B) $\dfrac{3^{n+1}-1}{n+1}$
(C) $\dfrac{3^n-1}{n}$
(D) $\dfrac{2^n-1}{n}$

Ans : (B)

Hints : $P = \sum\dfrac{2^{r+1}}{r+1}\,{}^nC_r$; $S_0 = \sum {}^nC_r.x^r$, $\int_0^2 S_0 = \int_0^2 \sum {}^nC_r.x^r$

$\therefore\ \dfrac{3^{n+1}-1}{n+1}$

8. Out of 7 consonants and 4 vowels, the number of words (not necessarily meaningful) that can be made, each consisting of 3 consonants and 2 vowels, is

(A) 24800
(B) 25100
(C) 25200
(D) 25400

Ans : (C)

Hints : ${}^7C_3 \times {}^4C_2 \times 5!$

9. The remainder obtained when 1! + 2! + 3! + ... + 11! is divided by 12 is

(A) 9
(B) 8
(C) 7
(D) 6

Ans : (A)

Hints : 12 divides 4!, 5! etc .

Remainder = 1 + 2 + 6 = 9

10. Let S denote the sum of the infinite series $1+\frac{8}{2!}+\frac{21}{3!}+\frac{40}{4!}+\frac{65}{5!}+......$. Then

(A) S < 8 (B) S > 12 (C) 8 < S < 12 (D) S = 8

Ans : (C)

Hints : n^{th} term of 1, 8, 21, 40, 65, ...= n(3n – 2)

$$\sum_{r=1}^{\infty}\frac{r.(3r-2)}{r!}=3e+3e-2e=4e$$

11. For every real number x, let $f(x) = \frac{x}{1!}+\frac{3}{2!}x^2+\frac{7}{3!}x^3+\frac{15}{4!}x^4+...$ .Then the equation $f(x) = 0$ has

(A) no real solution (B) exactly one real solution

(C) exactly two real solutions (D) inifinite number of real solutions

Ans : (B)

Hints : x = 0 is a solution

$$\sum_{r=1}^{\infty}\frac{x^r}{r!}(2^r-1)=e^{2x}-e^x$$

12. The coefficient of x^3 in the infinite series expansion of $\frac{2}{(1-x)(2-x)}$, for $|x| < 1$, is

(A) –1/16 (B) 15/8 (C) –1/8 (D) 15/16

Ans : (B)

Hints : Exp = $2(1-x)^{-1}\frac{1}{2}\left(1-\frac{x}{2}\right)^{-1}$ = $(1 + x + x^2 ......)\left(1+\frac{x}{2}+\frac{x^2}{2^2}+....\right)$

Coefficient = 15/8

13. If α, β are the roots of the quadratic equation $x^2 + px + q = 0$, then the values of $\alpha^3 + \beta^3$ and $\alpha^4 + \alpha^2\beta^2 + \beta^4$ are respectively

(A) $3pq - p^3$ and $p^4 - 3p^2q + 3q^2$ (B) $-p(3q - p^2)$ and $(p^2 - q)(p^2 + 3q)$

(C) $pq - 4$ and $p^4 - q^4$ (D) $3pq - p^3$ and $(p^2 - q)(p^2 - 3q)$

Ans : (D)

Hints : $\alpha^4 + \beta^4 + \alpha^2\beta^2$ $\quad$ $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= (\alpha^2 + \beta^2)^2 - \alpha^2\beta^2$ $\quad$ $= - p^3 + 3pq$

$= (p^2 - 2q)^2 - q^2$

14. A fair six-faced die is rolled 12 times. The probability that each face turns up twice is equal to

(A) $\frac{12!}{6!6!6^{12}}$ (B) $\frac{2^{12}}{2^6 6^{12}}$ (C) $\frac{12!}{2^6 6^{12}}$ (D) $\frac{12!}{6^2 6^{12}}$

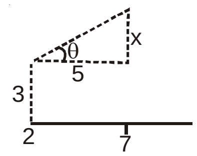
Ans : (C)

Hints : ${}^{12}C_2\,{}^{10}C_2..................{}^{2}C_2.\frac{1}{6^{12}}$

15. Let $f(x)$ be a differentiable function in [2, 7]. If $f(2) = 3$ and $f'(x) \le 5$ for all x in (2, 7), then the maximum possible value of $f(x)$ at x = 7 is

(A) 7 (B) 15 (C) 28 (D) 14

Ans : (C)

Hints : 

$\frac{x}{5} = \tan\theta = 5 \Rightarrow x = 25$

So $f(7) = 3 + 25 = 28$

16. The value of $\tan\frac{\pi}{5} + 2\tan\frac{2\pi}{5} + 4\cot\frac{4\pi}{5}$ is

(A) $\cot\frac{\pi}{5}$ (B) $\cot\frac{2\pi}{5}$ (C) $\cot\frac{4\pi}{5}$ (D) $\cot\frac{3\pi}{5}$

Ans : (A)

Hints : Add, subtract $\cot\frac{\pi}{5}$

17. Let $\mathbb{R}$ be the set of all real numbers and $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 3x^2 + 1$. Then the set $f^{-1}([1, 6])$ is

(A) $\left\{-\sqrt{\frac{5}{3}}, 0, \sqrt{\frac{5}{3}}\right\}$ (B) $\left[-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right]$ (C) $\left[-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right]$ (D) $\left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

Ans : (B)

Hints : $f'(x) = 6x > 0$ if $x > 0$, < 0 if $x < 0$

$f(0) = 1$ $f(\alpha) = 6$. So $\alpha = \pm\sqrt{\frac{5}{3}}$

[Note : $f(x)$ is is invertible either for x > 0 or for x < 0 so the right answer should be either $\left[-\sqrt{\frac{5}{3}}, 0\right]$ or $\left[0, \sqrt{\frac{5}{3}}\right]$

18. The area of the region bounded by the curves $y = x^2$ and $x = y^2$ is

(A) 1/3 (B) 1/2 (C) 1/4 (D) 3

Ans : (A)

Hints : $\int_0^1\left(\sqrt{x} - x^2\right)dx = \frac{1}{3}$

19. The point on the parabola $y^2 = 64x$ which is nearest to the line $4x + 3y + 35 = 0$ has coordinates

(A) (9, –24) (B) (1, 81) (C) (4, –16) (D) (–9, –24)

Ans : (A)

Hints : Normal at $P(am^2, -2am)$ has slope m. $a = 16$, $m = \frac{3}{4}$

20. The equation of the common tangent with positive slope to the parabola $y^2 = 8\sqrt{3}x$ and the hyperbola $4x^2 - y^2 = 4$ is

(A) $y = \sqrt{6}x + \sqrt{2}$ (B) $y = \sqrt{6}x - \sqrt{2}$ (C) $y = \sqrt{3}x + \sqrt{2}$ (D) $y = \sqrt{3}x - \sqrt{2}$

Ans : (A)

Hints : $y^2 = 4ax$, $a = 2\sqrt{3}$

$y = mx + \frac{a}{m}$; $m > 0$, $\left(\frac{a}{m}\right)^2 = 1.m^2 - 4$, $m^2 = 6$

21. Let p,q be real numbers. If α is the root of $x^2+3p^2x+5q^2 = 0$, β is a root of $x^2+9p^2x+15q^2 = 0$ and $0<\alpha<\beta$, then the equation $x^2+6p^2x+10q^2 = 0$ has a root γ that always satisfies

(A) $\gamma = \alpha/4+\beta$
(B) $\beta < \gamma$
(C) $\gamma = \alpha/2+\beta$
(D) $\alpha < \gamma < \beta$

Ans : (D)

Hints : Let, $f(x) = x^2+6p^2x+10q^2$

$f(\alpha) = \alpha^2+6p^2\alpha+10q^2 = (\alpha^2+3p^2\alpha+5q^2) + (3p^2\alpha+5q^2) = 0 + 3p^2\alpha+5q^2 > 0$

Again, $f(\beta) = \beta^2+6p^2\beta+10q^2 = (\beta^2+9p^2\beta+15q^2) - (3p^2\beta+5q^2) = 0 - (3p^2\beta + 5q^2) < 0$

So, there is one root γ such that, $\alpha < \gamma < \beta$

22. The value of the sum $({}^nC_1)^2 + ({}^nC_2)^2 + ({}^nC_3)^2 + ... + ({}^nC_n)^2$ is

(A) $({}^{2n}C_n)^2$
(B) ${}^{2n}C_n$
(C) ${}^{2n}C_n + 1$
(D) ${}^{2n}C_n - 1$

Ans : (D)

Hints : $(1+x)^n = C_0 + C_1x + C_2x^2 + \text{---------} + C_nx^n$, $(x+1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} + \text{------} + C_n$

So, coefficient of x^n in $[(1+x)^n \times (x+1)^n]$ i.e. $(1+x)^{2n}$ is $(C_0^2 + C_1^2 + \text{-------} + C_n^2)$, which is ${}^{2n}C_n$

So, ${}^{2n}C_n = C_0^2 + C_1^2 + C_2^2 + \text{-------} + C_n^2$, $\Rightarrow ({}^nC_1)^2 + ({}^nC_2)^2 + ({}^nC_3)^2 + \text{--------} + ({}^nC_n)^2 = {}^{2n}C_n - C_0^2 = {}^{2n}C_n - 1$

23. Ram is visiting a friend. Ram knows that his friend has 2 children and 1 of them is a boy. Assuming that a child is equally likely to be a boy or a girl, then the probability that the other child is a girl, is

(A) 1/2
(B) 1/3
(C) 2/3
(D) 7/10

Ans : (C)

Hints : Event that at least one of them is a boy $\rightarrow$ A, Event that other is girl $\rightarrow$ B, So, probability required P(B/A)

$= \frac{P(B \cap A)}{P(A)}$, Now, total cases are 3 (BG, BB, GG) $\therefore \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{3}}{\frac{2}{3}} = 1/2$

(As, $B \cap A = \{BG\}$ and $A = \{BG,BB\}$)

24. Let $n \geq 2$ be an integer, $A = \begin{pmatrix} \cos(2\pi/n) & \sin(2\pi/n) & 0 \\ -\sin(2\pi/n) & \cos(2\pi/n) & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and I is the identity matrix of order 3. Then

(A) $A^n = I$ and $A^{n-1} \neq I$
(B) $A^m \neq I$ for any positive integer m
(C) A is not invertible
(D) $A^m = 0$ for a positive integer m

Ans : (A)

Hints : $A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\Rightarrow A^n = \begin{pmatrix} \cos n\theta & \sin n\theta & 0 \\ -\sin n\theta & \cos n\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$, So, here, $A^n = \begin{pmatrix} \cos 2\pi & \sin 2\pi & 0 \\ -\sin 2\pi & \cos 2\pi & 0 \\ 0 & 0 & 1 \end{pmatrix}$, =

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, and $A^{n-1} \neq I$

25. Let I denote the 3 × 3 identity matrix and P be a matrix obtained by rearranging the columns of I. Then
(A) There are six distinct choices for P and det(P) = 1
(B) There are six distinct choices for P and det(P) = ±1
(C) There are more than one choices for P and some of them are not invertible
(D) There are more than one choices for P and $P^{-1} = I$ in each choice
Ans : (B)

Hints : $I_{3\times3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, 3 different columns can be arranged in, 3! i.e. 6 ways, In each case, if there are even number of interchanges of columns, determinant remains 1 and for odd number of interchanges, determinant takes the negative value i.e. –1

26. The sum of the series $\sum_{n=1}^{\infty} \sin\left(\frac{n!\pi}{720}\right)$ is

(A) $\sin\left(\frac{\pi}{180}\right)+\sin\left(\frac{\pi}{360}\right)+\sin\left(\frac{\pi}{540}\right)$
(B) $\sin\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{30}\right)+\sin\left(\frac{\pi}{120}\right)+\sin\left(\frac{\pi}{360}\right)$
(C) $\sin\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{30}\right)+\sin\left(\frac{\pi}{120}\right)+\sin\left(\frac{\pi}{360}\right)+\sin\left(\frac{\pi}{720}\right)$
(D) $\sin\left(\frac{\pi}{180}\right)+\sin\left(\frac{\pi}{360}\right)$

Ans : (C)

Hints : $\sum_{n=1}^{\infty} \sin\left(\frac{n!\pi}{720}\right), = \left(\sin\frac{1!\pi}{720}+\sin\frac{2!\pi}{720}+-----+\frac{\sin 5!\pi}{720}\right)+\sum_{n=6}^{\infty} \sin\frac{n!\pi}{720}$

$= \sin\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{30}\right)+\sin\left(\frac{\pi}{120}\right)+\sin\left(\frac{\pi}{360}\right)+\sin\left(\frac{\pi}{720}\right) + \sum_{n=6}^{\infty} \sin k_n \pi$, where $k_n \in \mathbb{N}$, so $\sin k_n \pi = 0 \ \forall\ k_n$

27. Let α,β be the roots of $x^2-x-1 = 0$ and $S_n = \alpha^n+\beta^n$, for all integers $n \ge 1$. Then for every integer $n\ge2$
(A) $S_n+S_{n-1} = S_{n+1}$
(B) $S_n-S_{n-1} = S_{n+1}$
(C) $S_{n-1} = S_{n+1}$
(D) $S_n+S_{n-1} = 2S_{n+1}$
Ans : (A)
Hints : $\alpha+\beta =1$, $S_n + S_{n-1}$, $= (\alpha^n+\alpha^{n-1})+(\beta^n+\beta^{n-1})$, $= \alpha^{n-1}(\alpha+1) + \beta^{n-1}(\beta+1)$, now since $\alpha^2-\alpha-1 = 0$ & $\beta^2-\beta-1=0$ $= \alpha^{n-1}.\alpha^2+ \beta^{n-1}.\beta^2 = \alpha^{n+1}+\beta^{n+1} = S_{n+1}$

28. In a ΔABC, a,b,c are the sides of the triangle opposie to the angles A,B,C respectively. Then the value of $a^3\sin(B-C) + b^3 \sin(C-A) + c^3\sin(A-B)$ is equqal to
(A) 0
(B) 1
(C) 3
(D) 2
Ans : (A)

29. In the Argand plane, the distinct roots of $1+z+z^3+z^4 = 0$ (z is a complex number) represent vertices of
(A) a square
(B) an equilateral triangle
(C) a rhombus
(D) a rectangle
Ans : (B)
Hints : $1+z+z^3+z^4 = 0, \Rightarrow (1+z)(1+z^3)= 0$, $z = -1, -1, -\omega, -\omega^2$, where ω is a cube root of unity, so, distinct roots are : $(-1,0)$, $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$, $\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$. Distance between each of them is $\sqrt{3}$. So, they form an equilateral triangle

30. The number of digits in 20^{301} (given $\log_{10}2 = 0.3010$) is
(A) 602
(B) 301
(C) 392
(D) 391
Ans : (C)
Hints : $\log 20^{301} = 301 \times \log 20 = 301 \times 1.3010 = 391.6010$, so, 392 digits

31. If $\sqrt{y}=\cos^{-1}x$, then it satisfies the differential equation $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=c$, where c is equal to

(A) 0 (B) 3 (C) 1 (D) 2

Ans : (D)

Hints : $\sqrt{y}=\cos^{-1}x \Rightarrow y=(\cos^{-1}x)^2$, $\therefore \frac{dy}{dx}=-\frac{2\cos^{-1}x}{\sqrt{1-x^2}}$, $\Rightarrow \frac{d^2y}{dx^2}=\frac{2-\frac{2x\cos^{-1}x}{\sqrt{1-x^2}}}{1-x^2}$, $=\frac{2+x\frac{dy}{dx}}{1-x^2}$, $\Rightarrow \frac{d^2y}{dx^2}(1-x^2)-x\frac{dy}{dx}=2 \therefore 2$

32. The integrating factor of the differential equation

$$(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x} \text{ is}$$

(A) $\tan^{-1}x$ (B) $1+x^2$ (C) $e^{\tan^{-1}x}$ (D) $\log_e(1+x^2)$

Ans : (C)

Hints : $\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{e^{\tan^{-1}}}{1+x}$, I.F $=e^{\int\frac{1}{1+x^2}dx}=e^{\tan^{-1}x}$

33. The solution of the equation

$$\log_{101}\log_7\left(\sqrt{x+7}+\sqrt{x}\right)=0 \text{ is}$$

(A) 3 (B) 7 (C) 9 (D) 49

Ans : (C)

Hints : $\log_{101}\log_7\left(\sqrt{x+7}+\sqrt{x}\right)=0$, $\Rightarrow \log_7\left(\sqrt{x+7}+\sqrt{x}\right)=1$, $\sqrt{x+7}+\sqrt{x}=7$, $\Rightarrow \sqrt{x+7}=7-\sqrt{x} \Rightarrow x+7=49-14\sqrt{x}+x$, $\Rightarrow \sqrt{x}=3$, $\Rightarrow x=9$

34. If α,β are the roots of $ax^2+bx+c=0$ $(a\neq0)$ and $\alpha+h$, $\beta+h$ are the roots of $px^2+qx+r=0$ $(p\neq0)$ then the ratio of the squares of their discriminants is

(A) $a^2:p^2$ (B) $a:p^2$ (C) $a^2:p$ (D) $a:2p$

Ans : (A)

Hints : $D_1=a^2(\alpha-\beta)^2$, $D_2=P^2(\alpha-\beta)^2$; $\frac{D_1}{D_2}=\frac{a^2}{p^2}$ [Note : Correct answer is $\frac{a^4}{p^4}$ for D_1^2/D_2^2]

35. Let $f(x)=2x^2+5x+1$. If we write f(x) as

$f(x)=a(x+1)(x-2)+b(x-2)(x-1)+c(x-1)(x+1)$ for real numbers a,b,c then

(A) there are infinite number of choices for a,b,c

(B) only one choice for a but infinite number of choices for b and c

(C) exactly one choice for each of a,b,c

(D) more than one but finite number of choices for a,b,c

Ans : (C)

Hints : $f(x)=(a+b+c)x^2+(-a-3b)x-2a+2b-c$, $a+b+c=2$, $-a-3b=5$, $-2a+2b-c=1$, $a=-4$, $b=-1/3$, $c=19/3$

36. Let $f(x)=x+1/2$. then the number of real values of x for which the three unequal terms f(x), f(2x), f(4x) are in H.P. is

(A) 1 (B) 0 (C) 3 (D) 2

Ans : (A)

Hints : $f(x) = x+\frac{1}{2} = \frac{2x+1}{2}$, $f(2x) = \frac{4x+1}{2}$, $f(4x) = \frac{8x+1}{2}$, f(x), f(2x), f(4x) are in H.P., So, $f(2x) = \frac{2f(x)f(4x)}{f(x)+f(4x)}$, $\Rightarrow$

$x=0,\ \frac{1}{4}$, at x= 0, terms are equal so only solution is $x=\frac{1}{4}$

37. The function $f(x)= x^2+bx+c$, where b and c real constants, describes

(A) one-to-one mapping (B) onto mapping

(C) not one-to-one but onto mapping (D) neither one-to-one nor onto mapping

Ans : (D)

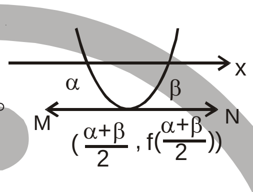
Hints : Upward parabola f(x) has a minimum value. So, it is not onto, also symmetric about its axis which is a straight line parallel to Y-axis, so it is not one-to-one

38. Suppose that the equation $f(x) = x^2+bx+c = 0$ has two distinct real roots α and β. The angle between the tangent to the curve y = f(x) at the point $\left(\frac{\alpha+\beta}{2}, f\left(\frac{\alpha+\beta}{2}\right)\right)$ and the positive direction of the x-axis is

(A) 0° (B) 30° (C) 60° (D) 90°

Ans : (A)

Hints : $f(x) = x^2+bx+c$ represents upward parabola which cuts x-axis at α and β. As the graph is symmetric, so, tangent at $\left(\frac{\alpha+\beta}{2}, f\frac{\alpha+\beta}{2}\right)$ parallel to x-axis. Hence, 0°



39. The solution of the differential equation $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\varphi\left(\frac{y^2}{x^2}\right)}{\varphi'\left(\frac{y^2}{x^2}\right)}\right]$ is (where c is a constant)

(A) $\varphi\left(\frac{y^2}{x^2}\right) = cx$ (B) $x\varphi\left(\frac{y^2}{x^2}\right) = c$ (C) $\varphi\left(\frac{y^2}{x^2}\right) = cx^2$ (D) $x^2\varphi\left(\frac{y^2}{x^2}\right) = c$

Ans : (C)

Hints : Let, $\frac{y}{x} = \nu$, $\Rightarrow y = \nu x \Rightarrow \frac{dy}{dx} = \nu + x\frac{d\nu}{dx}$ substituting, $\nu x\left(\nu + x\frac{d\nu}{dx}\right) = x\left(\nu^2 + \frac{\phi(\nu^2)}{\phi'(\nu^2)}\right)$, $\Rightarrow \int\frac{dx}{x} = \int\frac{\nu\phi'(\nu^2)}{\phi(\nu^2)}d\nu$,

[Let, $\phi(\nu^2)= z$, $\therefore 2\phi'(\nu^2)\nu\, d\nu = dz$], $\Rightarrow \frac{1}{2}\int\frac{dz}{z} = \ell nx$, $\Rightarrow \ell n\, z^{1/2} = \ell nx + k$, $\Rightarrow z = cx^2$, $\phi\left(\frac{y^2}{x^2}\right) = cx^2$

40. Let f(x) be a differentiable function and $f'(4) = 5$. Then $\lim\limits_{x\to 2}\frac{f(4)-f(x^2)}{x-2}$ equals

(A) 0 (B) 5

(C) 20 (D) –20

Ans : (D)

Hints : $\lim\limits_{x\to 2}\dfrac{f(4)-f(x^2)}{x-2}$, $\left(\dfrac{0}{0}\right.$ form, so using L′Hospital's rule), = $\lim\limits_{x\to 2}\dfrac{0-f'(x^2)\times 2x}{1}$, = –f′(4)×4 = –5×4 = –20

41. The value of $\lim\limits_{x\to 0}\dfrac{\int_0^{x^2}\cos(t^2)dt}{x\sin x}$ is

(A) 1 (B) –1 (C) 2 (D) $\log_e 2$

Ans : (A)

Hints : $\lim\limits_{x\to 0}\dfrac{\cos x^4\cdot 2x}{1\cdot\sin x+x\cos x}$

$$\lim_{x\to 0}\frac{2\cos x^4}{\frac{\sin x}{x}+\cos x}=\frac{2}{1+1}=1$$

42. The range of the function y = 3 sin $\left(\sqrt{\dfrac{\pi^2}{16}-x^2}\right)$ is

(A) $\left[0,\sqrt{3/2}\right]$ (B) [0, 1] (C) $\left[0,3/\sqrt{2}\right]$ (D) [0, ∞)

Ans : (C)

Hints : y = 3 sin $\sqrt{\dfrac{\pi^2}{16}-x^2}$

y_{max} = 3sin π/4 = $3/\sqrt{2}$, y_{min} = 0

43. There is a group of 265 persons who like either singing or dancing or painting. In this group 200 like singing, 110 like dancing and 55 like painting. If 60 persons like both singing and dancing, 30 like both singing and painting and 10 like all three activities, then the number of persons who like *only* dancing and painting is

(A) 10 (B) 20 (C) 30 (D) 40

Ans : (A)

Hints : n(S ∪ P ∪ D) = 265
n(S) = 200
n(D) = 110
n(P) = 55
n(S ∩ D) = 60
n(S ∩ P) = 30
n(S ∩ D ∩ P) = 10
n(S ∪ P ∪ D) = n(S) + n(D) + n(P) – n(S ∩ D) – n(D ∩ P) – n(P ∩ S) + n(S ∩ D ∩ P)
265 = 200 + 110 + 55 – 60 – 30 – n(P ∩ D) + 10
n(P ∩ D) = 285 – 265 = 20
n(P ∩ D) – n(P ∩ D ∩ S) = 20 – 10 = 10

44. The curve y = (cos x + y)$^{1/2}$ satisfies the differential equation

(A) $(2y-1)\dfrac{d^2y}{dx^2}+2\left(\dfrac{dy}{dx}\right)^2+\cos x=0$ (B) $\dfrac{d^2y}{dx^2}-2y\left(\dfrac{dy}{dx}\right)^2+\cos x=0$

(C) $(2y-1)\dfrac{d^2y}{dx^2}-2\left(\dfrac{dy}{dx}\right)^2+\cos x=0$ (D) $(2y-1)\dfrac{d^2y}{dx^2}-\left(\dfrac{dy}{dx}\right)^2+\cos x=0$

Ans : (A)

Hints : $y = (\cos x + y)^{1/2}$

$y^2 = \cos x + y$

$$2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$2\left(\frac{dy}{dx}\right)^2 + 2.y.\frac{d^2y}{dx^2} = -\cos x + \frac{d^2y}{dx^2},\ (2y-1)\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + \cos x = 0$$

45. Suppose that z_1, z_2, z_3 are three vertices of an equilateral triangle in the Argand plane. Let $\alpha = \frac{1}{2}\left(\sqrt{3}+i\right)$ and β be a non-zero complex number. The points $\alpha z_1 + \beta$, $\alpha z_2 + \beta$, $\alpha z_3 + \beta$ will be

(A) The vertices of an equilateral triangle
(B) The vertices of an isosceles triangle
(C) Collinear
(D) The vertices of an scalene triangle

Ans : (A)

Hints : $\frac{1}{(\alpha z_1+\beta)-(\alpha z_2+\beta)} + \frac{1}{(\alpha z_2+\beta)-(\alpha z_3+\beta)} + \frac{1}{(\alpha z_3+\beta)-(\alpha z_1+\beta)}$

$$= \frac{1}{\alpha(z_1-z_2)} + \frac{1}{\alpha(z_2-z_3)} + \frac{1}{\alpha(z_3-z_1)}$$

$$= \frac{1}{\alpha}\left[\frac{1}{(z_1-z_2)} + \frac{1}{(z_2-z_3)} + \frac{1}{(z_3-z_1)}\right] = 0$$

Hence, $\alpha z_1 + \beta$, $\alpha z_2 + \beta$, $\alpha z_3 + \beta$ are vertices of equilateral triangle.

46. If $\lim\limits_{x\to 0}\frac{2a\sin x - \sin 2x}{\tan^3 x}$ exists and is equal to 1, then the value of a is

(A) 2 (B) 1 (C) 0 (D) –1

Ans : (B)

Hints : $\lim\limits_{x\to 0}\frac{2a(x-\frac{x^3}{\underline{|3}}) - (2x - \frac{8x^3}{\underline{|3}}) + ...}{x^3 + ...}$

$$\lim_{x\to 0}\frac{2(a-1)x + \left(\frac{4}{3}-\frac{a}{3}\right)x^3 + ...}{x^3 + ...}$$

$\Rightarrow a - 1 = 0 \Rightarrow a = 1$

47. If $f(x) = \begin{cases} 2x^2+1, & x \le 1 \\ 4x^3-1, & x > 1 \end{cases}$, then $\int_0^2 f(x)dx$ is

(A) 47/3 (B) 50/3 (C) 1/3 (D) 47/2

Ans : (A)

Hints : $\int_0^1 f(x)dx + \int_1^2 f(x)dx$

$$= \int_0^1 (2x^2+1)dx + \int_1^2 (4x^3-1)dx$$

$$= \left(2\cdot\frac{x^3}{3} + x\right)_0^1 + \left(\cancel{4}\cdot\frac{x^4}{\cancel{4}} - 4\right)_1^2 = 5/3 + 14 = 47/3$$

48. The value of $|z|^2 + |z-3|^2 + |z-i|^2$ is minimum when z equals

(A) $2-\frac{2}{3}i$ (B) $45+3i$ (C) $1+\frac{i}{3}$ (D) $1-\frac{i}{3}$

Ans : (C)

Hints : $|z|^2 + |z-3|^2 + |z-i|^2 = x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-1)^2$

$$= 3x^2 + 3y^2 - 6x - 2y + 10$$

$$= 3\left[x^2 + y^2 - 2x - 2.y.\frac{1}{3}\right] + 10$$

$$= 3\left|z-\left(1+\frac{i}{3}\right)\right|^2 + \frac{20}{3}$$

49. The number of solution(s) of the equation $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$ is/are

(A) 2 (B) 0 (C) 3 (D) 1

Ans : (B)

Hints : $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$

Squaring,

$x + 1 + x - 1 - 2\sqrt{x^2-1} = 4x - 1$

$1 - 2x = 2\sqrt{x^2-1}$

$1 + 4x^2 - 4x = 4x^2 - 4$

$4x = 5$

$x = 5/4$

Which does not satisfies the equation.

Hence, no solution

50. The values of λ for which the curve $(7x+5)^2 + (7y+3)^2 = \lambda^2(4x+3y-24)^2$ represents a parabola is

(A) $\pm\frac{6}{5}$ (B) $\pm\frac{7}{5}$ (C) $\pm\frac{1}{5}$ (D) $\pm\frac{2}{5}$

Ans : (B)

Hints : $49\left[(x+5/7)^2 + (y+3/7)^2\right] = 25\lambda^2\left(\frac{4x+3y-24}{5}\right)^2$

$$\Rightarrow \frac{25\lambda^2}{49} = 1$$

$$\lambda^2 = \frac{49}{25}$$

$\lambda = \pm 7/5$

51. If $\sin^{-1}\left(\frac{x}{13}\right) + \operatorname{cosec}^{-1}\left(\frac{13}{12}\right) = \frac{\pi}{2}$, then the value of x is

(A) 5 (B) 4 (C) 12 (D) 11

Ans : (A)

Hints : $\sin^{-1}\left(\frac{x}{13}\right) = \frac{\pi}{2} - \operatorname{cosec}^{-1}\left(\frac{13}{12}\right)$

$= \sec^{-1}\left(\frac{13}{12}\right)$

$= \cos^{-1}\frac{12}{13}$

$\sin^{-1}\left(\frac{x}{13}\right) = \sin^{-1}\frac{5}{13}$

$x = 5$

52. The straight lines x + y = 0, 5x + y = 4 and x + 5y = 4 form

(A) an isosceles triangle (B) an equilateral triangle (C) a scalene triangle (D) a right angled triangle

Ans : (A)

Hints : Their point of intersection are (–1, 1) (1, –1) and (2/3, 2/3) which is the vertices of isoceles triangle.

53. If $I = \int_0^2 e^{x^4}(x-\alpha)dx = 0$, then α lies in the interval

(A) (0, 2) (B) (–1, 0) (C) (2, 3) (D) (–2, –1)

Ans : (A)

Hints : $I = \int_0^2 e^{x^4}(x-\alpha)dx = 0$

$e^{x^4} > 0$

$(x-\alpha)$ should be somewhere positive and somewhere negative so $\alpha \in (0, 2)$

Hence, $a \in (0, 2)$

54. If the coefficient of x^8 in $\left(ax^2 + \frac{1}{bx}\right)^{13}$ is equal to the coefficient of x^{-8} in $\left(ax - \frac{1}{bx^2}\right)^{13}$, then a and b will satisfy the relation

(A) ab + 1 = 0 (B) ab = 1 (C) a = 1 – b (D) a + b = –1

Ans : (A)

Hints : $\left(ax^2 + \frac{1}{bx}\right)^{13}$

Co-efficient of x^8 in $\left(ax^2 + \frac{1}{bx}\right)^{13}$

$^{13}C_6\, a^7 . \dfrac{1}{b^6}$ — (1)

Co-efficient x^{-8} in $\left(ax - \dfrac{1}{bx^2}\right)^{13} = {}^{13}C_7\, a^6 \times \left(-\dfrac{1}{b}\right)^7$

$$= -\,{}^{13}C_7\, a^6 . \frac{1}{b^7} \text{ — (2)}$$

Since, $^{13}C_6\, a^7/b^6 = -\,{}^{13}C_7\, a^6/b^7$

$a = -\dfrac{1}{b}$

$ab + 1 = 0$

55. The function $f(x) = a \sin|x| + be^{|x|}$ is differentiable at $x = 0$ when

(A) $3a + b = 0$ (B) $3a - b = 0$ (C) $a + b = 0$ (D) $a - b = 0$

Ans : (C)

Hints : $f(x) = a \sin|x| + be^{|x|}$

$f(x) = a \sin x + be^{x} \quad x \ge 0$

$= -a \sin x + be^{-x} \quad x < 0$

$f'(x) = a\cos x + be^{x} \quad x \ge 0$

$= -a\cos x - be^{-x} \quad x < 0$

at $x = 0$

$a + b = -a - b$

$a + b = 0$

56. If a, b and c are positive numbers in a G.P., then the roots of the quadratic equation $(\log_e a)x^2 - (2\log_e b)x + (\log_e c) = 0$ are

(A) -1 and $\dfrac{\log_e c}{\log_e a}$ (B) 1 and $-\dfrac{\log_e c}{\log_e a}$ (C) 1 and $\log_a c$ (D) -1 and $\log_c a$

Ans : (C)

Hints : $b^2 = ac \Rightarrow \log_e a - 2\log_e b + \log_e c = 0$

$(\log_e a)x^2 - (2\log_e b)x + \log_e c = 0$

Since, 1 satisfies the equation

Therefore 1 is one root and other root say β

$1.\beta = \dfrac{\log_e c}{\log_e a} = \log_c a$

$\beta = \dfrac{\log_e c}{\log_e a} = \log_a c$

57. Let $\mathbb{R}$ be the set of all real numbers and f: [–1, 1] $\to$ $\mathbb{R}$ be defined by f(x) = $\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$, Then

(A) f satisfies the conditions of Rolle's theorem on [–1, 1]

(B) f satisfies the conditions of Lagrange's Mean Value Theorem on [–1, 1]

(C) f satisfies the conditions of Rolle's theorem on [0, 1]

(D) f satisfies the conditions of Lagrange's Mean Value Theorem on [0, 1]

Ans : (D)

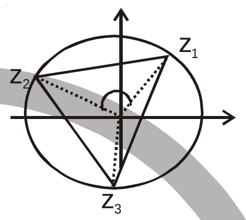
Hints : *f*(x) is nondifferentiable at x = 0

58. Let z_1 be a fixed point on the circle of radius 1 centered at the origin in the Argand plane and $z_1 \neq \pm 1$. Consider an equilateral triangle inscribed in the circle with z_1, z_2, z_3 as the vertices taken in the counter clockwise direction. Then $z_1z_2z_3$ is equal to

(A) z_1^2 (B) z_1^3 (C) z_1^4 (D) z_1

Ans : (B)

Hints : Let $z_1 = re^{i\alpha}$, $z_2 = r\,e^{i(\alpha+\frac{2\pi}{3})}$, $z_3 = r\,e^{i(\alpha+\frac{4\pi}{3})}$



$z_1z_2z_3 = r^3\,e^{i(\alpha+\alpha+\frac{2\pi}{3}+\alpha+\frac{4\pi}{3})}$

$= r^3e^{i(3\alpha + 2\pi)}$

$= r^3e^{i3\alpha}$

$= (re^{i\alpha})^3$

$= z_1^3$

59. Suppose that f(x) is a differentiable function such that f′(x) is continuous, f′(0) = 1 and f″(0) does not exist. Let g(x) = xf′(x). Then

(A) g′(0) does not exist (B) g′(0) = 0 (C) g′(0) = 1 (D) g′(0) = 2

Ans : (C)

Hints : g(x) = x.f′(x)

$$g'(x) = \lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$g'(x) = \lim_{h\to 0}\frac{(x+h)\cdot f'(x+h)-xf'(x)}{h}$$

$$g'(0) = 0 + \lim_{h\to 0} f'(0+h)$$

$= f'(0)$

$= 1$

60. Let [x] denote the greatest integer less than or equal to x for any real number x. Then $\lim_{n\to\infty}\frac{[n\sqrt{2}]}{n}$ is equal to

(A) 0 (B) 2 (C) $\sqrt{2}$ (D) 1

Ans : (C)

Hints : $\lim_{n\to\infty}\frac{[\sqrt{2}n]}{n}$

$$\sqrt{2}n \le [\sqrt{2}n] < \sqrt{2}n + 1$$

$$\sqrt{2} \le \frac{[\sqrt{2}n]}{n} < \sqrt{2} + \frac{1}{n}$$

$$\sqrt{2} \le \lim_{n\to\infty}\frac{[\sqrt{2}n]}{n} < \sqrt{2}$$

$$\Rightarrow \lim_{n\to\infty}\frac{[\sqrt{2}n]}{n} = \sqrt{2}$$

CATEGORY - II

Q.61 to Q.75 carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of 2/3 mark

61. We define a binary relation ~ on the set of all 3 × 3 real matrices as A ~ B if and only if there exist invertible matrices P and Q such that $B = PAQ^{-1}$. The binary relation ~ is

(A) Neither reflexive nor symmetric (B) Reflexive and symmetric but not transitive
(C) Symmetric and transitive but not reflexive (D) An equivalence relation

Ans : (D)

Hints : For Reflexive, A.I = IA, A = IAI^{-1}; so reflexive.
For Symmetric, $B = PAQ^{-1}$, BQ = PA, $P^{-1}BQ = A$ or $A = (P^{-1})\,B.\,(Q^{-1})^{-1}$,so symmetric.
For Transitive, $B = PAQ^{-1}$, $C = PBQ^{-1} = P.PAQ^{-1}.Q^{-1} = (PP)A(QQ)^{-1}$, so transitive

62. The minimum value of $2^{\sin x} + 2^{\cos x}$ is

(A) $2^{1-1/\sqrt{2}}$ (B) $2^{1+1/\sqrt{2}}$ (C) $2^{\sqrt{2}}$ (D) 2

Ans : (A)

Hints : $\frac{2^{\sin x}+2^{\cos x}}{2} \ge \sqrt{2^{\sin x+\cos x}} \ge \sqrt{2^{-\sqrt{2}}}$, $2^{\sin x}+2^{\cos x} \ge 2.2^{-\frac{1}{\sqrt{2}}}$, $2^{\sin x}+2^{\cos x} \ge 2^{1-\frac{1}{\sqrt{2}}}$

63. For any two real numbers θ and φ, we define θRφ if and only if $\sec^2\theta - \tan^2\varphi = 1$. The relation R is

(A) Reflexive but not transitive (B) Symmetric but not reflexive
(C) Both reflexive and symmetric but not transitive (D) An equivalence relation

Ans : (D)

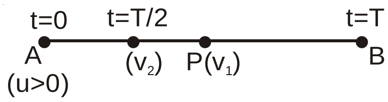
Hints: For reflexive, θ = ϕ so $\sec^2\theta - \tan^2\theta = 1$, Hence Reflexive
For symmetric, $\sec^2\theta - \tan^2\phi = 1$ so, $(1 + \tan^2\theta) - (\sec^2\phi - 1) = 1$ so, $\sec^2\phi - \tan^2\theta = 1$. Hence symmetric
For Transitive, let $\sec^2\theta - \tan^2\phi = 1$ and $\sec^2\phi - \tan^2\gamma = 1$ so, $1 + \tan^2\phi - \tan^2\gamma = 1$ or, $\sec^2\theta - \tan^2\gamma = 1$. Hence Transitive

64. A particle starting from a point A and moving with a positive constant acceleration along a straight line reaches another point B in time T. Suppose that the initial velocity of the particle is u > 0 and P is the midpoint of the line AB. If the velocity of the particle at point P is v_1 and if the velocity at time $\frac{T}{2}$ is v_2, then

(A) $v_1 = v_2$ (B) $v_1 > v_2$ (C) $v_1 < v_2$ (D) $v_1 = \frac{1}{2}v_2$

Ans : (B)

Hints :



Since the particle is moving with a positive constant acceleration hence it's velocity should increase. So the time taken to travel AP is more that the timetaken for PB. So the instant $T/2$ is before P. Hence $v_1 > v_2$ since velocity increases from A to B.

65. Let t_n denote the nth term of the infinite series $\frac{1}{1!}+\frac{10}{2!}+\frac{21}{3!}+\frac{34}{4!}+\frac{49}{5!}+$........... . Then $\lim\limits_{n\to\infty} t_n$ is

(A) e (B) 0 (C) e^2 (D) 1

Ans : (B)

Hints : $t_n = \frac{n^2+6n-6}{\lfloor n}$, $\lim\limits_{n\to\infty}\frac{n^2+6n-6}{\lfloor n} = 0$ since denominator is very large compared to numerator

66. Let α, β denote the cube roots of unity other than 1 and $\alpha \neq \beta$. Let $s = \sum\limits_{n=0}^{302}(-1)^n\left(\frac{\alpha}{\beta}\right)^n$. Then the value of s is

(A) Either -2ω or $-2\omega^2$ (B) Either -2ω or $2\omega^2$ (C) Either 2ω or $-2\omega^2$ (D) Either 2ω or $2\omega^2$

Ans : (A)

Hints : $a = \omega^2, \beta = \omega \Rightarrow \frac{\alpha}{\beta} = \omega$, $S = \sum\limits_{n=0}^{302}(-1)^n.(\omega)^n = \omega^0 - \omega^1 + \omega^2 - \omega^3 + \omega^4 \text{.........} + \omega^{302} = \frac{1-(-\omega)^{303}}{1-(-\omega)} = \frac{2}{-\omega^2} = -2\omega$

$\alpha = \omega, \beta = \omega^2 \Rightarrow \frac{\alpha}{\beta} = \frac{1}{\omega} = \omega^2$, $S = (\omega^2)^0 - (\omega^2)^1 + (\omega^2)^2 \text{.........} + (\omega^2)^{302} = \frac{1-(-\omega^2)^{303}}{1-(-\omega^2)} = \frac{2}{-\omega} = -2\omega^2$

67. The equation of hyperbola whose coordinates of the foci are (±8, 0) and the length of latus rectum is 24 units, is

(A) $3x^2 - y^2 = 48$ (B) $4x^2 - y^2 = 48$ (C) $x^2 - 3y^2 = 48$ (D) $x^2 - 4y^2 = 48$

Ans : (A)

Hints : $ae = 8$, $\frac{2b^2}{a} = 24$, $a^2e^2 = a^2 + b^2$ or, $64 = a^2 + 12a$ so $a = 4$, $b^2 = 48$, $\frac{x^2}{16} - \frac{y^2}{48} = 1$ so $3x^2 - y^2 = 48$

68. Applying Lagrange's Mean Value Theorem for a suitable function f(x) in [0, h], we have $f(h) = f(0) + hf'(\theta h)$, $0 < \theta < 1$. Then for $f(x) = \cos x$, the value of $\lim\limits_{h\to 0^+}\theta$ is

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Ans : (C)

Hints : For f(x) = cos x, cos h = 1 + h (–sin(θh)), $\sin\theta h = \frac{1-\cos h}{h}$, $\theta = \frac{\sin^{-1}\left(\frac{1-\cos h}{h}\right)}{h}$

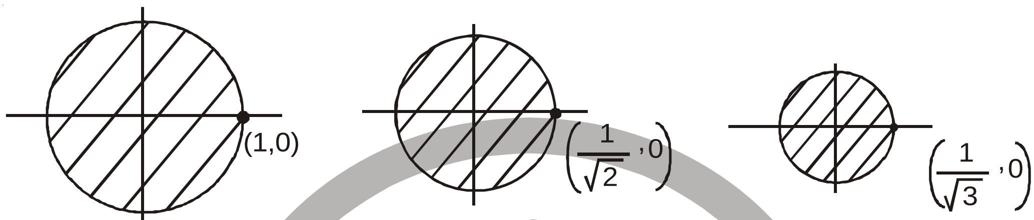
$$\lim_{h\to 0^+}\theta = \lim_{h\to 0^+}\frac{\sin^{-1}\left(\frac{1-\cos h}{h}\right)}{h} = \lim_{h\to 0^+}\frac{\sin^{-1}\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\frac{1}{2} = \frac{1}{2}$$

69. Let $X_n = \{z = x + iy : |z|^2 \le \frac{1}{n}\}$ for all integers n ≥ 1. Then $\bigcap_{n=1}^{\infty} X_n$ is

(A) A singleton set
(B) Not a finite set
(C) An empty set
(D) A finite set with more than one elements

Ans : (A)

Hints :



$X_1(|z| \le 1)$ $X_2\left(|z| \le \frac{1}{\sqrt{2}}\right)$ $X_3\left(|z| \le \frac{1}{\sqrt{3}}\right)$

The required regions are shaded for n = 1, 2, 3 so clearly $\bigcap_{n=1}^{\infty} X_n$ will be only the point circle origin. So a singleton set

70. Suppose $M = \int_0^{\pi/2}\frac{\cos x}{x+2}dx$, $N = \int_0^{\pi/4}\frac{\sin x\cos x}{(x+1)^2}dx$. Then the value of (M – N) equals

(A) $\frac{3}{\pi+2}$
(B) $\frac{2}{\pi-4}$
(C) $\frac{4}{\pi-2}$
(D) $\frac{2}{\pi+4}$

Ans : (D)

Hints : $N = \frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{\sin 2x}{(x+1)^2} = \frac{1}{2}\left[\sin 2x\times\left(-\frac{1}{x+1}\right)\Bigg|_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}}\frac{2\cos 2x}{(x+1)}dx\right] = \frac{-2}{\pi+4} + \int_0^{\frac{\pi}{4}}\frac{\cos 2x}{(x+1)}dx$

Replacing 2x = t, $\int_0^{\frac{\pi}{4}}\frac{\cos 2x}{(x+1)}dx = \int_0^{\frac{\pi}{2}}\frac{\cos t}{(t+2)}dt = M$. So M–N = $\frac{2}{\pi+4}$

71. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

(A) is equal to zero
(B) lies between 0 and 3
(C) is a negative number
(D) lies between 3 and 6

Ans : (c)

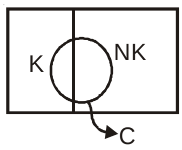
Hints : $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=\frac{\sin\frac{3\pi}{7}}{\sin\pi/7}\cdot\cos\frac{4\pi}{7}$. Clearly it is a negative no.

72. A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p, $0 < p < 1$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is

(A) $\frac{3p}{4p+3}$ (B) $\frac{5p}{3p+2}$ (C) $\frac{5p}{4p+1}$ (D) $\frac{4p}{3p+1}$

Ans : (c)

Hints : K = He knows the answers, NK = He randomly ticks the answers, C = He is correct



$$P\left(\frac{K}{C}\right)=\frac{P(K)\cdot P\left(\frac{C}{K}\right)}{P(K)\cdot P\left(\frac{C}{K}\right)+P(NK)\cdot P\left(\frac{C}{NK}\right)}=\frac{P\times 1}{P\times 1+(1-P)\times\frac{1}{5}}=\frac{5P}{4P+1}$$

73. A poker hand consists of 5 cards drawn at random from a well-shuffled pack of 52 cards. Then the probability that a poker hand consists of a pair and a triple of equal face values (for example, 2 sevens and 3 kings or 2 aces and 3 queens, etc.) is

(A) $\frac{6}{4165}$ (B) $\frac{23}{4165}$ (C) $\frac{1797}{4165}$ (D) $\frac{1}{4165}$

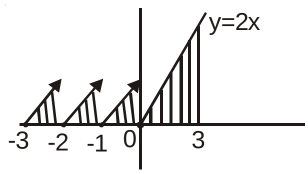
Ans : (A)

Hints : $\frac{{}^{13}C_1\times{}^4C_2\times{}^{12}C_1\times{}^4C_3}{{}^{52}C_5}=\frac{6}{4165}$.

74. Let $f(x)=\max\{x+|x|,x-[x]\}$, where [x] denotes the greatest integer $\le x$. Then the value of $\int_{-3}^{3}f(x)dx$ is

(A) 0 (B) 51/2 (C) 21/2 (D) 1

Ans : (c)



Hints : Required area $=3\times\frac{1}{2}+\frac{1}{2}\times 3\times 6=\frac{21}{2}$

75. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x\log_e x}=\frac{1}{x}$ under the condition y = 1 when x = e is

(A) $2y=\log_e x+\frac{1}{\log_e x}$ (B) $y=\log_e x+\frac{2}{\log_e x}$ (C) $y\log_e x=\log_e x+1$ (D) $y=\log_e x+e$

Ans : (A)

Hints : Integrating factor $=e^{\int\frac{dx}{x\log_e x}}=\log_e x$ $y\cdot\log_e x=\int\frac{\log_e x}{x}\cdot dx+c=\frac{(\log_e x)^2}{2}+c$, $c=\frac{1}{2}$

$$2y=(\log_e x)+\frac{1}{\log_e x}$$

CATEGORY - III

Q. 76 – Q. 80 carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question – irrespective of the number of correct options marked.

76. Let $f(x)=\begin{cases}\int_0^x |1-t|dt, & x>1\\ x-\dfrac{1}{2}, & x\le 1\end{cases}$ Then,

(A) f(x) is continuous at x = 1
(B) f(x) is not continuous at x = 1
(C) f(x) is differentiable at x = 1
(D) f(x) is not differentiable at x = 1

Ans : (A,D)

Hints : $\int\limits_0^x |1-t|\,dt=\int\limits_0^1 (1-t)\,dt+\int\limits_1^x (t-1)\,dt=\dfrac{x^2}{2}-x+1,$ $\quad f(x)=\begin{cases}\dfrac{x^2}{2}-x+1, & x>1\\ x-\dfrac{1}{2}, & x\le 1\end{cases}$ $\quad f'(x)=\begin{cases}x-1, & x>1\\ 1, & x\le 1\end{cases}$

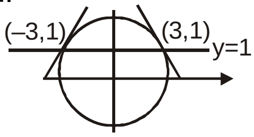
Clearly f(x) is continuous but not differentiable at x = 1.

77. The angle of intersection between the curves $y=\left[|\sin x|+|\cos x|\right]$ and $x^2+y^2=10$, where [x] denotes the greatest integer $\le x$, is

(A) $\tan^{-1}3$
(B) $\tan^{-1}(-3)$
(C) $\tan^{-1}\sqrt{3}$
(D) $\tan^{-1}\left(1/\sqrt{3}\right)$

Ans : (A,B)

Hints : $|\sin x|+|\cos x|=\sqrt{1+|\sin 2x|}$ So, $1\le|\sin x|+|\cos x|\le\sqrt{2}.$ $\quad y=\left[|\sin x|+|\cos x|\right]=1.$



$2x+2y\dfrac{dy}{dx}=0$, $\dfrac{dy}{dx}=\dfrac{-x}{y}$, So, angle is either $\tan^{-1}(-3)$ or $\tan^{-1}(3)$.

78. If u(x) and v(x) are two independent solutions of the differential equation $\dfrac{d^2y}{dx^2}+b\dfrac{dy}{dx}+cy=0,$ then additional solution(s) of the given differential equation is (are)

(A) y = 5 u(x) + 8 v(x)
(B) $y=c_1\{u(x)-v(x)\}+c_2v(x)$, c_1 and c_2 are arbitrary constants
(C) $y=c_1\,u(x)\,v(x)+c_2\,u(x)/v(x)$, c_1 and c_2 are arbitrary constants
(D) y = u(x) v(x)

Ans : (A,B)

Hints : Any linear combination of u(x) and v(x) will also be a solution.

79. For two events A and B, let P(A) = 0.7 and P(B) = 0.6. The necessarily false statements(s) is/are

(A) $P(A\cap B)=0.35$
(B) $P(A\cap B)=0.45$
(C) $P(A\cap B)=0.65$
(D) $P(A\cap B)=0.28$

Ans : (C,D)

Hints : $P(A\cup B)=1\cdot 3-P(A\cap B)$ now $P(A)\le P(A\cup B)\le 1,$ $0.7\le 1.3-P(A\cap B)\le 1,$ $0.3\le P(A\cap B)\le 0.6$

80. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts the three circles $x^2 + y^2 - 5 = 0$, $x^2 + y^2 - 8x - 6y + 10 = 0$ and $x^2 + y^2 - 4x + 2y - 2 = 0$ at the extremities of their diameters, then

(A) $C = -5$ (B) $fg = 147/25$ (C) $g + 2f = c + 2$ (D) $4f = 3g$

Ans : (A,B,D)

Hints : Common chords of the circle will pass through the centres.

$c = -5$, $8g + 6f = -35$, $4g - 2f = -7$ so, $g = \frac{-14}{5}$, $f = \frac{-21}{10}$

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