FUNDAMENTALS OF MODERN MATHS

PHI LAB

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Thematics

We have initiated Pegasus Thematics in PEX 2009! In MBA extended 2010 we are going ahead with the same academic philosophy with amendments based upon the feedbacks. The basic tenets of the program are as follows:

Theme Based Learning:
CL academics brings to you an offering, which incorporates theme-based learning that revolves around different concepts with diverse applications

Our integrated thematic methodology is driven by latest research, undertaken to enhance learning.

Our attempt has been to identify the basic concepts (or themes) that are required to solve different questions in MBA entrance examinations and, through our Class exercises integrate the different types of questions requiring application of these concepts around them. Each set of concepts along with relevant question types therefore, form a module; each unit of which is seamlessly integrated into the other. At the end of studying each module we expect the student to:

1. Clearly understand a concept through its repeated application in different question types.
2. Quickly and effectively apply the relevant concept to different question types in a time-bound examination scenario.
3. Develop long-lasting skills by imbibing each concept that is clearly covered through a module.
4. To develop an integrated perspective to “learning”

For example, a good vocabulary is required for solving both standard vocabulary related questions as well as for understanding a RC passage. Hence we have clubbed sessions on vocabulary and certain related sessions on reading comprehension into one module.

How to use this book

1. Before you enter the class read the topics that are to be covered beforehand. This will help you immensely in understanding the concepts when they are taught in the class.

2. After each class, once again go through the relevant topics very carefully, in order to understand the concepts and relate them to what was taught in the class.

3. Do not directly jump to the practice problems but go through the solved examples first as they will enhance your problem solving skills and help in further clarifying concepts.

4. After you are through with the fundamentals and the solved examples, move on to the unsolved problems given at the end of the book and the practice exercises.

5. Start with the Level - I problems as they are easier. Move to the Level - II problems, if and only if you have completely understood the concept used in every problem in Level - I. Similarly, move to the Level - III problems after you have completed all the problems in Level - II.

Modern Mathematics

Introduction

Modern mathematics is not so modern ! These are relatively new concepts which are only a few hundred years old. These concepts are normally perceived to be difficult and many students leave out these topics without even attempting it. It will make sense to try and understand the basics of these concepts. The level of questions asked from these topics in many cases is low and hence will be easy to crack !

Learning Objectives

By the end of all the chapters in the book, you should have

- Clarity in fundamental principle of counting.
- Basic knowledge of permutation and combination.
- Understood simple fundamentals of probability.
- Working knowledge in Set Theory.

Contents

1 Permutation and Combination

Introduction

The chapter covers permutation and combination which are efficient methods of counting numbers.

- Permutations are different ways of arranging things, we will learn to arrange different objects in different ways in this chapter.

  Do you know ?
  8 different books can be arranged in 40320 ways!!

- Combination deals with choosing the objects .

  Do you know in how many ways , we can select 3 books out of 8 different books ?
  56 ways!!

Learning Objectives

- Fundamental principles of counting
- Permutation - Linear and Geometrical
- Grouping and Distribution

Principal of Counting

Factorial:

Factorial of a natural number is defined as the product of all the consecutive natural numbers from 1 to that particular number. For example factorial of 5 is $1 \times 2 \times 3 \times 4 \times 5$. 'Factorial' word is represented with a symbol '!'. or 'L'. For example, factorial of 5 is written as L5 or 5!.

Example: $\frac{10!}{8!} = ?$

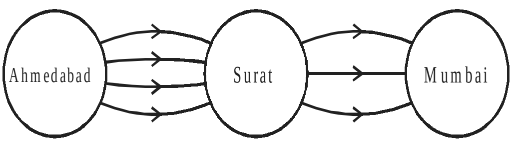
$$\frac{10!}{8!} = \frac{10 \times 9 \times 8!}{8!} = 10 \times 9 = 90.$$

Note: Factorial of zero is 1 (0! = 1)

Fundamental Principal of Counting

I. Product Rule:

In how many different ways can a person reach Mumbai from Ahmedabad (via Surat) if there are 4 different routes from Ahmedabad to Surat and 3 different routes from Surat to Mumbai?



Total number of ways = $4 \times 3 = 12$ ways.

So according to fundamental principle of counting, if there are m ways of doing a first thing and for each of them there are n ways of doing a second thing, then the total number of ways of doing both the things together is $m \times n$.

Most of the problems are based on the fundamental principle of counting.

Example: In how many different ways can 3 travellers stay in 4 hotels when each one should stay in different hotel?

Answer: For first traveller there are 4 choices; for second traveller 3 choices; for third traveller only 2 choices.

$\therefore$ Total ways = $4 \times 3 \times 2 = 24$ ways.

All problems of counting are based on fundamental principle of counting.

II. Addition rule

If there are 4 different ways from Surat to Ahmedabad and 3 different ways from Surat to Mumbai, then in how many different ways can a person go to Ahmedabad or Mumbai from Surat?

The answer is 4 + 3 = 7 ways.
The addition rule and Product rules signify the cases of "or" & "and".

Identify and understand clearly when addition rule is applied and when product rule is applied. Addition rule is applied when you take different cases and product rule is applied for the same case.

Example:

From Surat a person can go either to Mumbai OR to Ahmedabad. When we have OR it is two different cases hence the number of ways is 4 + 3. But to go from Ahmedabad to Mumbai you have to go from Ahmedabad to Surat AND from Surat to Mumbai. Hence they constitute single case. Hence number of ways = 4×3 and not 4 + 3.

Example 1:

There are 10 boys and 8 girls in a class . For the post of class monitor, the teacher wants to select either a boy or a girl. In how many ways can he do this function?

Solution:

He can select one boy out of 10 boys in 10 ways.
He can select one girl out of 8 girls in 8 ways.
He can select either a boy or a girl in 10 + 8 = 18 ways.

Note:

1. If all the functions are correlated, then basic principle of multiplication is used
2. If all the functions are independent, then basic principle of addition is used.

Example 2:
How many three-digit numbers are there?

Solution:
We know that there are 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
'0' cannot be at the hundreds place
So, 100th place can be filled in 9 ways.
Tens place can be filled in 10 ways.
Units place can be filled in 10 ways.
So the total number of three digit numbers
$= 9 \times 10 \times 10 = 900$

Example 3:
How many three-digit numbers are there in which all the digits are distinct?

Solution:
100th place can be filled in 9 ways.
10th place can be filled in 9 ways.
Units place can be filled in 8 ways because all the digits should be distinct.
So, the total number of three digit numbers in which all digits are distinct $= 9 \times 9 \times 8 = 648$

Example 4:
There are 5 multiple choice questions in an examination. First three questions have 4 choices each and the remaining two questions have 5 choices each. How many sequences of answers are possible?

Solution:
Each one of the first three questions can be solved in 4 ways, and each one of the last two questions can be solved in 5 ways.
So, the total number of different sequences of answers are $4 \times 4 \times 4 \times 5 \times 5 = 4^3 \times 5^2 = 1600$

Example 5:
How many even numbers less than 1000 can be formed by using the digits 2, 4, 3 and 5, if repetition of the digits is allowed?

Solution:
All the numbers of one digit, two digits and three digits are less than 1000. So take these cases one by one

1. Single-digit even numbers are 2 and 4
2. Two-digit even numbers:
   Unit's place can be filled in 2 ways, by 2 and 4 because unit's place digit must be an even number
   Ten's place can be filled in 4 ways.
   So the total number of two-digit even numbers $= 2 \times 4 = 8$

3. Three-digit even numbers
   Unit's place can be filled in 2 ways.
   Ten's place can be filled in 4 ways.
   Hundred's place can be filled in 4 ways
   So the total number of three-digit even numbers $= 2 \times 4 \times 4 = 32$
   Total number of three-digit even numbers (by using the digits 2, 4, 3 and 5) less then 1000 = 2 + 8 + 32 = 42

Permutations

Suppose there are three persons A, B and C contesting for the post of president and vice president of an organization and we have to select two persons. We can do it in 3 ! ways.
For example, (A, B), (B, C), (A, C) (B, A), (C, B) and (C, A). Here, the first person can be the president and the second person can be the vice president, means here we are talking about the order of arrangement.

The arrangements of a number of things taking some or all of them at a time are called permutations.

For example, if there are 'n' number of persons and we have to select 'r' persons at a time, then the total number of permutations is denoted by

${}^nP_r$ or by P(n, r).
First person can be selected in 'n' ways. Second person can selected in 'n –1' ways. Third person can be selected in 'n – 2' ways.
Similarly, the r^{th} person can be selected in '$n-(r-1)$' = '$(n-r+1)$' ways.
$\therefore$ Total number of ways of arranging these 'r' selected persons

$$= n\times(n-1)\times(n-2)\times....(n-r+1)$$

$$= \frac{n\times(n-1)(n-2)\times....1}{(n-r)(n-r-1)\times....1} = \frac{n!}{(n-r)!}$$

$$\therefore {}^nP_r = \frac{n!}{(n-r)\;!}.$$

Example 6:
There are four persons A, B, C and D and at a time we can arrange only two persons. Find the total number of arrangements.

Solution:
Total number of arrangements (**permutations**) is AB, BA, AC, CA, AD, DA, BC, CB, CD, DC, BD and DB or we can say that out of 4 persons we have to arrange only 2 at a time, so the total number of permutations is ${}^4P_2$.

$$^4P_2 = \frac{4!}{(4-2)\ !} = \frac{4!}{2!} = \frac{4\times3\times2}{2!} = 12$$

Example 7:
In the above question, if all the persons are selected at a time, then how many arrangements are possible?

Solution:
We have to arrange 4 persons, so this can be

$$^4P_4 = \frac{4!}{(4-4)\ !} = \frac{4!}{0!} = \frac{4!}{1}$$

$$= 4\times3\times2\times1 = 24$$

Example 8:
There are 4 flags of different colours. How many different signals can be given, by taking any number of flags at a time?

Solution:
Signals can be given either taking all or some of the flags at a time.
Number of signals that can be given by taking 1 flag $= {}^4P_1$
Number of signals that can be given by taking 2 flags $= {}^4P_2$
Number of signals that can be given by taking 3 flags $= {}^4P_3$
Number of signals that can be given by taking 4 flags $= {}^4P_4$
So the total number of signals

$$= {}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$$

$$= \frac{4!}{(4-1)\ !} + \frac{4!}{(4-2)\ !} + \frac{4!}{(4-3)\ !} + \frac{4!}{(4-4)\ !}$$

= 4 + 12 + 24 + 24 = 64

Example 9:
Find the number of ways in which 5 boys and 5 girls be seated in a row so that:

I. All the boys sit together and all the girls sit together.
II. Boys and girls sit at alternate positions.
III. No two girls sit together.
IV. All the girls always sit together.
V. All the girls are never together.

Solution:

I. All the boys can be arranged in 5! ways and all the girls can be arranged in 5! ways.
Now we have two groups (boys, girls) and these 2 groups can be arranged in 2 ! ways. [boys–girls and girls–boys]
So total number of arrangements is
5! × 5! × 2! = 28,800

II. Boys and girls sit alternately, this can be arranged like this
B G B G B G B G B G or G B G B G B G B G B
In the first case boys can be arranged in 5! and girls can be arranged in 5! ways.
In the second case also, the number of arrangement is same as first case
So the total number of arrangement
$= 5! \times 5! + 5! \times 5!$ or ${}^5P_5 \times {}^5P_5 + {}^5P_5 \times {}^5P_5$
= 120 × 120 + 120 × 120
= 14,400 + 14,400 = 28,800 ways

III. No two girls sit together - In this case __B__B__B__B__B__ there are 6 spaces where a girl can find her seat.
5 girls can be arranged in ${}^6P_5$
$$\frac{6!}{(6-5)!} = 6\times5\times4\times3\times2 = 720 \text{ ways}$$
5 boys can be arranged in ${}^5P_5$
$= 5\times4\times3\times2\times1 = 120$ ways
Total number of arrangements
= 720 × 120 = 86,400

IV. When all the girls are always together, then treat them as one group. So now we have 5 boys and 1 group of 6 girls and this can be permutated in 6! ways at the same time 5 girls in the group can be permutated in 5 ! ways, so total number of required ways is
6 ! × 5 ! = 720 × 120 = 86400

V. All the girls are never together
Total number of arrangements of 5 boys and 5 girls is 10!
Number of arrangements in which all the girls are always together
$= B_1, B_2, B_3, B_4, B_5$ [All 5 girls]
= 6! × 5! = 8,64,00
So number of arrangements in which all the girls are never together = total arrangement – number of arrangements when girls are always together.
= 10! – (6! × 5!) = 3,54,2400

Example 10:
Find the number of permutation of the letters of the word FOLDER taking all the letters at a time?

Solution:
Number of letters in the word FOLDER is 6
So the number of arrangements
$= {}^6P_6 = 6!$

Alternative method:
First place can be filled by any one of the six letters. The second place can be filled by any one of the five remaining letters, the third place can be filled by any one of the four remaining letters and so on. So the total number of arrangements is
6 × 5 × 4 × 3 × 2 × 1 = 720.

Example 11:
How many four-digit numbers greater than 5000 can be formed by using the digits 4, 5, 6 and 7? (Repetition of the digits is not allowed.)

Solution:
Total number of arrangements possible is
${}^4P_4 = 4!$
Total number of arrangements by using the digits 5, 6 and 7 is = 3!
So the total number of required arrangements is
4! – 3! = 24 – 6 = 18

Alternative method:
Thousand's place can be filled in 3 ways.
Hundred's place can be filled in 3 ways.
Ten's place can be filled in 2 ways.
Unit's place can be filled in 1 ways.
So total number of arrangements
= 3 × 3 × 2 × 1 = 18

Example 12:
In Q. 11, find the number of four-digit numbers that can be formed if the repetition of digits is allowed.

Solution:
If the repetition is allowed, then the total number of arrangements is
4 × 4 × 4 × 4 = 256 ways
Because on the first place any one of the four number can come, similarly on the 2nd, 3rd and 4th place also.
Total number of arrangements beginning with 4 is
4 × 4 × 4 = 64
So, total number of required arrangements = 256 – 64 = 192

Alternative method:
Thousand's place can be filled in 3 ways
Hundred's place can be filled in 4 ways.
Ten's place can be filled in 4 ways.
Unit's place can be filled in 4 ways.
So the total number of arrangements
= 3 × 4 × 4 × 4 = 192

Example 13:
There are 5 friends: A, B, C, D and E. They wanted to take a group photograph of all of them sitting in a single row.

a. How many distinctly different photographs can be clicked?

b. In how many of these photographs would A be sitting in the middle?

c. In how many of these photographs would A and B be sitting next to each other?

Solution:

a. There are 5 friends: A, B, C, D and E. The total number of photographs that can be taken each of which is distinctly different from the other is same as the total number of ways A, B, C, D and E can be permuted taken all at a time.

Hence, the total number of photographs
$= {}^5P_5 = 5! = 120$

b. If we fix the position of A in the middle, then the other 4 can be seated in 4! ways. Hence, the number of ways in which A is in the middle = 4!.

c. Take A and B as one unit.
Then there are 4 units that have to be arranged (A B) CDE. They can be arranged in 4! ways. A and B among themselves can be arranged in 2! ways. Hence, by the fundamental principle of counting we have 4! × 2! ways of arrangement.

$$^{n}C_r = \frac{n!}{r!(n-r)!}$$

But a simpler way to look at

$$^{10}C_3 = \frac{10\times9\times8}{1\times2\times3}$$

numerator has 3 numbers starting from 10 and denominator has 3 numbers starting from 1.

Similary, $^{n}P_3 = \frac{n!}{(n-r)!}$ $^{10}P_3 = 10\times9\times8$

This helps in faster calculation.

Example 14:
A letter lock contains 4 rings, each ring containing 5 letters. In how many different ways can the 4 rings be combined? If the lock opens in only one arrangement of 4 letters, how many unsuccessful events are possible?

Solution:
Each ring contains 5 letters. Therefore, for each of the ring we have 5 different ways of bringing a letter to the opening position.
∴ The number of ways in which the 4 rings can be combined = 5 × 5 × 5 × 5 = 625
But of these attempts to open the lock, only one will be successful.
Hence, the possible number of unsuccessful events = 625 – 1 = 624

If certain objects have to appear together, we can treat them as a single set in certain problems

Example 15:
A group of 6 students comprised of 3 boys and 3 girls. In how many ways could they be arranged in a straight line such that

a. the girls and the boys occupy alternate positions?
b. no two boys were sitting together?

Solution:
a. The positions could be BG BG BG or GB GB GB
Hence, the number of arrangements is 3! × 3! + 3! × 3! = 2 × 3! × 3!
b. First of all we will arrange 3 girls in 3! ways.
$| G_1 | G_2 | G_3 |$
Now we have 4 positions for 3 boys that can be filled in $^4P_3$ ways.
Hence, the total number of arrangements = $^4P_3 \times 3!$

Example 16:
The letters of the word FIGMENT are to be arranged in the following manner.
a. There is no restriction.
b. Start with F.
c. All vowels together
d. Vowels at first and last positions

Solution:
a. There are 7 letters which can be arranged at 7 positions in 7! ways = 5040 ways.

b. Starting with F, remaining 6 letters can be arranged in 6! ways = 720 ways.

c. Tying all vowels with a string, we have F, G, M, N, T and (I, E), i.e. 6 sets. These can be arranged in 6! ways and the 2 vowels can exchange their positions in 2! ways.
Total number of ways = 6! × 2! = 1440 ways.

d. For vowels at first and last positions, first place can be taken by I and last by E, or vice versa. Remaining 5 positions can be filled by 5 letters in 5! ways.
So total words formed = 5! × 2! = 240 words.

Combinations

Suppose three persons A, B and C are contesting for the post of president and vice president of an organization and we have to select two persons. We can select either (a, b) or (b, c) or (a, c) = 3 ways because here we are talking about the selection, not about the order. Whether 'a' is a president or 'b' is a vice president or vice-versa, doesn't matter.

Suppose there are 10 persons in class and we have to select any 3 persons at a point regardless of the order, it is a case of combination.

If there are n number of things and we have to select some or all of them it is called combinations. If out of n things we have to select r things ($1 \le r \le n$), then the number of combinations is denoted by $^nC_r = \frac{n!}{(n-r)!\,r!}$

We already know that the number of arrangements of 'r' things out of the 'n' things is given by $^nP_r = \frac{n!}{(n-r)!}$

Combination does not deal with the arrangements of the selected things.

$\therefore$ 'r' selected things can be arranged in r! ways.

$\therefore (r!) \times (^nC_r) = {^nP_r}$

$\Rightarrow {^nC_r} = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$

Difference between permutations and combinations

Suppose that there are five persons A, B, C, D and E and we have to choose two persons at a time then in

Permutation	Combinations
Number of required ways $= \frac{5!}{(5-2)!}$ $= \frac{5!}{3!} = 5 \times 4 = 20$	$= \frac{5!}{2!(5-2)!}$ $= \frac{5!}{2! \times 3!} = \frac{5 \times 4}{2} = 10$

So it is clear that in permutations (rearrangement) order matters but in combinations (selections) order does not matter.

Example 17:

In a class there 5 boys and 6 girls. How many different committees of 3 boys and 2 girls can be formed?

Solution:

Out of 5 boys we have to select 3 boys, this can be done in $^5C_3$ ways.

Out of 6 girls we have to select 2 girls, this can be done in $^6C_2$ ways.

So, selection of 3 boys and 2 girls can be done in $(^5C_3) \times (^6C_2)$ ways

[Basic rule of multiplication]

$$= \left(\frac{5!}{3!(5-3)!}\right) \times \left(\frac{6!}{2!\ (6-2)!}\right)$$

$$= \left(\frac{5 \times 4}{2}\right) \times \left(\frac{6 \times 5}{2}\right) = 10 \times 15 = 150 \text{ ways}$$

Example 18:

If there are 10 persons in a party, and each person shake hands with all the persons in the party, then how many hand shakes took place in the party?

Solution:

It is very obvious that when two persons shake hands, it is counted as one handshake. So we can say that there are 10 hands and every combination of 2 hands will gives us one handshake.

So, the number of handshakes

$$= {^{10}C_2} = \frac{10!}{2!\ (10-2)\ !}$$

$$= \frac{10 \times 9 \times 8!}{2! \times 8!} = 45$$

Example 19:

For the post of Maths faculty in Career Launcher there are 6 vacant seats. Exactly 2 seats are reserved for MBA's. There are 10 applicants out of which 4 are MBA's. In how many ways the selection can be made?

Solution:

There are 4 MBA's and 6 other candidates.

So we have to select 2 candidates out of the 4 MBA's and the rest 4 candidates out of 6 other candidates.

So the total number of ways of selection $= (^4C_2) \times (^6C_4)$

$$= \left(\frac{4!}{2!\times(4-2)\,!}\right)\times\left(\frac{6!}{4!(6-4)\,!}\right)$$

$$= \left(\frac{4\times3\times2!}{2\times1\times2!}\right)\times\left(\frac{6\times5\times4!}{4!\times2\times1}\right)$$

$$= 6\times15 = 90 \text{ ways}$$

Example 20:
There are 10 points out of which no three are collinear. How many straight lines can be formed using these 10 points?

Solution:
By joining any two points we will get one line.
So the total number of lines formed

$$= {}^{10}C_2 = \frac{10\times9\times8!}{2\times(10-2)\,!} = \frac{10\times9\times8!}{2\times8!} = 45$$

Example 21:
Find the number of diagonals that can be drawn by joining the vertices of a decagon.

Solution:
In decagon there are 10 vertices and by joining any two vertices we will get one line.
So in a decagon total number of lines formed

$$= {}^{10}C_2 = \frac{10!}{2!\ (10-2)!} = \frac{10\times9\times8!}{2!\times8!} = 45$$

But out of these 45 lines, 10 lines will be the sides of the decagon. So total number of diagonals = 45 – 10 = 35

Example 22:
In the above question how many triangles can be formed?

Solution:
We know that in a triangle there are three vertices and by joining any three points we will get a triangle.
So number of triangles formed

$$= {}^{10}C_3 = \frac{10\times9\times8\times7!}{3!\times(10-3)!} = \frac{10\times9\times8\times7!}{3!\times7!}$$
$$= 120$$

Example 23:
There are 5 boys and 6 girls. A committee of 4 is to be selected so that it must consist at least one boy and at least one girl?

Solution:
The different possibilities are
I. 1 boy and 3 girls
II. 2 boys and 2 girls
III. 3 boys and 1 girl
In the first possibility total number of combinations is ${}^5C_1\times{}^6C_3$
In the second possibility total number of combinations is ${}^5C_2\times{}^6C_2$
In the third possibility total number of combinations is ${}^5C_3\times{}^6C_1$
So the total number of combinations are ${}^5C_1\times{}^6C_3+{}^5C_2\times{}^6C_2+{}^5C_3\times{}^6C_1 = 310$

Permutation of things when some are identical

Above was the case when all letters in the word were different. What if some letters are identical?

$\Rightarrow$ If out of n things, p are exactly alike of one kind, q exactly alike of second kind and r exactly alike of third kind and the rest are different, then the number of permutations of n things taken all at a time = $\frac{n!}{p!q!r!}$

Example 24:
How many different words can be formed using the letter of "HALLUCINATION"

i. Using all the letters.
ii. If all vowels are together?
There are 6 vowels: two **A**s, two **I**s, one **U**, one **O**.
iii. All vowels occupy odd places only.

Solution:
i. Total letters in the word are 13 and the identical letters are 2L, 2A, 2I, 2N.
So total number of arrangements possible

$$= \frac{13!}{2!\,2!\,2!\,2!}$$

ii. Tie all vowels together and considering as a single letter, now we have 8 letters, out of these 8 letters 2L and 2N are identical. These 8 letters can be arranged in $\frac{8!}{2!\,2!}$ ways .

In group of 6 vowels, 6 letters can be arranged themselves in $\frac{6!}{2!\,2!}$ ways.

So total number of words formed

$$= \frac{8!}{2!\,2!} \times \frac{6!}{2!\,2!}$$

iii. Out of 7 odd places (1, 3, 5, 7, 9, 11, 13), 6 odd places for 6 vowels can be selected in ${}^7C_6$ ways. On these 6 places, 6 letters can be arranged in $\frac{6!}{2!\,2!}$ ways.

Remaining 7 letters can be arranged in 7 remaining places in $\frac{7!}{2!\,2!}$ ways.

Total words formed = ${}^7C_6 \times \frac{6!}{2!\,2!} \times \frac{7!}{2!\,2!}$

Note that you would apply the formula for arrangement of objects some of which are identical, only if all the objects are permuted. If "r" objects are selected, we will apply the rule to those "r" objects that are selected.

Example 25:

In how many ways can the letters of the word SUCCESSFUL be arranged? In how many of them will (i) all Ss come together, (ii) all Ss not come together, (iii) the Ss come together and Us also come together?

Solution:

The word contains 10 letters of which 3 are Ss, 2 are Cs, and 2 are Us and the rest all are different.

∴ The letters of the word SUCCESSFUL can be arranged in $\frac{10!}{3!\,2!\,2!} = 1\,51\,200$ ways.

i. Since the Ss are to come together, treat 3 Ss as one letter. Now with this restriction there will be 8 letters of which 2 are Cs and 2 are Us and the rest all are different.

∴ The arrangement in which Ss will come together $= \frac{8!}{2!\,2!} = 10080$

ii. The arrangements in which all Ss will not come together
= Total number of arrangements – The number of arrangements in which all the Ss will come together
= 151200 – 10080 = 141120

iii. Since the Ss and Us are to come together, treat 3 Ss as one letter and 2 Us as one letter. Now there will be 7 letters of which 2 are Cs and the rest all are different.

∴ The arrangements in which Ss and Us will come together $= \frac{7!}{2!} = 2520$

Note that in the case of repetition of digits all those cases where no digits are repeated are also included.

Example 26:

There are 10 digits from 0 to 9 in the decimal system. Find the following using this data.

a. How many 5-digit numbers can be formed, such that no 2 digits are the same?

b. How many 4-digit numbers can be formed using these 10 digits?

c. How many numbers more than 1,000 and less than 10,000 can be formed such that they are divisible by 5 and no 2 digits are the same?

d. What is the number of arrangements in which 3 appears exactly twice in part (b)?

Solution:

a. The total number of digits that can occupy the 1st place = 9 (zero cannot occupy the first place).
Consequently, the number of digits that can fill the 2nd, 3rd, 4th and 5th places are 9, 8, 7 and 6 respectively.
So total number of 5-digit numbers with distinctly different digits is
9 × 9 × 8 × 7 × 6 = 27216

b. The number of digits that can occupy the first place = 9
For the 2nd, 3rd and 4th places any of the 10 digits can occupy the distinct places.
Hence, the total number of 4-digit numbers is $9 \times 10 \times 10 \times 10 = 9000$
Note: Repetition of digits occur in these arrangements.

c. The number has to be a 4-digit number. Since the number is divisible by 5. It has to end in either a 5 or a 0.
Take each of these cases separately.

Case 1: If it ends in a 5, the 1st, 2nd, 3rd places could be filled in 8, 8 and 7 ways respectively.
Hence, the number of 4-digit numbers with distinctly different digits and ending in a 5 is $8 \times 8 \times 7 = 448$

Case 2: If the number ends in a 0, the 1st, 2nd, 3rd places could be filled in 9, 8 and 7 ways respectively.Hence, the total number of such numbers possible is $9 \times 8 \times 7 = 504$
So total number of 4-digit numbers divisible by 5 and having distinctly different digits is $448 + 504 = 952$

d. Out of the 4-digit numbers formed with repetition we need to find how many of them have two 3s.
The cases are:
(i) When one of the 3s is in the first place.
(ii) When none of the 3s is in the first place.

Case (i)
If we fix a 3 in the first position, then the total number of ways of forming the remaining 3 digits is $9 \times 9 \times {}^3C_1 = 243$
[The second 3 can occupy 3 possible position.]

Case (ii)
If we let the first digit be anything other than 3, then the number of arrangements = $8 \times 9 \times {}^3C_2 = 216$
Total such numbers = 243 + 216 = 459

In (2 + 3 + 4 + 5 + 6) (1111) (4!)
(2 + 3 + 4 + 5 + 6) ⇒ Sum of the digits
(11111) ⇒ As many 1's as there are digits in the number.
(4!) ⇒ Indicates number of times any digit appears in any place.

Example 27:
Using the digits 2, 3, 4, 5 and 6, find the following.
a. Sum of all 5-digit numbers that can be formed such that no 2 digits are the same.
b. Sum of all 4-digit numbers that can be formed such that no 2 digits are the same.
c. Sum of all 4-digit numbers that can be formed such that digits can be repeated.

Solution:
a. Each of the numbers would be in any place 4! times.
Hence, their contribution when in the ten thousand's place is $(2 + 3 + 4 + 5 + 6)(10000) \times 4!$
Similarly, when in thousand's place they contribute $(2 + 3 + 4 + 5 + 6)(1000) \times 4!$.
For hundred's, ten's and unit's places the contributions are
$(2 + 3 + 4 + 5 + 6)(100) \times 4!$,
$(2 + 3 + 4 + 5 + 6)(10) \times 4!$,
$(2 + 3 + 4 + 5 + 6)(1) \times 4!$ respectively.
Hence, the total contribution to the sum is $(2 + 3 + 4 + 5 + 6)(11111)(4!)$.

b. The method is similar as in the previous question.
Each of these digits would be in any place in ${}^4P_3$ times.
Hence, the sum of all 4-digit numbers, with no repetitions, is
$(2 + 3 + 4 + 5 + 6)(1111)({}^4P_3)$

c. Each of these digits would appear in the thousand's place 5^3 times.
Hence, their total contribution when in that position is $(2 + 3 + 4 + 5+ 6)(1000)(5^3)$
Extending the same principle to the rest of the problem, we get the sum of all such numbers with repetitions is
$(2 + 3 + 4 + 5 + 6)(1111)5^3$.

Example 28:
Using the digits 0, 1, 2 and 4, find the sum of all four-digit numbers that can be formed. (Repetition of digits is not allowed.)

Solution:
Using the principle as in # 8 (b), we would have sum of all 4-digit numbers
= (0 + 1 + 2 + 4)(1111)(3!)
But some of these numbers will have
0 in the thousand's place and such cases are to be taken away. Such numbers would be same as the 3-digit numbers formed using digits 1, 2 and 4.
Hence, the sum of all such 3-digit numbers are (1 + 2 + 4)(111)(2!).
Total sum = (0 + 1 + 2 + 4)(1111)(3!) – (1 + 2 + 4)(111) (2!)

Example 29:
Find the total number of triangles that can be formed by joining the vertices of the polygon of *n* sides. If the polygon has the same number of diagonals as its sides, find the number of triangles?

Solution:
The triangle is formed by joining any 3 vertices of the polygon of *n* vertices.
∴ The number of triangles formed by *n* vertices = $^nC_3$
The number of diagonals = $^nC_2 - n$
If a polygon with *n* sides has the same number of diagonals as sides, we have $n = {}^nC_2 - n$ Solving for n, we get n = 0 or 5.
Since $n \neq 0$, n = 5. Hence, the number of triangles = $^5C_3 = 10$

Example 30:
There are 10 points in a plane. Except for 4 points which are collinear no three points are in a straight line. Find
(i) the number of straight lines obtained by joining these points,
(ii) number of triangles that can be formed with the vertices as these points.

Solution:
i. Two points form a straight line.
∴ Number of lines formed by joining 10 points = $^{10}C_2$

$$= \frac{10.9}{2!} = 45$$

Number of straight lines formed by joining 4 points = $^4C_2 = 6$
But 4 collinear points give only one line. So these lines should be excluded.
∴ Required number of straight lines
= 45 – 6 + 1 = 40

ii. Number of triangles formed by joining the points taking 3 at a time

$$= {}^{10}C_3 = \frac{10.9.8}{3!} = 120$$

Number of triangles formed by 4 points
= $^4C_3 = 4$
But 4 collinear points cannot form any triangle.
∴ Required number of triangles = 120 – 4
= 116

The number of different relative arrangement for n different things arranged on a circle is (n – 1)!

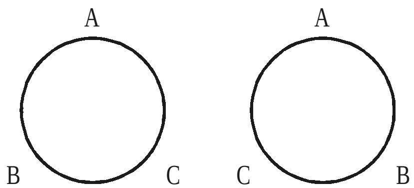
Example 31:
In how many ways can the letters of the word 'PROPORTION' be arranged without changing the relative positions of the vowels and consonants.

Solution:
In the word PROPORTION, there are 6 consonants of which 2 are Ps, 2 are Rs and the rest are different and there are 4 vowels of which 3 are Os and one I. The positions originally occupied by vowels must be occupied by vowels and those occupied by consonants, by consonants only. The vowels must be permuted among themselves and similarly the consonants.
∴ The consonants can be permuted among themselves in $\frac{6!}{2!\,2!}$ ways and the vowels can be permuted among themselves in $\frac{4!}{3!}$.
Since the two operations are independent, the required number of ways is $\frac{6!}{2!\,2!} \times \frac{4!}{3!}$.

Geometrical Arrangements

Sitting in a circle is not same as sitting in a straight line. A circle has no straight point and no ending point. We will also talk about relative arrangements in a circle, which means the positions of others relative to a point being the same or different. The moment we label the positions on a circle the relative arrangements though same can yield "n" circular arrangements.

Circular Permutation: Number of circular permutation of n different things taken all at a time = (n – 1)! ways. Fix any one as reference point, other (n – 1) things can be arranged in (n – 1)! ways. Three persons around a circular table can be arranged in 2 ways, i.e. (3 – 1)! ways



Necklace: In case of the necklace or garland, anticlockwise and clockwise arrangements are same. So total number of arrangements of n beads for forming a necklace is $\frac{1}{2}(n-1)!$

Example 32:
In how many ways can 7 boys be seated at a round table so that 2 particular boys are
- **i.** next to each other,
- **ii.** separated.

Solution:
Number of boys = 7
- **i.** Let the 2 particular boys be taken together as one unit.
  Then the number of units will be 6. They can sit around the table in 5! ways. For each of this arrangement, the 2 can be interchanged in 2! ways.
  $\therefore$ The total number of arrangements = 5! 2!
- **ii.** The arrangements that the 2 persons are separated = 6! – 5! 2!

Example 33:
There are 25 gangsters including 2 brothers, 'Munna Mobile' and 'Pappu Pager'. In how many ways can they be arranged around the circular table if
- **a.** there is exactly one person between these 2 brothers,
- **b.** the 2 brothers are always separated?

Solution:
- **a.** One person between 2 brothers can be selected in 23 ways.
  Remaining 22 persons can be arranged in 22! ways.
  2 brothers can interchange their positions.
  So total number of ways $= 2 \times 23 \times 22!$
  = 2 × 23! ways
- **b.** Total ways of arranging 25 people = 24!
  Subtract those ways in which 2 brothers are together = 2 × 23!
  $\therefore$ Number of ways when 2 brothers are always separated = 24! – 2 × 23!

Arrangement around a regular polygon:

If N people are to be arranged around a K sided regular polygon, such that each side of that polygon contains same number of people, then the number of arrangements will be $\frac{N!}{K}$

For example, 24 people are to be arranged around a square table having six people on each side of the table, number of arrangements will be $\frac{24!}{4}$.

Please remember if the polygon is not regular, i.e., if the sides of that polygon are uneven in length, then the number of arrangements will be just N!, whatever be the number of sides of that polygon.

Special case of a rectangular table:

If N people are to be arranged around a rectangular table, such that there are 6 people on each side of the table, then total number of arrangements will be $\frac{N!}{2}$. Here '2' signifies the degree of symmetry of a rectangle.

Example 34:
A group of 11 people went to a party. There were 5 girls and 6 boys. They were seated on a rectangular table with 6 chairs on either side of the longer edge.

a. What is the total number of ways the group could be seated?
[Sides are indistinguishable.]

b. What is the number of ways they can be seated so that all the 5 girls were sitting on the same side?

Solution:

a. The total number of ways we can form 2 groups of 6 and 5 is ${}^{11}C_6$ or ${}^{11}C_5$. The total number of ways these 2 groups can be seated on either side is ${}^{11}C_6 \times 6! \times {}^6P_5$.

b. There will be 2 cases here.

Case (i)
When there are 5 girls, and a guy is sitting on one side and the remaining 5 guys are on the other side:
This is possible in ${}^6C_1 \times 6! \times 6!$ ways.

Case (ii) When there are 5 girls on one side and all the guys are on the other side:
This is possible in ${}^6P_5 \times {}^6P_6 = 6! \times 6!$ ways.
$\therefore$ Total number of required arrangement
$= 6 \times 6! \times 6! + 6! \times 6!$
$= 7 \times 6! \times 6!$

Grouping and Distribution

This is another very important concept of permutation and combination. To distribute something, first grouping is done. Then permute these groups if required.

To illustrate the difference take the example of a case where you have 2 items I_1, I_2, of you have to split into 2 groups there is only 1 way of doing it. I_1 goes into one group and I_2 into another group. If you have to distribute among 2 people A, B then these 2 groups can be permuted in 2! ways. Similarly if there are 3 items I_1, I_2, I_3 the number of ways of splitting into 2 groups is ${}^3C_2$ i.e. $(I_1, I_2), (I_3)(I_1, I_3), (I_2)$(or) $(I_2, I_3), (I_1)$

They can be distributed among 2 people in 2! ways. So it is important to distinguish between grouping and distribution.

Important points for grouping:

(i) The number of ways in which (m + n) things can be divided into two groups containing m and n things respectively = $\frac{(m+n)!}{m!\,n!}$

(ii) If the numbers of things are equal, say m = n, total ways of grouping = $\frac{(2\,m!)}{2!(m!)^2}$

It means divide by p! if there are p groups having same number of things or in other words, p groups are identical.

Example 35:

I. In how many ways can 15 soldiers be divided into 3 groups equally?

Answer: $\frac{15!}{3!\,(5!)^3}$. Here we are dividing by 3! because 3 groups are having same number of persons.

II. But if the question is, in how many ways can 15 soldiers be drafted into 3 regiments (JAT, SIKH, GORKHA)?

Answer: $\frac{15!}{3!\,(5!)^3} \times 3! = \frac{15!}{(5!)^3}$ i.e. the concept is same. Dividing or grouping first, then permutating if groups are named, i.e. if groups are different. All questions of distributions can be solved easily if you are very clear about grouping.

Example 36:
In how many different ways can 5 different balls be distributed to 3 different boxes, when each box can hold any number of balls?

Solution:

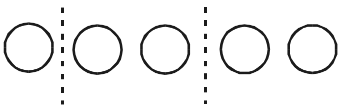
Every ball has 3 ways of distribution. It can go to any of 3 boxes. So applying fundamental principle of counting, we get
$3 \times 3 \times 3 \times 3 \times 3 = 3^5$ ways.

Note: We cannot say every box has 5 ways of chosing a ball. So 5^3 is wrong.

Example 37:
In how many different ways can 5 identical balls be distributed to 3 different boxes, when each box can have any number of balls?

Solution:
In this question, now the balls are identical. So number of balls in each box will matter. This question is exactly same as find non-negative integral solution of the equation
$x_1 + x_2 + x_3 = 5$

These 3 variables are representing the number of balls in 3 different boxes. Insert 2 partitions in between these 5 balls.



These 2 partitions will divide these 5 balls in 3 groups. Total number of ways of arranging these (5 + 2) things is $= \frac{(5+2)!}{2!\,5!}$
(Because 2 partitions are alike, 5 balls are identical.) $= {}^{7}C_2$

So, distributing 'n' identical things in 'r' different boxes = ${}^{n+r-1}C_{r-1}$

The number of ways of picking up any number of items from n different items is 2^n. Here the case of not picking up any item is also considered

Example 38:
If x + y + z = 12, then what is the total number of positive integral solutions?

Solution:
The difference in this question from above question is that it is asking for positive integral solution. It means now none of the variables can take 0 value. So giving one ball to each of 3 boxes initially will ensure positive integral solution of x + y + z = 12. Total non-negative integral solutions of x + y + z = 9 is ${}^{9+3-1}C_{3-1} = {}^{11}C_2$

Note that from 10 identical items the number of distinct ways of choosing r items is not ${}^{10}C_r$ but just 1. The key word here is <u>distinct</u>.

Example 39:
What is the total number of ways of selecting at least one object from 2 sets of

i. 10 distinctly different objects?

ii. 10 identical objects?

iii. 10 distinctly different objects picking at least one from each set?

iv. 10 identical objects picking at least one from each set?

Solution:

i. Number of ways of selecting an item from 10 distinctly different items of one set

$= {}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + \cdots + {}^{10}C_{10} = (1+1)^{10}$
$= 2^{10}$

Since there are 2 sets, the total number of selections = $(2^{10}) \times (2^{10}) = 2^{20}$
Since at least one has to be selected, deduct the case where none has been selected from either sets, i.e. $2^{20} - 1$ cases.

ii. If all the objects are identical, then the number of ways is 11. (Select 0 or 1 or 2 or 3 ... or 10. Each one of these selections can be made in only 1 way.)

Since there are 2 sets there would be 11 × 11 = 121 cases. One of these cases would involve 0 selections from either of the sets. Hence, the total number of ways = 121 – 1 = 120

iii. If we have to pick at least 1 from each set, there are

$({}^{10}C_1 + {}^{10}C_2 + \cdots + {}^{10}C_{10})\ ({}^{10}C_1 + {}^{10}C_2 + \cdots + {}^{10}C_{10}) = (2^{10} - 1)^2$

cases.

iv. If at least one item has to be selected from either of the sets, the total number of ways = 10 × 10 = 100
[The case of 0 selection from each of the sets is not considered.]

2 Probability

Introduction

- Probability is concerned with random outcomes, such as flipping coins or rolling dice.
- Probabilty is used to determine the possible outcome of a coin toss or a genetic sequence

Learning Objectives

- Probability
- Conditional Probability

Probability

Probability is the measure of the likelihood of occurrence of an event. Now we may define the probability of an event as follows:

Probability of an event

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

An event is any outcome or a set of outcomes from an experiment.

1. If an event E is sure to occur, we say that the probability of the event E is equal to 1 and we write P (E) = 1. Such events are known as certain events.

2. If an event E is sure not to occur, we say that the probability of the event E is equal to 0 and we write P (E) = 0. Such events are known as impossible events.

Therefore, for any event E, $0 \le P(E) \le 1$.

For example, if we toss a coin, is it more likely for a 'head' or a 'tail' to come up? If the coin is unbiased, we find that there is an equal chance for a 'head' or a 'tail' to come up. Thus, the chance for a 'head' (or a 'tail') to come up is $\frac{1}{2}$. An alternative word used for 'chance' is 'probability' and it is generally represented by 'P'.

Mathematical definition of probability:

A. If the outcome of an operation can occur in n equally likely, mutually exclusive and exhaustive ways, and if m of these ways are favourable to an event E, then probability of E, denoted by P (E), is given by $P(E) = \frac{m}{n}$

B. As $0 \le m \le n$, therefore for any event E, we have $0 \le P(E) \le 1$.

C. The probability of E not occuring, denoted by P (not E), is given by P (not E) or P ($\bar{E}$) = 1 – P (E)

D. Odds in favour

$$= \frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}}$$

E. Odds against

$$= \frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}}$$

Mutually exclusive events and addition law

(A) Mutually exclusive events:

If two events are said to be mutually exclusive then if one happens, the other cannot happen and vice versa. In other words, the events have no simultaneous occurence. For example,

1. In rolling a die:
E : – The event that the number is odd
F : – The event that the number is even
G : – The event that the number is a multiple of three.

2. In drawing a card from a deck of 52 cards:
E : – The event that it is a spade.
F : – The event that it is a club.
G : – The event that it is a king.

In general P(A or B) = P(A) + P(B) – P($A \cap B$*)*
If A, B are mutually exclusive then $P(A \cap B) = 0$
If A, B are independent then

$P(A \cap B) = P(A) \cdot P(B)$

In the above 2 cases events E and F are mutually exclusive but the events E and G are not mutually exclusive or disjoint since they may have common outcomes.

(B) Additional law of probability:

If E and F are two mutually exclusive events, then the probability that either event E or event F will occur in a single trial is given by:

P(E or F) = P(E) + P(F)

If the events are not mutually exclusive, then

P(E or F) = P(E) + P(F) – P(E and F together).

Note: Compare this with of set theory.

Similarly, **P (neither E nor F) = 1 – P(E or F).**

Independent Events And Multiplication Law

(A) Two events are independent if the occurence of one has no effect on the occurence of the other.

For example,

1. On rolling a die and tossing a coin together:
   E : – The event that number 6 turns up.
   F : – The event that head turns up.
2. In shooting a target:
   E : – Event that the first trial is missed.
   F : – Event that the second trial is missed.
   In both these cases events E and F are independent.
3. In drawing a card from a well–shuffled pack:
   E : – Event that first card is drawn.
   F : – Event that second card is drawn without replacing the first .
   G : – Event that second card is drawn after replacing the first.
   In this case, E and F are not independent but E and G are independent.

(B) Multiplication law of probability:

If the events E and F are independent, then

P(E and F) = P (E) × P (F)

Example 1:

In a single throw of a fair dice what is the probability that the number the appearing on the top face of the dice is more than 2?

Solution:

In a dice there are 6 faces numbered 1, 2, 3, 4, 5 and 6.

So, the total number of possible events are 1, 2, 3, 4, 5 and 6 = 6 and the total number of favourable events are 3, 4, 5 and 6 = 4

So, the required probability is $\frac{4}{6} = \frac{2}{3}$

Example 2:

If two fair dice are thrown simultaneously, then what is the probability that the sum of the numbers appearing on the top faces of the dice is less than 4?

Solution:

Total number of possible events = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2) ….and so on. There will be 6 × 6 = 36 possible events.

Number of favourable events
= (1, 1), (1, 2) and (2, 1) = 3 events

So, the required probability

$= \frac{3}{36} = \frac{1}{12}$

Example 3:

If out of the first 20 natural numbers Mr. X selects a number at random, then what is the probability that this number will be a multiple of 4?

Solution:

Total number of possible events = 1, 2, 3, …, 20 = 20 such numbers

Total number of favourable events
= 4, 8, 12, 16 and 20 = 5 such numbers

So, the required probability = $\frac{5}{20} = \frac{1}{4}$

Example 4:

In the example 3, what is the probability that this number will be a multiple of 4 or 7?

Solution:

Total number of possible events
= 1, 2 … 20 = 20 such numbers

Numbers divisible by 4 = 4, 8, 12, 16, 20 = 5 such numbers

Number divisible by 7 = 7 and 14 = 2 such numbers

Since from 1 to 20 there is no number which is divisible by both 4 and 7. It is a case of mutually exclusive events.

So number of possible outcomes
= 5 + 2 = 7

So, the required probability is = $\frac{7}{20}$

Example 5:
In the example 3, what is the probability that the selected number is divisible by 2 and 4?

Solution:
The total number of possible events
= 20 such numbers
Number divisible by 2 and 4 means the number should be divisible by 4 (LCM of 2 and 4 is 4)
= 4, 8, 12, 16, 20 = 5 such numbers

So, the required probability is $\frac{5}{20} = \frac{1}{4}$

Example 6:
In the example 3, what is the probability that this number is divisible by 2 or 4?

Solution:
The total number of possible outcomes = 20 in number
Number divisible by 2 = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 = 10 such numbers
Number divisible by 4 = 4, 8, 12, 16 and 20 = 5 such numbers
There are certain numbers which are divisible by both 2 and 4, so it is case of non mutually exclusive events.
Number divisible by both 2 and 4 are 4, 8, 12, 16 and 20 = 5 such number
So, the required probability= P(A) + P(B) – P(C)

$$= \frac{10}{20} + \frac{5}{20} - \frac{5}{20} = \frac{10}{20} = \frac{1}{2}$$

Conditional Probability

Let A and B are two dependent events, then probability of occurence of event A when B has already occurred is given by $P(A|B)$

$$= \frac{P(A \cap B)}{P(B)}$$

Example 7:
From a pack of 52 cards, 4 cards were picked one at a time.

a. If the card picked is not replaced, find the probability that all the cards were aces.

b. If the card picked was replaced, what is the probability that all the 4 pickings were aces?

c. If the cards were picked all at a time, find the probability that all the 4 cards were aces.

Solution:

a. Probability that the first card is an ace is

$$\frac{{}^{4}C_1}{{}^{52}C_1}.$$

Probabilities that the 2nd, 3rd and 4th cards are all aces are

$$\frac{{}^{3}C_1}{{}^{51}C_1},\ \frac{{}^{2}C_1}{{}^{50}C_1} \text{ and } \frac{{}^{1}C_1}{{}^{49}C_1} \text{ respectively.}$$

Hence, the total probability is

$$\frac{{}^{4}C_1}{{}^{52}C_1} \times \frac{{}^{3}C_1}{{}^{51}C_1} \times \frac{{}^{2}C_1}{{}^{50}C_1} \times \frac{{}^{1}C_1}{{}^{49}C_1}$$

$$= \frac{4 \times 3 \times 2 \times 1}{52 \times 51 \times 50 \times 49} = \frac{1}{{}^{52}C_4}.$$

b. With replacement, the probability is

$$\left(\frac{{}^{4}C_1}{{}^{52}C_1}\right)^4 = \frac{1}{13^4}$$

c. If all the 4 cards were picked simultaneously, then the required probability is $\frac{{}^{4}C_4}{{}^{52}C_4} = \frac{1}{{}^{52}C_4}$.

Compare the cases (a) and (c). You would note that they are one and the same.

Example 8: One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that the card drawn is

i. a king,
ii. either red or king,
iii. red and a king.

Solution:
Out of 52 cards, one card can be drawn in ${}^{52}C_1$ ways. Therefore, exhaustive number of cases = ${}^{52}C_1$ = 52

i. There are 4 kings in a pack of cards, out of which one can be drawn in $^4C_1$. Therefore, favourable number of cases = $^4C_1 = 4$.

So, the required probability = $\frac{4}{52} = \frac{1}{13}$

ii. There are 28 cards in a pack of cards which are either a red or a king. Therefore, one can be drawn in $^{28}C_1$ ways. Therefore, favourable number of cases = $^{28}C_1 = 28$

So the required probability = $\frac{28}{52} = \frac{7}{13}$

iii. There are 2 cards which are red and king, i.e. red kings. Therefore, favourable number of cases = $^2C_1 = 2$.

So, the required probability = $\frac{2}{52} = \frac{1}{26}$

Example 9:

Three unbiased coins are tossed. What is the probability of getting the following?

i. All heads
ii. 2 heads
iii. Exactly 1 head

Solution:

If 3 coins are tossed together, we can obtain any one of the following as an outcome.

HHH, HHT, HTH, THH, TTH, THT, HTT, TTT

So exhaustive number of cases = 8

i. All heads can be obtained in only one way, i.e. HHH.
So, the favourable number of cases = 1

Thus, the required probability = $\frac{1}{8}$

ii. Two heads can be obtained in any one of the following ways: HHT, THH, HTH. So favourable number of cases = 3. Thus,

required probability = $\frac{3}{8}$

iii. Required probability = $\frac{3}{8}$. The probability of exactly 1 head is same as probability of exactly 1 tail (or 2 heads) since the coin is unbiased.

Example 10:

An urn contains 9 red, 7 white and 4 black balls. If 2 balls are drawn at random, find the probability that

i. both the balls are red,
ii. one ball is white.

Solution:

There are 20 balls in the bag out of which 2 balls can be drawn in $^{20}C_2$ ways. So the exhaustive number of cases = $^{20}C_2 = 190$

i. There are 9 red balls out of which 2 balls can be drawn in $^9C_2$ ways. Therefore, favourable number of cases = $^9C_2 = 36$.

So, the required probability = $\frac{36}{190} = \frac{18}{95}$

ii. There are 7 white balls out of which one white can be drawn in $^7C_1$ ways. One ball from the remaining 13 balls can be drawn in $^{13}C_1$ ways. Therefore, one white and one other colour ball can be drawn in $^7C_1 \times {}^{13}C_1$ ways. So the favourable number of cases
$= {}^7C_1 \times {}^{13}C_1 = 91$

So, the required probability = $\frac{91}{190}$

Let p be the probability of getting a head, q be the probability of not gettinga head (i.e. a tail).If n coins are tossed simultaneously or one coin is tossed n times, $^nC_r \cdot p^r \cdot q^{n-r}$ gives the probability of having r heads and (n – r) tails.

Example 11:

Four coins were tossed. What is the probability that

a. all the 4 coins showed a head?
b. exactly 3 coins showed a head and the fourth showed a tail?

Solution:
The problem is based on binomial distribution of probabilities.

If p + q = 1, then the term $^nC_r . p^r . q^{n-r}$ in the expansion $(p + q)^n$ gives the probability that when n such experiments are conducted r events are favourable and n–r events are unfavourable.

a. The probability that 4 heads occur when a coin is tossed 4 times is $^4C_4.\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4$

b. The probability that there are 3 heads and 1 tail corresponds to $^4C_3 \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) = \frac{1}{4}$

Example 12:
Two dice were thrown. What is the probability that

a. both of them showed a 6?

b. the sum of the numbers on the dice was 10?

Solution:

a. The probability that 1 die shows a 6 is $\frac{1}{6}$.

The probability that both the dice show a 6 is $\frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$.

b. The total number of cases in the sample space = 6 × 6 = 36

The cases satisfying a sum of 10 is {(4, 6) (5, 5)(6, 4)}, i.e. 3 events. Hence, the probability of having a sum of 3 is $\frac{3}{36}$

$= \frac{1}{12}$

Example 13:
Ramesh and Geeta were in the same class. The probability of Ramesh attending the class is 0.6. The probability of Geeta attending the class is 0.4. (Assume they behave independent of each other)

a. What is the probability that both of them attended the class?

b. What is the probability that at least one of them attended the class?

Solution:

a. Since the 2 events happen independent of each other, the probability of both Ramesh and Geeta attending the class simultaneously is 0. 6 × 0. 4 = 0.24

b. The probability of at least one of them attending the class is

P (Ramesh attends) + P (Geeta attends) – P (Both attend)

$\Rightarrow$ 0.6 + 0.4 – 0.24 = 0.76

Example 14:
Two machines A and B produce 100 and 200 items every day. Machine A produce 10 defective items and machine B produces 40 defective items. On one particular day the supervisor of the shop floor picked up an item and found that it was defective. Find the probability that it came from machine A.

Solution:
Method 1: The total number of defective items produced on any single day = 50

The number of defective items from machine A = 10

Hence, probability of that item having come from machine $A = \frac{10}{50} = \frac{1}{5}$

Method 2: Probability of finding a defective item = Probability that it is from machine A and is defective + Probability that it is from machine B and is defective

$$\Rightarrow \frac{100}{300}\times\frac{10}{100}+\frac{200}{300}\times\frac{40}{200}=\frac{50}{300}=\frac{1}{6}$$

Hence, probability that the defective item is from machine

$$A = \frac{\frac{100}{300}\times\frac{10}{100}}{\frac{1}{6}} = \frac{1}{5}$$

Method 2, Example 14

This illustrates Bay's theorem.
P (finding defective) = P (Def from A) + P (Def from B) = y + z (say)

If given that you have found a defective the probability that it is produced by

machine $A = \frac{y}{y+z}$.

Example 15:
Three black marketers A, B and C were selling the tickets of Jerry Maguire. The odds in favour of their selling all the tickets was 1 : 4, 2 : 3 and 4 : 1 respectively. What is the probability that at least one of them could sell all his tickets?

Solution:
Probabilities of the three selling all their tickets are $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{4}{5}$.

Hence, probability that at least one of them sells all the ticket is equal to 1 – (None of them sells all his tickets)

$$= 1-\frac{4}{5}\times\frac{3}{5}\times\frac{1}{5} = 1-\frac{12}{125} = \frac{113}{125}$$

Example 16:
A drawer contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted.
If one item is chosen at random, what is the probability that it is rusted or a bolt?

Solution:
Let A be the event that the item chosen is rusted and B be the event that the item chosen is a bolt.
Clearly, there are 200 items in all, out of which 100 are rusted.

$$\therefore P(A) = \frac{100}{200},\ P(B) = \frac{50}{200} \text{ and } P(A\cap B) = \frac{25}{200}$$

Required probability

$$= P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \left(\frac{100}{200}\right)+\left(\frac{50}{200}\right)-\left(\frac{25}{200}\right)=\frac{5}{8}$$

Example 17: An urn contains 5 white and 8 black balls. Two successive drawings of 3 balls at a time are made such that the balls are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and second draw gives 3 black balls.

Solution:
Consider the following events.
A = Drawing 3 white balls in first draw,
B = Drawing 3 black balls in the second draw
Required probability

$$= P(A\cap B) = P(A)\ P(B\,|\,A) \quad \ldots(i)$$

Now $P(A) = \frac{^{5}C_3}{^{13}C_3} = \frac{10}{286} = \frac{5}{143}$

After drawing 3 white balls in first draw,
10 balls are left in the bag, out of which 8 are black balls.

$$\therefore P(B\,|\,A) = \frac{^{8}C_3}{^{10}C_3} = \frac{56}{120} = \frac{7}{15}$$

Hence, the required probability **=**

$$P(A\cap B) = P(A)\ P(B\,|\,A)$$

$$= \left(\frac{5}{143}\right)\times\left(\frac{7}{15}\right)=\frac{7}{429}$$

Example 18:
A dart is thrown at a dart board whose dimensions are 5 m × 5 m. If the probability of missing the dart board 0.25, find the probability of hitting the board at a point that is at a maximum distance of 2 m from the centre of the board.

Solution:

Probability of hitting the dart board = 1 – 0.25 = 0.75

If the dart hits, then probability of hitting within the circle of radius 2 m

$$= \frac{\pi r^2}{a^2} = \frac{\pi(2)^2}{5^2} = \frac{4\pi}{25}$$

Hence, the resultant probability

$$= 0.75 \times \frac{4\pi}{25} = \frac{3\pi}{25}$$

Example 18 is a case of infinistic probability, we cannot count the number of favourable outcomes because they are infinite. Hence, we take the ratio of favourable area to total area.

Example 19:

If n persons are seated on a round table, what is the probability that 2 of them are always together?

Solution:

Total number of ways in which n persons can sit on a round table is (n – 1)! Therefore, exhaustive number of cases = (n – 1)!. Considering 2 individuals as one persons there are (n – 1) persons who can sit on a round table in (n – 2)! ways. But the 2 individuals can be seated together in 2! ways. Therefore, favourable number of cases = (n – 2)! × 2!

So required probability $= \frac{(n-2)! \times 2!}{(n-1)!} = \frac{2}{n-1}$

Example 20:

Three different prizes have to be distributed among 4 different students. Each student could get 0 to 3 prizes. If all the prizes were distributed, find

a. the number of ways the prizes are distributed,

b. the probability that exactly 2 students did not receive a prize.

Solution:

a. Each of the prizes could have been given to any of the 4 students.
Hence, the total number of ways of distributing the prizes $= 4^3$

Note: This will include all the cases when the prizes are distributed among 3 or 2 or only 1 student.

b. The total number of ways of distributing the prizes among exactly 2 students is
$\left({}^4C_2\right)(2^3 - 2)$ ways.
${}^4C_2$ gives the selection of 2 boys.
$2^3 - 2$ gives the total number of ways of distributing 3 prizes among those 2 students. The subtraction of the 2 cases is to take care of those cases when all the prizes are distributed to only one among the two.

$\therefore$ The required probability $= \frac{36}{64} = \frac{9}{16}$

3 Set Theory

Introduction

A few disconnected topics make their appearances in the management entrance examinations. Usually, a few questions only are asked from these topics. Among these, set theory is an important topic to study. It is also not very complicated and a lot of day-to-day applications of set theory are there.
The treatment of trigonometry given here is elementary as, usually, problems related to only height and distance are known to be asked in these examinations.

Learning Objectives

At the end of this chapter you would have learnt:

- Basic definitions in set theory
- Venn diagram representation of sets

Miscellaneous Topics

We will cover the following topics in this chapter.

1. Set theory
2. Trigonometry
3. Stocks and shares

Set Theory

Definition: A set is a well-defined collection of objects.

If A is a set and 'a' is an element of this set, we say that 'a' belongs to A or $a \in A$. A set 'A' which has only a finite number of elements is called a finite set. The number of elements in a finite set is denoted by n(A).

The universal set is the set containing all the elements under consideration.

The empty set or null set (ϕ) is the set which has no element.

If a is an element of set A, then we write $a \in A$ (read a belongs to A or a is a member of set A). If a does not belong to A, then we write $a \notin A$. It is assumed that either $a \in A$ or $a \notin A$ and the two possibilities are mutually exclusive.

Some important definitions:

Subset: If every element of A is an element of B, then A is called a subset of B and we write $A \subseteq B$. Every set is a subset of itself and the empty set is a subset of every set. A subset A of set B is called a proper subset of B if $A \neq B$ and we write $A \subset B$. If a set has n elements, then number of its subsets = 2^n.

Superset: If A is a subset of B, then B is known as the superset of A and we write $B \supseteq A$.

Power set: Let A be a set. Then the collection or family of all subsets of A is called the power set of A and is denoted by P(A).

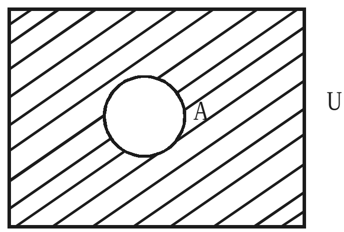
Example: Let A = {1, 2, 3}
Then the subsets of A are ϕ, {1}, {2}, {3}, {1, 2} {1, 3}, {2, 3} and {1, 2, 3}.
Hence P(A) = { ϕ, {1}, {2}, {1, 2} {1, 3}, {2, 3} {1, 2, 3}}

Universal set: A set that contains all the sets in a given context is called the universal set, i.e. It is the super set of all the sets under consideration e.g.
if A = {1, 2, 3} and B = {2, 4, 5, 6}, then a set of all natural numbers (N) can be taken as a universal set.

Introduction to Venn diagrams

The sets can be illustrated by means of Venn diagrams. A universal set U is represented by a rectangle and a subset by a circle within it.

Complement of a set



Let U and A be 2 sets such that $A \subseteq U$, then (U – A) is simply called the complement of A.

It is denoted by $\overline{A}$ or A'.
e.g. U is the set of natural numbers, the complement of odd numbers will be a set of even numbers.

The following letter sets are standard notations:
N: Set of natural numbers
Z: Set of integers

Q: Set of all rational numbers.
R: Set of all real numbers.
C: Set of all complex numbers.

Example 1:
U = {1, 2, 3, 4}, A = {3}. What is the complement of A?

Solution:
A' = {1, 2, 4}

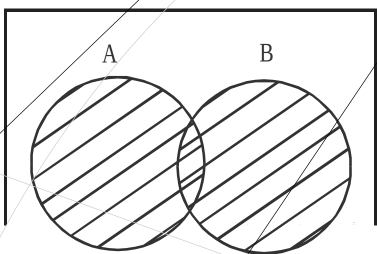
Remember the following

(i) $U' = \phi, \phi' = U$

(ii) (A')' = A

Union of Sets

If A and B are 2 sets, then the union of A and B, denoted by $A \cup B$, is the set of all elements which are **either** in A **or** in B or in both A and B.



$A \cup B$

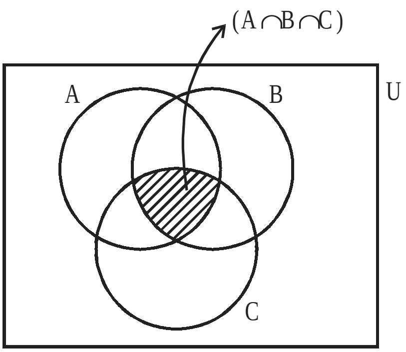
Solution:
A – B = {1, 2, 5}

Is A – B = B – A?
Find out for the sets A and B given in the example.

e.g. A = {1, 2, 3, 4, 5} and B = {3, 4, 6, 7}, find A – B.

Venn Diagrams

For three sets, the following diagram is valid.



$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap C) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ If $n(A \cup B)$ is not given and $n(A \cap B)$ is to be found, then we get a range of values for both in general. In specific cases, it might be a unique answer.

Example 4:
In a group of 800 persons, 600 can speak English and 400 can speak Telugu. If all the people speak at least one of the two languages, then find
a. how many can speak both the languages?
b. how many can speak exactly one language?

Solution:
a. $n(E \cup T) = n(E) + n(T) - n(E \cap T)$
$800 = 600 + 400 - n(E \cap T) \Rightarrow n(E \cap T) = 200$
b. People speaking exactly one language is equal to $n(E \cup T) - n(E \cap T)$ = 800 – 200 = 600

Example 5:
A survey shows that 63% of the Americans like apples whereas 76% like guns. What percentage of Americans like both apples and guns?

Solution:
The solution for the question cannot be determined. This is because we do not have the information whether all Americans like at least one of these.
(If we assume that 100% Americans like at least one of these)
Then n (A) = 63, n (G) = 76
and $n(A \cup G) = 100$
$\Rightarrow n(A \cap G) = n(A) + n(G) - n(A \cup G)$
$= 63 + 76 - 100 = 39$
Thus, 39% Americans like both guns and apples.

Example 6: In a certain city only 2 newspapers A and B are published. It is known that 25% of the city population reads A and 20% reads B, while 8% read both A and B. It is also known that 30% of those who read A but not B, look into advertisements and 40% of those who read B but not A, look into advertisements while 50% of those who read both A and B look into advertisements. What percentage of the population look into an advertisement?

Solution: Let A and B denote sets of people who read newspaper A and newspaper B respectively. Then
n(A) = 25, n(B) = 20, $n(A \cap B)$ = 8;
n(A – B) = n(A) – $n(A \cap B)$ = 25 – 8 = 17;
n (B – A) = n (B) – $n(A \cap B)$ = 20 – 8 = 12
Percentage of people reading an advertisement
= [(30% of 17) + (40% of 12) + (50% of 8)]%
= 13.9%

Concept of maximum or minimum

Type I:

Example 7:

In a school there are 200 students. 100 play cricket, 50 play hockey and 60 play basketball. 30 students play both cricket and hockey, 35 play both hockey and basketball, and 45 play both basketball and cricket.

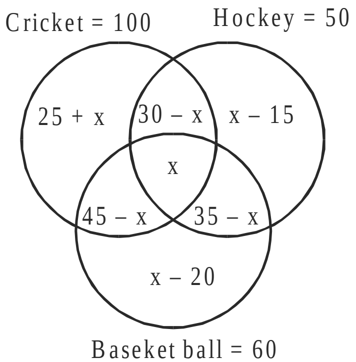
a. What is the maximum possible number of students who play at least one game?

b. What is the maximum possible number of students who play all the 3 games?

c. What is the minimum possible number of students playing at least one game?

d. What is the minimum possible number of students playing all the 3 games?

Solution:



Let x be the number of students playing all the 3 games.

Converting all values in terms of variable x, the number of students cannot be negative in any cell.

$\therefore\ x - 20 \geq 0$

$\therefore$ For minimum possible number of students playing all three games, i.e. x = 20

For maximum possible value of x, again none of the categories should have negative number of students.

$\therefore\ 30 - x \geq 0$

$x \leq 30$

If x is more than 30, (30 – x) would be negative which is not possible.

$\therefore\ 20 \leq x \leq 30$

Total number of students playing at least one game.

$= 100 + (x - 15) + (35 - x) + (x - 20) = 100 + x$

$\therefore$ Minimum possible number of students playing at least one game = 100 + 20 = 120

Maximum possible number of students playing at least one game

= 100 + 30 = 130

Alternatively (for Example 8) the minimum value can be found by:

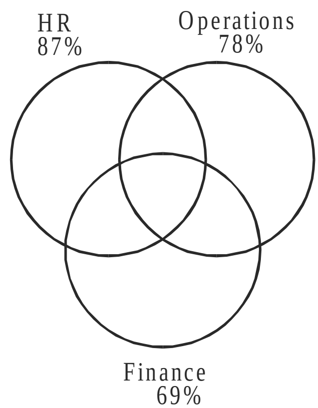
(78 + 69 + 87) – 200 = 34%

Type II:

Example 8:

In an office, where working in at least one department is mandatory, 78% of the employees are in operations, 69% are in finance and 87% are in HR. What are the maximum and minimum percentages of employees that could have been working in all three departments?

Solution:



Let the total number of employees in the office be 100.

Lets assume that x, y and z number of people are in exactly one, two and three departments of the office respectively.
Therefore, $x + 2y + 3z = 78 + 69 + 87 = 234$ and $x + y + z = 100$
$\Rightarrow (x + 2y + 3z) - (x + y + z) = 134$.
$\Rightarrow y + 2z = 134$.

Maximum possible value of z is $\frac{134}{2} = 67$

Therefore, maximum possible percentage of employees who could be working in all the three departments is 67%.

To minimize the value of z, we need to maximize the value of y, keeping in mind that $x + y + z = 100$.

Maximum possible value of y could be 66 and for this value of y, z = 34 and x = 0.

If we take a value of y greater than 66, lets say 68, then value of z comes out be 33, but here x + y + z is getting greater than 100, which is not possible.

Therefore, minimum possible percentage of employees who could be working in all the three departments is 34%.

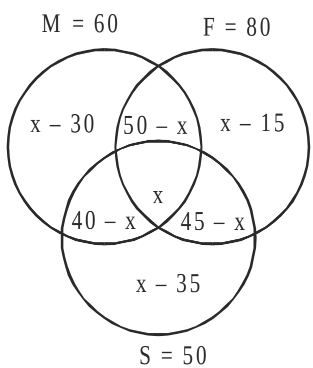
Alternative method:
Minimum: 87% are in HR, it means at least 13% are in operations or finance or operations and finance both. In the same way, 78% are in operations.
So at least 22% are in HR or finance or HR and finance both.
Similarly, 31% are in HR, or operations or HR and operations.
Adding all three, 13% + 22% + 31% = 66%
It means that if there is no intersection among these three sets, 66% would be maximum number of employees in A, B, C alone or
$(A \cap B), (B \cap C), (C \cap A)$.
This gives that at least 34% would be in all 3 departments.

The formula is $\left(\overline{A} + \overline{B} + \overline{C}\right)$

Type III:

Example 9:
There are 3 electives offered to the students in class of 92 (the students have a choice of not choosing any electives).
60 students opted for marketing, 80 for finance and 50 for systems, 40 students opted for both marketing and systems, 50 for both marketing and finance and 45 for both finance and systems. What is the maximum and the minimum possible number of students who opted for all 3 electives?

Solution:



Total number of students = 60 + x – 5 = 55 + x
The minimum possible value of x, so that none of the categories becomes negative = 35
Now applying the same concept, maximum possible value of x = 40. For this value of x, total number of students = 55 + 40 = 95
Which exceeds the total number of students, i.e. 92 by 3, which is not possible.
So to make it equal to 92 the maximum possible value of x = 37
$\therefore 35 \le x \le 37$

4 Practice Exercises

Introduction

There are 5 practice exercises out of which 2 are of level-1, 2 are of level 2 and 1 is of level 3 apart from the non MCQ to strengthen you fundamentals. While solving the exercises make sure that each and every concept is understood properly.

Problems for Practice (Non MCQ)

Level – 1

1. (a) Find r if
   (i) ${}^{10}P_r = 720$ (ii) ${}^{9}P_r = 3024$
   (b) Find n and r if
   (i) ${}^{n}P_r = 1680$ (ii) ${}^{n}P_r = 5040$

2. (a) Find n if ${}^{n}P_5 : {}^{n}P_3 = 2 : 1$
   (b) Find r if ${}^{9}P_5 + 5 \cdot {}^{9}P_4 = {}^{10}P_r$

3. In how many ways can 3 scholarships of unequal value be awarded to 17 candidates, such that no candidate gets more than one scholarship?

4. A man has 4 sons. There are 6 schools near his house. In how many ways can he send his sons to school, if no 2 of his sons are to study in the same school?

5. How many different 7-digit numbers can be formed from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9?

6. There are 15 railway stations between Bangalore and Hyderabad. How many different kinds of second class tickets must be printed so as to enable a passenger to travel from every place in the route to other?

7. In how many ways can 7 letters be posted in 4 letter boxes?

8. How many natural numbers can be formed by using any number of digits from 0, 1, 2, 3, 4? (Repetition is not allowed.)

9. Five persons are to address a meeting. If a specified speaker is to speak before another specified speaker, find the number of ways in which this can be scheduled.

10. In how many permutations of 10 things taken 4 at a time will one particular thing (i) always occur and (ii) never occur?

11. The letters of the word LABOUR are permuted in all possible ways and the words thus formed are arranged as in a dictionary. What is the rank of the word LABOUR?

12. In how many ways can 17 billiard balls be arranged in a row if 7 of them are black, 6 red and 4 white?

13. A round table conference is to be held between 20 delegates of 20 countries. In how many ways can they be seated if 2 particular delegates always sit together?

14. In how many ways can a committee of 6 men and 3 women be formed from a group of 10 men and 7 women?

15. Out of 8 gentlemen and 5 ladies a committee of 5 is to be formed. Find the number of ways in which this can be done so as to include at least 3 ladies.

16. There are 20 points in a plane. Five of them are collinear.
    i. How many triangles can be made using these points as the vertices?
    ii. How many straight lines can be drawn passing through at least 2 of these points?

17. What is the total number of 4-digit numbers that can be formed using the digits 0 to 5 without repetition, such that the number is divisible by 9?

Directions for questions 18 to 29: Answer the questions based on the information given below. There are 5 different boxes B1, B2, B3, B4, B5, and 5 different hats H1, H2, H3, H4, H5. The hats are to be distributed among the different boxes. Each box can accomodate all the hats.

18. If any box can have any number of hats, in how many ways can all the hats be distributed?

19. If all the hats are identical, in how many ways can the hats be arranged in the different boxes such that no box is without a hat?

20. If all the hats have different colours and each box can have only one hat, in how many ways can you arrange all the hats among the different boxes?

21. If the hats have to be arranged such that any box can have a maximum of one hat only, in how many ways can you arrange the hats among the 5 boxes? (At least one hat has to be distributed.)

22. If hats H1 and H2 are similar in all aspects, in how many ways can you arrange the hats in such a way that all the boxes have one hat?

23. If B1 can keep only hat H1 or H2, in how many ways can you arrange the hats such that all boxes have one hat?

24. What is the probability that B1 has either H1 or H2?

25. If B1 and B2 have the hats H1 and H2 among themselves, in how many ways can you arrange the hats among the 5 boxes?

26. In how many arrangements does B3 have hat H3?

27. If another hat H6 is also there, such that H6 has a different colour in comparison to all the other hats, in how many ways can you arrange the hats such that all the boxes have only one hat?

28. In question 30, if hat H6 has the same colour as H5, how many arrangements are there?

29. If it is known that hat H6 has the same colour as one of the other 5 hats, how many arrangements are possible in question 30?

30. In how many ways can 3 prizes be given to 4 contestants, if any contestant can receive any number of prizes?

31. m parallel lines in a plane are intersected by a family of n parallel lines. How many parallelograms are formed in the network thus formed?

32. In how many ways can 100 scouts be divided into squads of 50, 30 and 20 respectively?

33. There are 10 identical mangoes. In how many ways can you divide them among 3 brothers?

34. A person has to climb 10 steps. He climbs either a single step or 2 steps at a time. In how many ways can he do it?

35. The odds in favour of India winning a match against England is 4 : 3 and the odds against South Africa winning a match against Pakistan is 7: 5. Find the probability that at least one of them will win their respective matches.

36. A problem in mathematics is given to 3 students whose chance of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem is solved?

37. A bag contains 3 red and 5 black balls and a second bag contains 6 red and 4 black balls. A ball is drawn from each bag. Find the probability that both are (i) red, and (ii) black.

38. In a group there are 30 guys. 10 of them wear earrings, 15 of them wear ponytails. If the percentage of guys who wear neither is 30%, how many guys wear both earrings and ponytails?

39. The ratio of the par value of 2 shares X and Y is 2 : 1. Their market prices is in the ratio 3 : 1. If a person invests in them in the ratio 4 : 1, what is the ratio of the incomes if the dividends (in percentage terms) are in the ratio 5 : 1?

40. A person bought a share (par value = Rs. 50) at the rate of Rs. 150 from the market. If it yields a dividend of 20%, what is his effective return on investment?

41. Par values of two kinds of shares were Rs. 10 and Rs. 20 respectively. The market values are Rs. 150 and Rs. 200 respectively. Find the ratio in which the shares have to be purchased such that the profit from selling the shares in the market value is the same.

Level – 2

42. How many 5-digit numbers exist having exactly two 4s in them?

43. What is the sum of all 5-digit numbers formed using the digits 0, 2, 3, 4, 5?

44. There are 9 books of different subjects.
    i. What is the total number of selections of 3 books that can be made?
    ii. What is the total number of ways can 3 of these books be arranged on a shelf?
    iii. What is the total number of ways of dividing them into groups of 3 each?

45. What is the probability that when 2 dice and 4 coins are thrown simultaneously, there is a sum of 9 on the dice and at least 2 heads on the coins?

46. The probabilities of A, B, C solving a problem are $\frac{1}{3}$, $\frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

47. A gangster fires 4 bullets at the police inspector. The probability that the inspector will be killed by a bullet is 0.4. What is the probability that the inspector survives?

48. Two integers are selected at random from first 11 natural numbers. If the sum is even, find the probability that both the numbers are odd.

49. Twenty friends, including the host decide to party on a festive Saturday night in a bar. The host knows that 15 of them can have whisky and 10 of them can have rum. He also knows that each of them have 4 pegs of any drink. None of them has both whisky and rum simultaneously. Every peg of whisky costs Rs. 60 and every peg of rum costs Rs. 30. What is the maximum and the minimum budget that the host would have planned for? [Assume that atmost 5 people can go without drink in that party]

50. In a class of 50 students, a test for 2 subjects was conducted. 30 passed in the first subject and 40 passed in the second subject.
    (a) What is the maximum number of people who passed in both the subjects?
    (b) What is minimum number of people who passed in both the subjects?

Level – 3

51. A survey was conducted on a car brand KHATARA. It was found that 60% of vehicles had a problem with their engines, 60% of vehicles had a problem with the doors, 50% of vehicles had a problem with the tyres. 20% of the vehicles had no problem. What is the minimum percentage of vehicles which had all the 3 problems?

52. In question 51, what is the maximum percentage of vehicles which could have had all the problems?

Practice Exercise 1 - Level 1

1. Find the value of $^8P_6$

   a. 33425 b. 20160 c. 18972 d. 6625 e. 6620

2. Find the value of $^8C_6$

   a. 33 b. 32 c. 30 d. 28 e. 35

3. Find the number of ways in which the letters of the word BIHAR can be rearranged.

   a. 99 b. 129 c. 119 d. 125 e. 130

4. Find the number of ways in which the letters of the word AMERICA can be rearranged.

   a. 2519 b. 2620 c. 1250 d. 2500 e. 2000

5. Find the number of ways in which the letters of the word CALCUTTA can be rearranged.

   a. 3000 b. 5009 c. 5029 d. 5039 e. 5150

6. In how many ways can you arrange the letters of the word AKSHAY such that vowels do not start the words?

   a. $\frac{6!}{2!}-1$ b. $\frac{6!}{2!}-2$ c. $2 \times 5!$ d. 248 e. 120

7. In how many ways can 2 cards be drawn from a full pack of 52 cards such that both the cards are red?

   a. 275 b. 325 c. 350 d. 375 e. 300

8. How many four-digit numbers each consisting of 4 different digits can be formed with the digits 0, 1, 2, 3?

   a. 10 b. 12 c. 18 d. 20 e. 16

9. In a tournament 7 teams are participating. Each team plays with every other participating team once and the winner is decided by the total points accumulated by the teams at the end of all these matches. Find the total number of matches in the tournament.

   a. 7! b. 7! – 1 c. 20 d. 21 e. 25

10. Ram buys 7 novels from a book fair. Shyam buys 8 novels from the fair, none of which is common with those bought by Ram. They decide to exchange their books one for one. In how many ways can they exchange their books for the first time?

    a. 7! × 8! b. 7 × 8! c. 7! × 8 d. 56 e. None of these

11. In an Olympic 100 m race, 7 athletes are participating. Then the number of ways in which the first 3 prizes can be won is

    a. 7! b. 7^3 c. 3^7 d. 210 e. 320

12. After group discussion and interview 6 candidates were selected for admission in a college. But unfortunately the number of seats left is 2. So it was left to the principal to select 2 candidates out of them. In how many ways can he select 2 candidates?

    a. $^6P_2$ b. $\frac{6!}{2!}$ c. 15 d. 20 e. 18

13. In an examination 10 questions are to be answered choosing at least 4 from each of part A and part B. If there are 6 questions in part A and 7 in part B, in how many ways can 10 questions be answered?

    a. 212 b. 280 c. 272 d. 312 e. 266

14. There are 2 parallel line segments AB and CD in a plane. AB contains 12 marked points whereas CD contains 8 marked points. How many triangle can be formed by using these marked points as vertices?

    a. 12! × 28 + 8! × 66 b. 12! × 8! c. $^{20}C_3$ d. 864 e. 1024

15. In a box there are 5 distinct white and 6 distinct black balls. A person has to pick up 2 balls from the box such that there is one each of both the colours. In how many ways can he pick up the balls?
a. 25 b. 30 c. 35 d. 40 e. 45

16. The product of any r consecutive positive integers must be divisible by
a. r^2 b. r! c. (r – 1)!
d. ${}^{r-1}C_{r+1}$ e. None of these

17. Two cards are drawn together from a pack of 52 cards at random. What is the probability that both the cards are spades?
a. $\frac{{}^{4}C_{2}}{{}^{52}C_{2}}$ b. $\frac{{}^{13}C_{2}}{{}^{52}C_{2}}$
c. $\frac{{}^{26}C_{2}}{{}^{52}C_{2}}$ d. $\frac{{}^{8}C_{2}}{{}^{52}C_{2}}$
e. $\frac{{}^{13}C_{2}}{{}^{51}C_{2}}$

18. In question number 17, what is the probability that both the cards are kings?
a. $\frac{{}^{8}C_{2}}{{}^{52}C_{2}}$ b. $\frac{{}^{13}C_{2}}{{}^{52}C_{2}}$
c. $\frac{{}^{26}C_{2}}{{}^{52}C_{2}}$ d. $\frac{{}^{4}C_{2}}{{}^{52}C_{2}}$
e. $\frac{{}^{10}C_{2}}{{}^{52}C_{2}}$

19. In question number 17, what is the probability that one card is a spade and one card is a heart?
a. $\frac{{}^{13}C_{1}\times{}^{13}C_{2}}{{}^{52}C_{2}}$ b. $\frac{{}^{13}C_{1}\times{}^{26}C_{1}}{{}^{52}C_{2}}$
c. $\frac{13}{52}\times\frac{13}{52}$ d. $\frac{{}^{13}C_{1}\times{}^{13}C_{1}}{{}^{52}C_{2}}$
e. $\frac{{}^{13}C_{2}\times{}^{13}C_{2}}{{}^{52}C_{2}}$

20. In question number 17, what is the probability that exactly one card is a king ?
a. $\frac{{}^{52}C_{1}}{{}^{52}C_{2}}$ b. $\frac{4}{{}^{58}C_{2}}$
c. $\frac{{}^{4}C_{1}\times{}^{48}C_{1}}{{}^{52}C_{2}}$ d. $\frac{1}{2}$
e. $\frac{3}{{}^{52}C_{2}}$

Practice Exercise 2 - Level 1

1. A telegraph has 5 arms and each arm is capable of 4 distinct positions including the position of rest. What is the total number of signals that can be made?
   a. 1,024 b. 1,021 c. 1,020
   d. 1,022 e. 1,023

2. If A and B are 2 independent events and P(A) = 0.5 and P(B) = 0.4, find $P\left(\frac{A}{B}\right)$.
   a. 0.5 b. 0.4 c. 0.88
   d. 0.6 e. 0.74

3. A set of cards bearing the numbers 100-199 is used in a game. If a card is drawn at random, what is the probability that it is divisible by 3?
   a. $\frac{2}{3}$ b. 0.33 c. $\frac{32}{99}$
   d. $\frac{1}{5}$ e. None of these

4. A box contain 6 red balls, 7 green balls and 5 blue balls. Each ball is of a different size. The probability that the red ball being selected is the smallest red ball, is
   a. $\frac{1}{18}$ b. $\frac{1}{3}$ c. $\frac{1}{6}$
   d. $\frac{2}{3}$ e. $\frac{1}{5}$

5. A flagpost of $2\sqrt{3}$ m long casts a shadow of 2 m long at some point of time. What is the angle of elevation of sun with respect to the pole at that point of time?
   a. 25° b. 45° c. 60°
   d. 75° e. None of these

6. In a shooting competition a shooter has to hit any point on the target board in his last shot to win the tournament. His gun deviates by 30° in left or right when he shoots. If he is standing 15 m away from the board and direction of his gun is normal to the centre of the target board, what should be the diameter of the board so that he surely wins?
   a. 10 m b. $10\sqrt{3}$ m c. $11\sqrt{3}$ m
   d. $15\sqrt{3}$ m e. $12\sqrt{3}$ m

7. If θ and α are complementary angles and $\sin\theta = \frac{p}{q}$ and $\cos\alpha = \frac{r}{s}$, then which of the following gives the relation between p, q, r and s?
   a. pr = qs b. pr + qs = 0
   c. ps = qr d. p + q + r + s = 0
   e. pq = rs

8. tan 1° tan 2° tan 3° .... tan 87° tan 88° tan 89° =
   a. tan 90° b. 0 c. 1
   d. 2 e. None of these

9. If $\sin\theta + \cos\theta = \frac{7}{5}$, then $\sin\theta\cos\theta$ =
   a. $\frac{1}{25}$ b. $\frac{11}{45}$ c. $\frac{12}{25}$ d. $\frac{12}{19}$ e. $\frac{13}{25}$

10. If $\sec\theta = \frac{5}{4}$, then $\text{cosec}\,\theta$ = 0
    a. $\frac{4}{5}$ b. $\frac{3}{5}$ c. $\frac{6}{7}$ d. $\frac{4}{9}$ e. $\frac{5}{3}$

Directions for questions 11 to 13: Read the passage given below and answer the questions.
In a class of 33 students, 20 play cricket, 25 play football and 18 play table tennis. 15 play both cricket and football, 12 play football and table tennis, 10 play table tennis and cricket. Each student plays at least one game. Find the number of students.

11. How many students play only cricket?
    a. 5 b. 6 c. 2 d. 3 e. 7

12. How any students play all the 3 games?
    a. 5 b. 7 c. 2 d. 3 e. 4

13. How many students play only 2 games?
    a. 16 b. 18 c. 20 d. 22 e. 24

14. In a group of 500 students, selected for admission in a business school, 64% opted for finance and 56% for operations as specialisations. If dual specialisation is allowed, how many have opted for both? Each student opts for at least one of the two specialisations.
    a. 200 b. 100 c. 150 d. 125 e. 140

Directions for questions 15 to 17: Read the passage given below and answer the questions.
In a locality, 30% of the residents read *The Times of India* and 75% read *The Hindustan Times*. 3 people read neither of the papers and 6 read both. Only *The Times of India* and *The Hindustan Times* newspapers are available.

15. How many people are there in the locality?
    a. 60 b. 120 c. 126 d. 130 e. 90

16. What is the percentage of people who read only *The Times of India*?
    a. 15% b. 20% c. 25% d. 30% e. 35%

17. What percentage of residents read only one newspaper?
    a. 11% b. 43% 0c. 85% d. 20% e. 60%

Directions for questions 18 and 19: Answer the questions based on the following information.
In a sports centre 70 students play cricket, 50% play hockey, 25% play both hockey and cricket and 5% play none.

18. How many students are there in the class?
    a. 80 b. 85 c. 95 d. 100 e. 110

19. How many students play only one game?
    a. 65 b. 70 c. 75 d. 80 e. 90

Directions for questions 20 and 21: Read the following information and answer the questions.
In an examination there are 150 candidates. 40 candidates passed in papers A and B; 40 candidates passed in papers B and C; 30 candidates passed in papers C and A ; and 10 candidates passed in all the 3 papers.

20. How many students passed in paper B only?
    a. 40 b. 20 c. 15
    d. 25 e. Cannot be determined

21. If no students failed in all the 3 subjects, what is the total number of students who passed in exactly one paper?
    a. 25 b. 45 c. 60 d. 50 e. 55

Practice Exercise 3 - Level 2

1. How many distinct 4 letter words can be formed by using the letters a, b, c and d? (Repetition of the letters is allowed).
   a. 296 b. 346 c. 440 d. 256 e. 361

2. How many four digit numbers can be formed by using the digits 2, 3, 4 and 5?
   a. 58 b. 512 c. 64
   d. 256 e. None of these

3. How many numbers greater than 4000 can be made by using the digits 2, 3, 4 and 5? (Repetition of the digits is not allowed).
   a. 12 b. 14 c. 20 d. 24 e. 30

4. How many numbers greater than 4000 can be made by using the digits 2, 3, 4 and 5? (Repetition of digits is allowed).
   a. 120 b. 128 c. 138 d. 130 e. 125

5. If 4 dices and 3 coins are tossed simultaneously, then find the number of elements in the sample space.
   a. $2^4 \times 6^3$ b. $6^4 \times 2^3$ c. 2156
   d. $4^2 \times 3^6$ e. $4^6 \times 3^2$

6. There are 3 roads from A to B, 4 roads from B to C , and 1 road from C to D. How many combinations of roads are there from A to D?
   a. 11 b. 15 c. 14 d. 12 e. 10

7. There are 5 questions in a question paper. In how many ways a candidate can attempt at least 1 question?
   a. 30 b. 34 c. 32 d. 31 e. 20

8. In how many ways can the letters of the word ‘POSSESS’ be arranged so that the four Ss are in alternate positions only?
   a. 8 b. 6 c. 12 d. 10 e. 16

9. In how many ways can a committee of 3 men and 2 women be formed out of a total of 4 men and 4 women?
   a. 15 b. 16 c. 20 d. 28 e. 24

10. A six-face die, an eight-face die and a ten-face die are thrown together. What is the probable number of outcomes?
    a. 286 b. 320 c. 480 d. 492 e. 360

11. In an entrance test, a candidate is required to attempt a total of 4 questions which are to be attempted from 2 sections each containing 5 questions. The maximum number of questions that he can attempt from any section is 3. In how many ways can he answer in the test?
    a. 150 b. 175 c. 200 d. 250 e. 240

12. In a cultural festival, 6 programmes are to be staged, 3 on a day for 2 days. In how many ways could the programmes be arranged?
    a. 320 b. 360 c. 675
    d. 720 e. None of these

13. All the odd numbers from 1 to 9 are written in every possible order. How many numbers can be formed if repetition is not allowed?
    a. 60 b. 120 c. 150 d. 180 e. 90

14. How many numbers lying between 3000 and 4000 and made with the digits 3, 4, 5, 6, 7 and 8 are divisible by 5? Repetitions are not allowed.
    a. 5! b. 4! c. 12 d. 6 e. 20

15. Five persons A, B, C, D and E occupy seats in a row such that A and B sit next to each other. In how many possible ways can these 5 people sit?
    a. 24 b. 48 c. 72 d. 96 e. 40

16. Ten distinguishable balls are distributed into 4 distinct boxes such that a specified box contains exactly 2 balls. Find the number of such distributions.
    a. 3^8 b. 3^{10} c. 3^6
    d. 45×3^8 e. None of these

17. Five speakers A, B, C, D and E are to be scheduled to speak such that A must speak immediately before B. In how many ways can their speeches be scheduled?
a. 32 b. 48 c. 72 d. 96 e. 24

18. A production unit produces 10 articles of which 4 are defective. A quality inspector allows release of the products if he finds none out of the 3 articles he chooses at random to be defective. In how many ways can he pick up the 3 articles such that he clears the release?
a. 10 b. 6! c. 20 d. 18 e. 24

19. $P_i = {}^iP_i$, where i is an integer. Then $1 + 1P_1 + 2P_2 + 3P_3 + \cdots + nP_n =$
a. n! b. (n + 1)! c. (n + 2)!
d. $\frac{(n+2)!}{n-1}$ e. None of these

20. From a class of 12 students, 5 are to be chosen for an excursion. But 3 very close friends decide among themselves that either all 3 of them will go or none of them will go. In how many ways can the excursion party be chosen?
a. 150 b. 156 c. 162 d. 169 e. 184

21. How many different words can be made from the word 'EDUCATION' so that all the vowels are always together? (Do not bother about many meaningless words.)
a. 12,320 b. 13,460 c. 14,400
d. 16,200 e. 18,400

22. How many numbers are there between 100 and 1000 such that every digit is either 4 or 5?
a. 1 b. 6 c. 5 d. 4 e. 8

23. A tea-expert claims that he can easily find out whether milk or tea leaves were added first to water just by tasting the cup of tea. In order to check this claim 10 cups of tea are prepared, 5 in one way and 5 in the other. Find the different possible ways of presenting these 10 cups to the expert.
a. 100 b. 10! c. $\frac{10!}{(5!)^2}$
d. 300 e. 240

24. In how many ways can a leap year have 53 Sundays?
a. ${}^{365}C_7$ b. 7 c. 4
d. 2 e. None of these

25. On their 10th wedding anniversary a Bengali couple bought 10 different sweets and then distributed it between 2 of their family friends such that both of them got 5 sweets each. Find the number of different ways in which this distribution can be done.
a. 126 b. 252 c. 350
d. 729 e. None of these

26. In the country Utopia, the language contains only 4 letters. Find the maximum number of words that can exist in the Utopian dictionary if no letter can be repeated in a word.
a. 26 b. 4! c. 40 d. 64 e. 80

27. A company could advertise about its new product in 4 magazines, 3 newspapers and 2 television channels. But in a later move it decided to give advertisements in only 2 of the magazines, one of the newspapers and one of the TV channels. In how many ways can they advertise their product?
a. 30 b. 36 c. 44 d. 40 e. 48

28. The first 5 odd natural numbers are written in every possible order. How many numbers can be formed if no repetition is allowed and what is their sum?
a. 5!, 6666600 b. ${}^5C_1$, 10^5 c. 51, 55555
d. 50, 666660 e. None of these

29. In how many ways one or more than one fruit can be selected from 6 varieties of fruits, given that there are 5 fruits of each variety? (All the fruits of one variety are identical.)
a. 6^6 b. 5^6 c. $6^6 - 1$
d. 5^6 e. None of these

Practice Exercise 4 - Level 2

1. In a staircase there are 4 steps. A person can jump one step, 2 steps, 3 steps or all 4 steps. In how many ways can he reach the top?
a. 2 b. 4 c. 6 d. 8 e. 10

2. Kapil wishes to pay Rs. 255 with hundred notes. In how many ways can this task be performed if Kapil had hundred notes of value Re 1 and Rs. 5 only?
a. More than 100
b. More than 50 but less than 100
c. 50
d. 10
e. 0

3. A committee is to be formed comprising of 7 members such that there is a majority of men and at least 1 woman in the committee. The shortlisting for the committee is done out of 9 men and 6 women. In how many ways can this be done?
a. 3,724 b. 3,630 c. 3,526
d. 4,914 e. 4312

4. A committee is to be formed comprising 7 members such that there is a simple majority of men and at least 1 woman. The shortlist consists of 9 men and 6 women. In how many ways can this committee be formed?
a. 3,724 b. 3,630 c. 4,914
d. 5,670 e. 3,824

5. For the BCCI, a selection committee is to be chosen consisting of 5 ex-cricketers. Now there are 12 representatives from four zones. It has further been decided that if Srikanth is selected, Mohinder Amarnath will not be selected and vice versa. In how many ways can this be done?
a. 572 b. 372 c. 672 d. 472 e. 362

6. How many five-digit positive integers have the product of their digits equal to 2000?
a. 15 b. 20 c. 22 d. 30 e. 36

7. A bag contains 6 white balls and 4 red balls. Three balls are drawn one by one with replacement. What is the probability that all the 3 balls are red?
a. $\frac{8}{125}$ b. $\frac{1}{20}$ c. $\frac{1}{30}$
d. $\frac{1}{120}$ e. $\frac{7}{20}$

8. In the above question, if 3 balls are drawn one by one with replacement, then what is the probability that 2 balls are white and 1 ball is red?
a. $\frac{54}{125}$ b. $\frac{1}{4}$ c. $\frac{1}{3}$
d. $\frac{1}{2}$ e. $\frac{53}{125}$

9. In question 7, if the balls are drawn without replacement. What is the probability that 2 balls are red and 1 ball is white?
a. 0.1 b. 0.2 c. 0.3
d. 0.4 e. 0.5

10. The probability that A will pass the examination is $\frac{1}{3}$ and the probability that B will pass the examination is $\frac{1}{2}$. What is the probability that both A and B will pass the examination?
a. $\frac{1}{6}$ b. $\frac{1}{4}$ c. $\frac{2}{3}$
d. $\frac{1}{3}$ e. $\frac{1}{2}$

11. In Q. No. 10, what is the probability that only one person [either A or B] will pass the examination?

a. $\frac{1}{6}$ b. $\frac{1}{2}$ c. $\frac{1}{3}$

d. $\frac{2}{3}$ e. $\frac{1}{4}$

12. In Q. No. 10, what is the probability that at least one person will pass the examination?

a. 1 b. $\frac{1}{2}$ c. $\frac{1}{3}$

d. $\frac{2}{3}$ e. $\frac{1}{4}$

13. In Q. No. 10, what is the probability that no one will pass the examination?

a. $\frac{2}{3}$ b. $\frac{1}{2}$ c. $\frac{1}{4}$

d. $\frac{1}{3}$ e. $\frac{1}{6}$

14. When 2 fair dice are thrown simultaneously, what is the probability that one die will show more value than the other?

a. $\frac{1}{6}$ b. $\frac{1}{16}$ c. $\frac{5}{6}$

d. $\frac{1}{2}$ e. $\frac{15}{16}$

15. A basket contains 20 apples and 10 oranges out of which 2 oranges and 5 apples are defective. If a person takes out 2 at random, what is the probability that either both are apples, or both are good?

a. $\frac{119}{435}$ b. $\frac{338}{435}$

c. $\frac{841}{870}$ d. $\frac{217}{870}$

e. None of these

16. What is the probability of getting a 2 in the roll of 2 fair dice, given that the sum is 7?

a. $\frac{2}{3}$ b. $\frac{25}{36}$ c. $\frac{11}{36}$

d. $\frac{1}{4}$ e. $\frac{1}{3}$

17. In an urn there are 6 red, 4 black and 3 white balls. 3 balls are drawn out of it simultaneously. What is the probability that all the three are of the same colour?

a. $\frac{7}{220}$ b. $\frac{9}{44}$ c. $\frac{25}{286}$

d. $\frac{35}{286}$ e. $\frac{21}{44}$

18. When 3 fair coins are tossed together, what is the probability of getting at least 2 tails?

a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. $\frac{1}{3}$

d. $\frac{2}{3}$ e. None of these

19. The probability that a bullet fired from a point will hit the target is $\frac{1}{3}$. Three such bullets are fired simultaneously towards the target from that very point. What is the probability that the target will be hit?

a. $\frac{1}{27}$ b. $\frac{1}{8}$ c. $\frac{19}{27}$

d. $\frac{8}{27}$ e. $\frac{7}{8}$

20. In a management entrance examination there are 200 questions with four alternatives each. A student marks first alternative as the answer to all the questions. What is his probable net score if each right answer fetches +1 and each wrong answer fetches $-\frac{1}{4}$ marks?

a. 0 b. 10 c. 12.5
d. –12.5 e. None of these

21. There are 2 positive integers a and b. What is the probability that a + b is odd?

a. $\frac{1}{4}$ b. $\frac{1}{3}$ c. $\frac{1}{2}$
d. $\frac{1}{5}$ e. $\frac{2}{3}$

22. A 5-digit number is formed by the digits 1, 2, 3, 4 and 5 without repetition. What is the probability that the number formed is a multiple of 4?

a. $\frac{1}{4}$ b. $\frac{3}{5}$ c. $\frac{2}{5}$
d. $\frac{4}{5}$ e. $\frac{1}{5}$

23. If n is any positive integer greater than 1, then $\left(2^{3n} - 7n - 1\right)$ must be divisible by

a. 2 b. 5 c. 36
d. 49 e. 40

24. In a class of 60 boys, there are 45 boys who play cards and 30 boys play carrom. Find how many boys play both the games. (assuming that every boy plays either cards or carrom or both)

a. 15 b. 17 c. 20
d. 21 e. 16

25. In question number 24, find the number of boys who only play cards.

a. 27 b. 30 c. 32
d. 25 e. 35

26. In question number 24, find the number of boys who only play carrom.

a. 10 b. 12 c. 15
d. 20 e. 14

27. Each student in a class of 40, studies at least one of the subjects namely English, Mathematics and Economics. 16 study English, 22 study Economics and 26 study Mathematics, 5 study English and Economics, 14 study Mathematics and Economics and 2 study English, Economics and Mathematics. Find the number of students who study English and Mathematics.

a. 10 b. 7 c. 17
d. 27 e. None of these

28. In question number 27, find the number of students who study English and Mathematics but not Economics.

a. 8 b. 12 c. 7
d. 5 e. 6

29. If $\tan\theta + \cot\theta = 2$ and $0 < \theta < 90°$, then $\sin\theta + \operatorname{cosec}\theta =$

a. 1 b. $\frac{3}{\sqrt{2}}$ c. $\frac{1}{\sqrt{2}}$
d. $\frac{3}{5}$ e. None of these

30. If $\sin\theta$ and $\cos\theta$ are 2 roots of the equation $ax^2 + bx + c = 0$, then which of the following is true?

a. $a^2 + b^2 = 2c$ b. $a + b = c$
c. $a^2 - b^2 + 2ac = 0$ d. $a + b = 2c$
e. None of these

Practice Exercise 5 - Level 3

1. A garland is to be prepared with 10 different flowers such that 2 particular flowers will be next to each other. Find the number of different garlands that can be formed.
   a. 8! b. 80,640
   c. 26,880 d. 40,000 e. None of these

2. In how many ways can 6 identical rings be worn in 4 fingers of one hand assuming any number of rings can be worn in one finger?
   a. 6^4 b. 4^6 c. $4! \times 6!$
   d. 84 e. 196

3. In a global conference there are 16 delegates who are to be seated along 2 sides of a long table with 8 chairs on each side. Four delegates having same views wish to sit on one particular side whereas 2 delegates having views opposite to them wish to sit on the other side of the table. In how many ways can these 16 delegates be seated?
   a. ${}^8C_4 \times {}^8C_2 \times 10!$
   b. ${}^8P_4 \times {}^8P_2 \times 10!$
   c. $(8!)^4 \times (10!)^2$
   d. 48
   e. None of these

4. In how many ways can 3 children in a family have all different birthdays in a leap year?
   a. ${}^{365}C_3$
   b. ${}^{365}C_2 - 1$
   c. $365^2 \times 364 \times 363$
   d. $364 \times 363 \times 362$
   e. None of these

5. A box contains 20 tickets of identical appearance, the tickets being numbered 1, 2, 3, ..., 20. In how many ways can 3 tickets be chosen such that the numbers on the drawn tickets are in arithmetic progression?
   a. 18 b. 33 c. 56 d. 90 e. 84

6. An intelligence agency decide on a code of 2 digits selected from 0, 1, 2, …, 9. But the slip on which the code is handwritten, allows confusion between the top and the bottom, because these are indistinguishable. Thus, for example, the code 81 could be confused with 18. How many codes are there such that there is no possibility of any confusion?
   a. 25 b. 75 c. 80
   d. 70 e. None of these

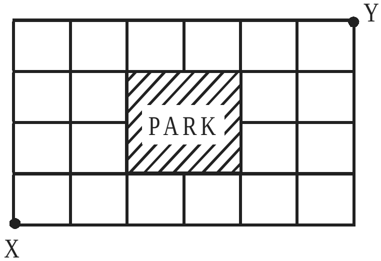
7. For the BCCl, a selection committee is to be chosen consisting of 5 ex-cricketers. Now there are 10 representatives from various zones. It has further been decided that if Kapil Dev is selected, Sunil Gavaskar will not be selected and vice versa. In how many ways can this be done?
   a. 140 b. 112 c. 196 d. 56 e. 80

8. An executive wrote 5 letters to 5 people A, B, C, D and E and asked his secretary to place them in 5 envelopes also marked A, B, C, D and E. The secretary, however, placed the letters at random into the envelopes such that each envelope received exactly one letter. Then the number of arrangements in which exactly 2 letters are placed in correct envelopes is
   a. 10 b. 15 c. 30 d. 25 e. 20

9. From a pack of 52 playing cards, 4 cards are removed at random. In how many ways can the 1st place and 3rd place cards be drawn out such that both are black?
   a. 64,974 b. 62,252 c. 69,447
   d. 15,92,500 e. 64,256

10. If the vowels A, E, I, O and U are given by 0the symbols !, @, #, $ and % respectively, then the message given by the defence code 2$%8!5@ will be
    a. YOU VASE b. YOU SAVE
    c. YIU SAFE d. YIU SAVE
    e. YIU VASE

11. The following diagram shows the road map of a city. The lines through the city indicate roads but there is no road through the park. All the roads are either parallel or perpendicular to each other. Peter wants to go from X to Y travelling the minimum possible distance. In how many ways can he make his journey?



a. 55 b. 100 c. 166 d. 220 e. 110

12 How many five-digit numbers can be formed such that it has the following properties:
I. It has at least one zero and at most three zeros.
II. The non-zero digits are non-repeating.
a. 9962 b. 17378 c. 12570
d. 14398 e. 15408

13. If two dices are thrown simultaneously, then what is the probability that the product of the numbers appearing on the top faces of the dice is less than 36?

a. $\frac{35}{36}$ b. $\frac{1}{6}$ c. $\frac{23}{36}$ d. $\frac{32}{36}$ e. $\frac{34}{36}$

14. Two urns contain 3 white and 4 black balls, and 2 white and 5 black balls. One ball is transferred to the second urn and then one ball is drawn from the second urn. Find the probability that the first ball transferred is black, given that the ball drawn is black.

a. $\frac{15}{39}$ b. $\frac{39}{56}$ c. $\frac{8}{13}$ d. $\frac{10}{39}$ e. $\frac{5}{13}$

15. In a pack of cards having numbers between 100 and 999 (both inclusive), what is the probability of drawing a multiple of 3, the number should comprises of digits 1, 0, 2, 3, 4?

a. $\frac{24}{900}$ b. $\frac{33}{900}$ c. $\frac{1}{30}$ d. $\frac{20}{900}$ e. None of these

16. If we pick a number between 1 and 999 s(both inclusive) randomly, what is the probability that the number is an even number with no digit repeated?

a. $\frac{662}{999}$ b. $\frac{631}{999}$ c. $\frac{337}{999}$ d. $\frac{320}{999}$ e. $\frac{373}{999}$

17. Σn is written for n = 1 to n = 99 on cards. What is the probability of drawing a card with an even number written on it?

a. $\frac{1}{2}$ b. $\frac{49}{100}$ c. $\frac{49}{99}$ d. $\frac{50}{99}$ e. $\frac{2}{3}$

18. In the previous question, what is the probability that the number is more than 100?

a. $\frac{13}{99}$ b. $\frac{86}{99}$ c. $\frac{14}{100}$ d. $\frac{85}{100}$ e. $\frac{73}{99}$

19. There are 1001 red balls and 1001 black balls in a box. Two balls are drawn without replacement one after the other. Let 'P_s' be the probability that two balls drawn at random from the box are of the same colour, and let 'P_d' be the probability that they are of different colours. The difference between P_s and P_d is

a. 0 b. $\frac{1}{2002}$ c. $\frac{1}{2001}$

d. $\frac{2}{2001}$ e. $\frac{1}{1001}$

20. Sudip thought of a two-digit number and divided the number by the sum of the digits of the number. He found that the remainder is 3. Sonal also thought of a two-digit number and divided the number by the sum of the digits of the number. She also found that the remainder is 3. Find the probability that the two-digit number thought by Sudip and Sonal is same.

a. $\frac{1}{11}$ b. $\frac{1}{12}$ c. $\frac{1}{13}$ d. $\frac{1}{14}$ e. $\frac{1}{15}$

Answers Key

Practice Exercise (Non MCQ)

Level – 1

1. *(a)* (i) 3 (ii) 4
(b) (i) 8, 4 (ii) 10 and 4

2. *(a)* 5 *(b)* 5

3. $^{17}P_3$

4. $^{6}P_4$

5. 9×10^6

6. $^{17}P_2$

7. 4^7

8. 260

9. $\frac{1}{2}(5!)$

10. *(i)* $4 \times {}^{9}P_3$ *(ii)* $^{9}P_4$

11. 242

12. $\frac{17!}{7!\ 6!\ 4!}$

13. $18! \times 2$

14. $^{10}C_6 \times {}^{7}C_3$

15. 321

16. *(i)* $^{20}C_3 - {}^{5}C_3$ *(ii)* $^{20}C_2 - {}^{5}C_2 + 1$

17. 36

18. 5^5

19. 1

20. $^{5}P_5$

21. $^{5}C_1 \times {}^{5}P_1 + {}^{5}C_2 \times {}^{5}P_2 + {}^{5}C_3 \times {}^{5}P_3 + {}^{5}C_4 \times {}^{5}P_4 + {}^{5}C_5 \times {}^{5}P_5$

22. $\frac{5!}{2!}$

23. $2 \times 4!$

24. $\frac{9}{25}$

25. 4×5^3

26. 5^4

27. $^{6}P_5$

28. $5! + \frac{5!}{2!} \times {}^{4}C_3$

29. $5! + \frac{5!}{2!} \times {}^{4}C_3 \times 5$

30. 4^3 ways

31. $^{m}C_2 \times {}^{n}C_2$

32. $^{100}C_{50} \times {}^{50}C_{30} \times {}^{20}C_{20}$

33. 66 ways

34. 89

35. $\frac{3}{4}$

36. $\frac{3}{4}$

37. *(i)* $\frac{9}{40}$ *(ii)* $\frac{1}{4}$

38. 4

39. 40 : 3

40. $\frac{20}{3}\%$

41. $\frac{9}{7}$

Level – 2

42. 9400

43. 625(2 + 3 + 4 + 5) (10000) + 500 (2 + 3 + 4 + 5) (1111) = 95277000

44. *(i)* $^{9}C_3$ *(ii)* $^{9}P_3$

(iii) $\left({}^{9}C_3 \times {}^{6}C_3 \times {}^{3}C_3\right)\left(\frac{1}{3!}\right)$

45. $\frac{11}{144}$

46. $\frac{25}{56}$

47. 0.1296

48. $\frac{3}{5}$

49. Rs. 4200, Rs. 2400

50. *(a)* 30 *(b)* 20

Level – 3

51. 10%

52. 45%

Chapter 4

Practice Exercise 1 – Level 1

1	b	**2**	d	**3**	c	**4**	a	**5**	d	**6**	c	**7**	b	**8**	c	**9**	d	**10**	d
11	d	**12**	c	**13**	e	**14**	d	**15**	b	**16**	b	**17**	b	**18**	d	**19**	d	**20**	c

Practice Exercise 2 – Level 1

1	e	**2**	a	**3**	b	**4**	c	**5**	c	**6**	b	**7**	c	**8**	c	**9**	c	**10**	e
11	e	**12**	b	**13**	a	**14**	b	**15**	a	**16**	b	**17**	c	**18**	d	**19**	b	**20**	e
21	c																		

Practice Exercise 3 – Level 2

1	d	**2**	d	**3**	a	**4**	b	**5**	b	**6**	d	**7**	d	**8**	b	**9**	e	**10**	c
11	c	**12**	d	**13**	b	**14**	c	**15**	b	**16**	d	**17**	e	**18**	c	**19**	b	**20**	c
21	c	**22**	e	**23**	c	**24**	d	**25**	b	**26**	d	**27**	b	**28**	a	**29**	c		

Practice Exercise 4 – Level 2

1	d	**2**	e	**3**	d	**4**	c	**5**	c	**6**	d	**7**	a	**8**	a	**9**	c	**10**	a
11	b	**12**	d	**13**	d	**14**	c	**15**	b	**16**	e	**17**	c	**18**	b	**19**	c	**20**	c
21	c	**22**	e	**23**	d	**24**	a	**25**	b	**26**	c	**27**	b	**28**	d	**29**	b	**30**	c

Practice Exercise 5 – Level 3

1	a	**2**	d	**3**	b	**4**	e	**5**	d	**6**	c	**7**	c	**8**	e	**9**	d	**10**	b
11	e	**12**	e	**13**	a	**14**	c	**15**	b	**16**	e	**17**	c	**18**	b	**19**	c	**20**	d