ICSE Board
Class X Mathematics
Board Paper – 2015 Solution

SECTION A

1.

(a) Cost price of the article = Rs. 3,450

(i) Marked price of the article = Cost price + 16% of Cost price

$$= 3450 + \frac{16}{100}\times 3450$$
$$= 3450 + 552$$
$$= \text{Rs.}4002$$

(ii) Price paid by the customer = Marked price + Sales Tax

$$= 4002 + \frac{10}{100}\times 4002$$
$$= 4002 + 400.2$$
$$= \text{Rs. } 4402.2$$

(b) $13x - 5 < 15x + 4 < 7x + 12, \quad x \in R$

Take $13x - 5 < 15x + 4$ and $15x + 4 < 7x + 12$

$13x < 15x + 9$ and $15x < 7x + 8$

$0 < 2x + 9$ and $8x < 8$

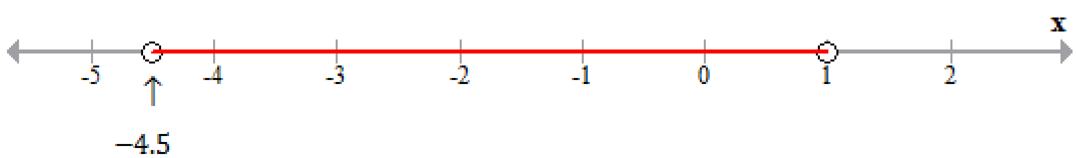
$-9 < 2x$ and $x < 1$

$-\frac{9}{2} < x$ and $x < 1$

$\therefore \ -\frac{9}{2} < x < 1$

i.e. $-4.5 < x < 1$

The required line is



(c) $\frac{\sin 65°}{\cos 25°}+\frac{\cos 32°}{\sin 58°}-\sin 28°.\sec 62°+\operatorname{cosec}^2 30°$

$=\frac{\sin(90°-25°)}{\cos 25°}+\frac{\cos(90°-58°)}{\sin 58°}-\sin 28°\times\frac{1}{\cos(90°-28°)}+\frac{1}{\sin^2 30}$

$=\frac{\cos 25°}{\cos 25°}+\frac{\sin 58°}{\sin 58°}-\sin 28°\times\frac{1}{\sin 28°}+\left(\frac{1}{\left(\frac{1}{4}\right)^2}\right)$

$=1+1-1+4$

$=5$

2.

(a) Given : $A=\begin{bmatrix}3 & x\\ 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}9 & 16\\ 0 & -y\end{bmatrix}$ and $A^2=B$

Now, $A^2=A\times A$

$=\begin{bmatrix}3 & x\\ 0 & 1\end{bmatrix}\times\begin{bmatrix}3 & x\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}9 & 3x+x\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}9 & 4x\\ 0 & 1\end{bmatrix}$

We have $A^2=B$

Two matrices are equal if each and every corresponding element is equal.

Thus, $\begin{bmatrix}9 & 4x\\ 0 & 1\end{bmatrix}=\begin{bmatrix}9 & 16\\ 0 & -y\end{bmatrix}$

$\Rightarrow 4x=16$ and $1=-y$

$\Rightarrow x=4$ and $y=-1$

(b) Population after 2 years $=$ Present population$\times\left(1+\frac{r}{100}\right)^n$

Present population $=2,00,000$

After first year, population $=2,00,000\times\left(1+\frac{10}{100}\right)^1$

$=2,00,000\times\frac{11}{10}$

$=2,20,000$

$$\text{Population after two years} = 2{,}20{,}000 \times \left(1 + \frac{15}{100}\right)^{1}$$
$$= 2{,}53{,}000$$

Thus, the population after two years is 2,53,000.

(c) Three vertices of a parallelogram ABCD taken in order are $A(3, 6)$, $B(5, 10)$ and $C(3, 2)$.

(i) We need to find the co-ordinates of D.

We know that the diagonals of a parallelogram bisect each other.

Let (x, y) be the co-ordinates of D.

$$\therefore \text{ Mid-point of diagonal AC} = \left(\frac{3+3}{2}, \frac{6+2}{2}\right) \equiv (3, 4)$$

$$\text{And, mid-point of diagonal BD} = \left(\frac{5+x}{2}, \frac{10+y}{2}\right)$$

Thus, we have

$$\frac{5+x}{2} = 3 \text{ and } \frac{10+y}{2} = 4$$
$$\Rightarrow 5 + x = 6 \text{ and } 10 + y = 8$$
$$\Rightarrow x = 1 \text{ and } y = -2$$

(ii) $$\text{Length of diagonal BD} = \sqrt{(1-5)^2 + (-2-10)^2}$$
$$= \sqrt{(4)^2 + (-12)^2}$$
$$= \sqrt{16 + 144}$$
$$= \sqrt{160} \text{ units}$$

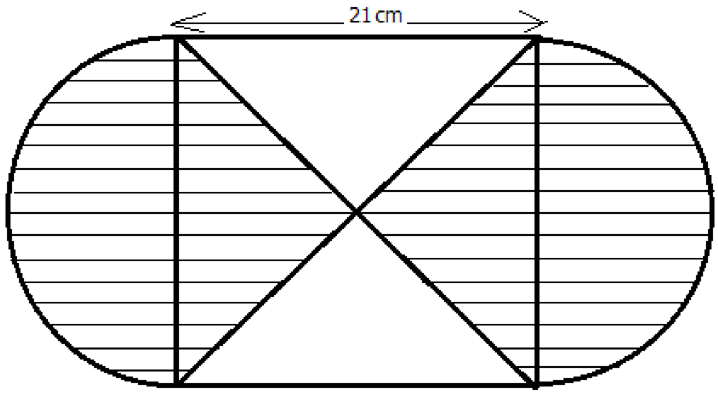
(iii) Equation of the side joining $A(3,6)$ and $D(1,-2)$ is given by

$$\frac{x-3}{3-1} = \frac{y-6}{6+2}$$
$$\Rightarrow \frac{x-3}{2} = \frac{y-6}{8}$$
$$\Rightarrow 4(x-3) = y - 6$$
$$\Rightarrow 4x - 12 = y - 6$$
$$\Rightarrow 4x - y = 6$$

Thus, the equation of the side joining $A(3,6)$ and $D(1,-2)$ is $4x - y = 6$.

3.

(a)



Area of one semi-circle = $\frac{1}{2}\times\pi\times\left(\frac{21}{2}\right)^2$

Area of both semi-circles = $2\times\frac{1}{2}\times\pi\times\left(\frac{21}{2}\right)^2$

Area of one triangle = $\frac{1}{2}\times 21\times\frac{21}{2}$

Area of both triangles = $2\times\frac{1}{2}\times 21\times\frac{21}{2}$

Area of shaded portion

$=2\times\frac{1}{2}\times\pi\times\left(\frac{21}{2}\right)^2+2\times\frac{1}{2}\times 21\times\frac{21}{2}$

$=\frac{22}{7}\times\frac{441}{4}+\frac{441}{2}$

$=\frac{693}{2}+\frac{441}{2}$

$=\frac{1134}{2}=567\text{cm}^2$

(b)

Marks	0	1	2	3	4	5
No. of Students	1	3	6	10	5	5
Cumulative Frequency	1	4	10	20	25	30

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{0\times1+1\times3+2\times6+3\times10+4\times5+5\times5}{1+3+6+10+5+5} = \frac{90}{30} = 3$$

$\therefore$ 3 is the mean.

There are a total of 30 observations in the data.

The median is the arithmetic mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observation

in case of even number of observations.

$= \text{Arithmetic mean of } \left(\frac{30}{2}\right)^{th} \text{ and} \left(\frac{30}{2}+1\right)^{th}$

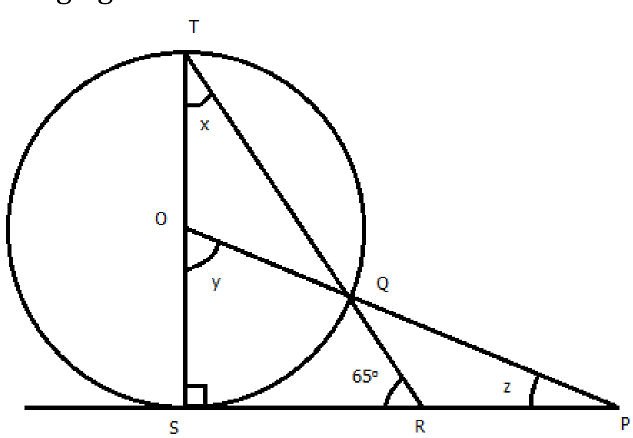
$=$ Arithmetic mean of 15^{th} and 16^{th} observation will be the median.

$$\text{Median} = \frac{3+3}{2} = 3$$

Frequency is highest for the observation $x_i = 3$

Mode $= 3$

(c) Consider the following figure:



TS ⊥ SP,

m∠TSR = m∠OSP = 90°

In Δ TSR,

m∠TSR + m∠TRS + m ∠RTS = 180°

$\Rightarrow 90° + 65° + x = 180°$

$\Rightarrow x = 180° - 90° - 65°$

$\Rightarrow x = 25°$

$y = 2x$

[Angle subtended at the centre is double that of the angle subtended by the arc at the same centre]

$\Rightarrow y = 2 \times 25°$

$\therefore y = 50°$

In ΔOSP,

$m\angle OSP + m\angle SPO + m\angle POS = 180°$

$\Rightarrow 90° + z + 50° = 180°$

$\Rightarrow z = 180° - 140°$

$\therefore z = 40°$

4.

(a) Given,

p = Rs. 1000

n = 2 years = 24 months

r = 6%

(i) $\text{Interest} = p \times \frac{n(n+1)}{2} \times \frac{r}{12 \times 100}$

$= 1000 \times \frac{24(25)}{2} \times \frac{6}{12 \times 100}$

$= 1500$

Thus, the interest earned in 2 years is Rs. 1500.

(ii) Sum deposited in two years $= 24 \times 1000 = 24{,}000$

Maturity value = Total sum deposited in two years + Interest

= 24,000 + 1,500

= 25,500

Thus, the maturity value is Rs. 25,500.

(b) $(k + 2)x^2 - kx + 6 = 0$ ....(1)

Substitute x = 3 in equation (1)

$(k + 2)(3)^2 - k(3) + 6 = 0$

$\therefore 9(k + 2) - 3k + 6 = 0$

$\therefore 9k + 18 - 3k + 6 = 0$

$\therefore 6k + 24 = 0$

$\therefore k = -4$

Now, substituting k = – 4 in equation (1), we get

$(-4 + 2)x^2 - (-4)x + 6 = 0$

$\therefore -2x^2 + 4x + 6 = 0$

$\therefore x^2 - 2x - 3 = 0$

$\therefore x^2 - 3x + x - 3 = 0$

$\therefore x(x - 3) + 1(x - 3) = 0$

$\therefore (x + 1)(x - 3) = 0$

So, the roots are x = –1 and x = 3.

Thus, the other root of the equation is x = –1.

(c) Each side of the regular hexagon is 5 cm.

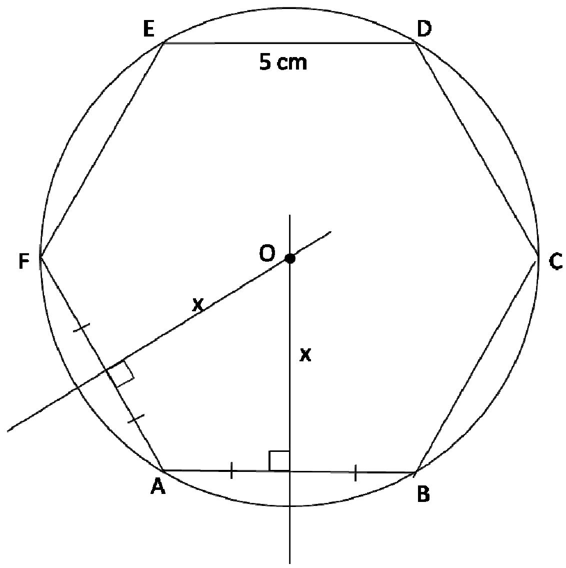
$$\therefore \text{ Each interior angle of the regular hexagon} = \frac{(2n-5)}{n} \times 90^\circ$$
$$= \frac{(2\times6-5)}{6} \times 90^\circ$$
$$= 105^\circ$$

Steps of construction:

i. Using the given data, construct the regular hexagon ABCDEF with each side equal to 5 cm.

ii. Draw the perpendicular bisectors of sides AB and AF and make them intersect each other at point O.

iii. With O as the centre and OA as the radius draw a circle which will pass through all the vertices of the regular hexagon ABCDEF.



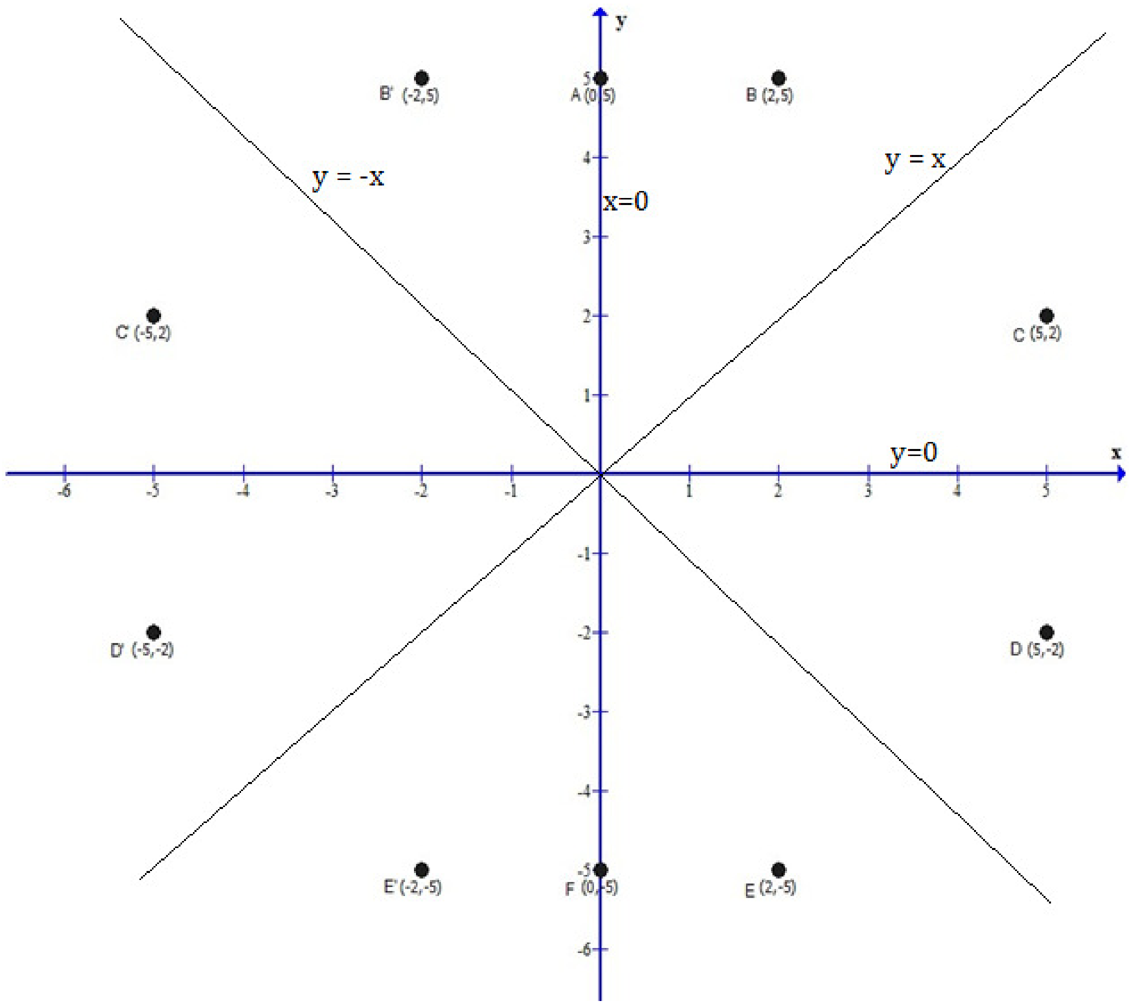
SECTION B (40 Marks)

Attempt any four questions from this section

5.

(a)

(i)



(ii) Reflection of points on the y-axis will result in the change of the x-coordinate

(iii) Points will be B'(−2, 5), C'(−5, 2), D'(−5, −2), E'(−2, −5).

(iv) The figure BCDEE'D'C'B' is a octagon.

(v) The lines of symmetry are x-axis, y-axis, Y = X and Y = -X

(b) Virat opened his savings bank account on the 16th of April, 2010.

Minimum amount to his credit from 16th April 2010 to 30th April 2010 = Rs. 2500.

Principal for the month of April = Rs. 2500

Principal for the month of May = Rs. 5250

Principal for the month of June = Rs. 4750

Principal for the month of July = Rs. 8950

Sum of all the balances = Rs. (2500 + 5250 + 4750 + 8950) = Rs. 21,450

Principal = Rs. 21,450

Time = $\frac{1}{12}$ year

Rate = 4%

S.I. = $\frac{21450 \times 1 \times 4}{100 \times 12} = \frac{143}{2} = 71.5$

Sum that Virat will get after he closes the account on 1st August, 2010

= Rs. (8950 + 71.5) = Rs. 9021.50

6.

(a) Given that a, b and c are in continued proportion.

$\therefore \frac{a}{b} = \frac{b}{c}$

$\therefore b^2 = ac$

To prove: $(a+b+c)(a-b+c) = a^2 + b^2 + c^2$

$\text{L.H.S.} = (a+b+c)(a-b+c)$

$= a(a-b+c) + b(a-b+c) + c(a-b+c)$

$= a^2 - ab + ac + ab - b^2 + bc + ac - bc + c^2$

$= a^2 + ac - b^2 + ac + c^2$

$= a^2 + b^2 - b^2 + b^2 + c^2 \quad \left[\because b^2 = ac\right]$

$= a^2 + b^2 + c^2$

$= \text{R.H.S.}$

(b)

i. The line intersects the x–axis where, y = 0.
   The co-ordinates of A are (4, 0).

ii. Length of AB $= \sqrt{\left(4-(-2)\right)^2 + (0-3)^2} = \sqrt{36+9} = \sqrt{45}$

   Length of AC $= \sqrt{\left(4-(-2)\right)^2 + (0+4)^2} = \sqrt{36+16} = \sqrt{52}$

iii. Let k be the required ratio which divides the line segment joining the co-ordinates A(4, 0) and C(– 2, – 4).
   Let the co-ordinates of Q be x and y.

$\therefore x = \frac{k(-2) + 1(4)}{k+1}$ and $y = \frac{k(-4) + 0}{k+1}$

Q lies on the y-axis where x = 0.

$\Rightarrow \frac{-2k+1}{k+1} = 0$

$\Rightarrow 2k = 1$

$\Rightarrow k = \frac{1}{2}$

The ratio is 1 : 2.

iv. The equation of line AC is

$$\frac{x-4}{4+2}=\frac{y-0}{0+4}$$

$$\Rightarrow \frac{x-4}{6}=\frac{y}{4}$$

$$\Rightarrow \frac{x-4}{3}=\frac{y}{2}$$

$$\Rightarrow 2(x-4)=3y$$

$$\Rightarrow 2x-8=3y$$

$$\Rightarrow 2x-3y=8$$

The equation of the line AC is $2x-3y=8$.

(c) Consider the following distribution:

Class Interval	Frequency f_i	Class mark x_i	f_ix_i
0 – 10	8	5	40
10 – 20	5	15	75
20 – 30	12	25	300
30 – 40	35	35	1225
40 – 50	24	45	1080
50 – 60	16	55	880
Total	$\sum f_i = 100$		$\sum f_ix_i = 3600$

$$\text{Mean} = \frac{\sum f_ix_i}{\sum f_i} = \frac{3600}{100} = 36$$

7.

(a) Radius of small sphere = r = 2 cm

Radius of big sphere = R = 4 cm

$$\text{Volume of small sphere} = \frac{4}{3}\pi r^3 = \frac{4\pi}{3}\times(2)^3 = \frac{32\pi}{3}\text{cm}^3$$

$$\text{Volume of big sphere} = \frac{4}{3}\pi R^3 = \frac{4\pi}{3}\times(4)^3 = \frac{256\pi}{3}\text{cm}^3$$

$$\text{Volume of both the spheres} = \frac{32\pi}{3} + \frac{256\pi}{3} = \frac{288\pi}{3}\text{cm}^3$$

$$\text{Volume of the cone} = \frac{1}{3}\pi R_1{}^2 h$$

We need to find R_1.

$h = 8$ cm (Given)

$$\text{Volume of the cone} = \frac{1}{3}\pi R_1{}^2 \times (8)$$

Volume of the cone = Volume of both the sphere

$$\Rightarrow \frac{1}{3}\pi R_1{}^2 \times (8) = \frac{288\pi}{3}$$

$$\Rightarrow R_1{}^2 \times (8) = 288$$

$$\Rightarrow R_1{}^2 = \frac{288}{8}$$

$$\Rightarrow R_1{}^2 = 36$$

$$\Rightarrow R_1 = 6 \text{ cm}$$

(b) The given polynomials are $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$.

Let $p(x) = ax^3 + 3x^2 - 9$ and $q(x) = 2x^3 + 4x + a$

Given that p(x) and q(x) leave the same remainder when divided by (x + 3),

Thus by Remainder Theorem, we have

$p(-3) = q(-3)$

$\Rightarrow a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$

$\Rightarrow -27a + 27 - 9 = -54 - 12 + a$

$\Rightarrow -27a + 18 = -66 + a$

$\Rightarrow -27a - a = -66 - 18$

$\Rightarrow -28a = -84$

$$\Rightarrow a = \frac{84}{28}$$

$\therefore a = 3$

(c) $\text{L.H.S.} = \dfrac{\sin\theta}{1-\cot\theta} + \dfrac{\cos\theta}{1-\tan\theta}$

$= \dfrac{\sin\theta}{1-\dfrac{\cos\theta}{\sin\theta}} + \dfrac{\cos\theta}{1-\dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\sin^2\theta}{\sin\theta-\cos\theta} + \dfrac{\cos^2\theta}{\cos\theta-\sin\theta}$

$= \dfrac{\sin^2\theta}{\sin\theta-\cos\theta} - \dfrac{\cos^2\theta}{\sin\theta-\cos\theta}$

$= \dfrac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$

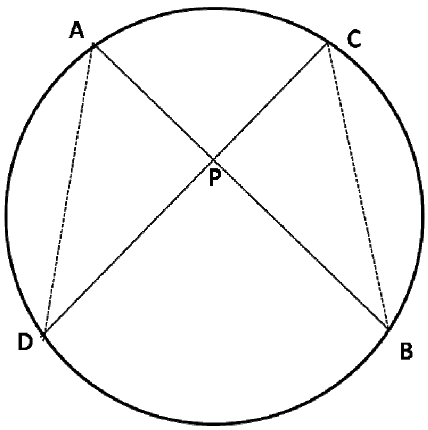
$= \dfrac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta}$

$= \sin\theta+\cos\theta$

$= \text{R.H.S.}$

8.

(a) Construction: Join AD and CB.



In ΔAPD and ΔCPB

$\angle A = \angle C$ .....(Angles in the same segment)

$\angle D = \angle B$ .....(Angles in the same segment)

$\Rightarrow \Delta APD \sim \Delta CPB$ ....(By AA Postulate)

$\Rightarrow \dfrac{AP}{CP} = \dfrac{PD}{PB}$ ....(Corresponding sides of similar triangles)

$\Rightarrow AP \times PB = CP \times PD$

(b) $P(\text{Green ball}) = \dfrac{\text{Number of Green balls}}{\text{Total number of balls}}$

$= \dfrac{9}{20}$

$P(\text{White ball or Red ball}) = P(\text{White ball}) + P(\text{Red ball})$

$= \dfrac{\text{Number of White balls}}{\text{Total number of balls}} + \dfrac{\text{Number of Red balls}}{\text{Total number of balls}}$

$= \dfrac{5}{20} + \dfrac{6}{20}$

$= \dfrac{11}{20}$

$P(\text{Neither Green ball nor White ball}) = P(\text{Red ball})$

$= \dfrac{\text{Number of Red balls}}{\text{Total number of balls}}$

$= \dfrac{6}{20}$

(c)

(i) 100 shares at Rs. 20 premium means:

Nominal value of the share is Rs. 100

and its market value = 100 + 20 = Rs. 120

Money required to buy 1 share = Rs. 120

$\therefore$ Number of shares $= \dfrac{\text{Money Invested}}{\text{Market Value of 1 Share}}$

$= \dfrac{9600}{120}$

$= 80$

(ii) Dividend on 1 share = 8% of N.V.

= 8% of 100

= 8

Total dividend on 80 shares = 80 × 8 = 640

Each share is sold at Rs. 160.

$\therefore$ The sale proceeds (excluding dividend)

= 80 × 160 – 640

= 12800 – 640

= 12160

(iii) New investment = Rs. 12160

Dividend = 10%

Net Value = 50

Market Value = Rs. 40

$\therefore$ Number of shares $= \dfrac{\text{Investment}}{\text{Market Value}}$

$= \dfrac{12160}{40}$

$= 304$

(iv) Now, dividend on 1 share = 10% of N.V.

= 10% of 50

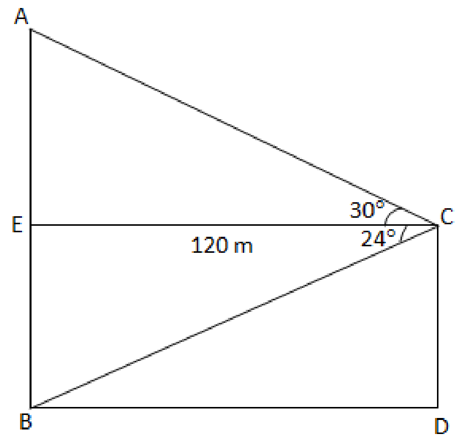
= 5

$\therefore$ Dividend on 304 shares = 1520

Change in two dividends = 1520 – 640 = 880

9.

(a) Consider the following figure:



$\tan 30° = \dfrac{AE}{EC}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{AE}{120}$

$\Rightarrow \dfrac{120}{\sqrt{3}} = AE$

$\therefore AE = 69.282 \text{ m}$

$\tan 24° = \dfrac{EB}{EC}$

$\Rightarrow 0.445 = \dfrac{EB}{120}$

$\therefore EB = 53.427 \text{ m}$

Thus, height of first tower, $AB = AE + EB$

$= 69.282 + 53.427$

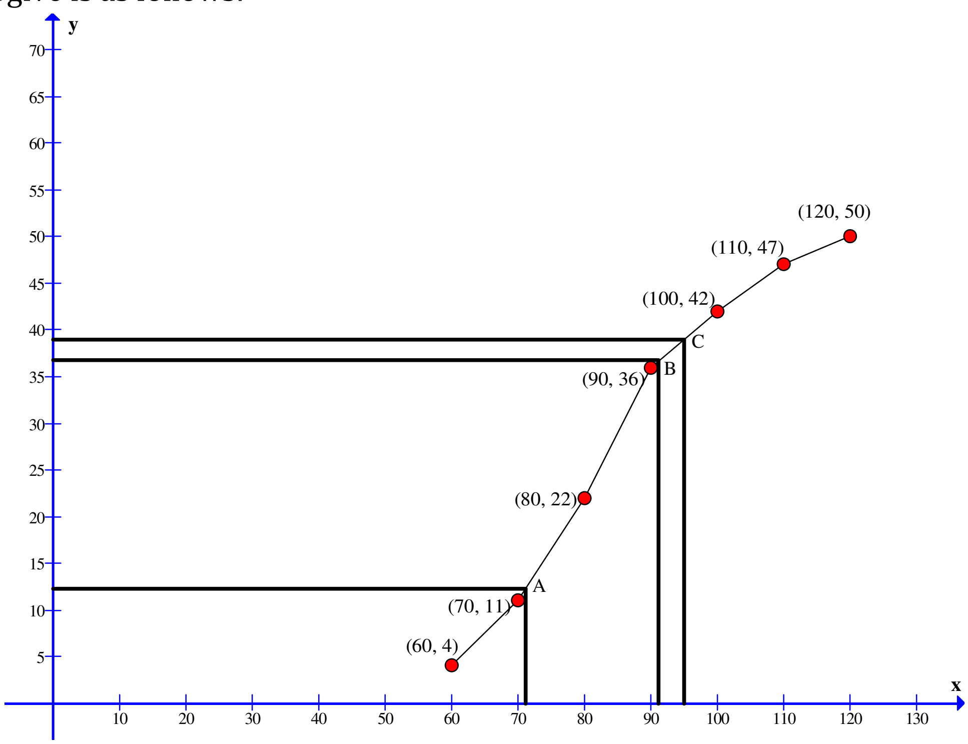
$= 122.709 \text{ m}$

And, height of second tower, $CD = EB = 53.427 \text{ m}$

(b) The cumulative frequency table of the given distribution table is as follows:

Weight in Kg	Number of workers	Cumulative frequency
50-60	4	4
60-70	7	11
70-80	11	22
80-90	14	36
90-100	6	42
100-110	5	47
110-120	3	50

The ogive is as follows:



Number of workers = 50

(i) Lower quartile $(Q_1) = \left(\frac{50}{4}\right)^{th}$ term $= (12.25)^{th}$ term $= 71.1$

Upper quartile $(Q3) = \left(\frac{3\times 50}{4}\right)^{th}$ term $= (36.75)^{th}$ term $= 91.1$

(ii) Through mark of 95 on the x – axis, draw a verticle line which meets the graph at point C.

Then through point C, draw a horizontal line which meets the y-axis at the mark of 39.

Thus, number of workers weighing 95 kg and above = $50 - 39 = 11$

10.

(a) Selling price of the manufacturer = Rs. 25000

Marked price of the wholesaler

$$= 25000 + \frac{20}{100} \times 25000$$
$$= 25000 + 5000$$
$$= \text{Rs. } 30,000$$

Selling Price of the wholesaler

$$= 30000 - \frac{10}{100} \times 30000$$
$$= 30000 - 3000$$
$$= \text{Rs. } 27,000$$

Tax for the wholesaler $= \frac{8}{100} \times 27000 = \text{Rs. } 2160$

(i) Marked price = Rs. 30,000

(ii)Total Cost Price of the retailer including tax = Rs. (27000 + 2160) = Rs. 27160

(iii)VAT paid by the wholesaler = $\frac{8}{100} \times 25000 = \text{Rs. } 2000$

(b) $AB = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3\times0+7\times5 & 3\times2+7\times3 \\ 2\times0+4\times5 & 2\times2+4\times3 \end{bmatrix}$$
$$= \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix}$$
$$= \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$5C = 5\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

(c)

(i)Consider ΔADE and ΔACB.

$\angle A = \angle A$ [Common]

$m\angle B = m\angle E = 90°$

Thus by Angle-Angle similarity, triangles, $\Delta ACB \sim \Delta ADE$.

(ii)Consider ΔADE and ΔACB.

Since they are similar triangles, the sides are proportional.

Thus, we have,

$$\frac{AE}{AB}=\frac{AD}{AC}=\frac{DE}{BC}...(1)$$

Consider ΔABC.

By applying Pythagoras Theorem, we have,

$$AB^2+BC^2=AC^2$$

$$\Rightarrow AB^2+5^2=13^2$$

$$\Rightarrow AB=12 \text{ cm}$$

From equation (1), we have,

$$\frac{4}{12}=\frac{AD}{13}=\frac{DE}{5}$$

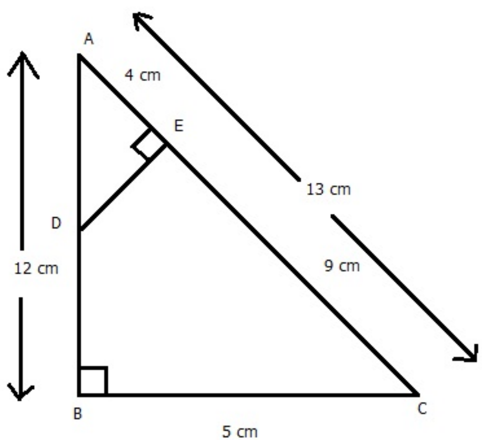
$$\Rightarrow \frac{1}{3}=\frac{AD}{13}$$

$$\Rightarrow AD=\frac{13}{3} \text{ cm}$$

Also $\frac{4}{12}=\frac{DE}{5}$

$$\Rightarrow DE=\frac{20}{12}=\frac{5}{3} \text{ cm}$$

(iii) We need to find the area of ΔADE and quadrilateral BCED.



Area of ΔADE = $\frac{1}{2}\times AE\times DE$

$$=\frac{1}{2}\times 4\times\frac{5}{3}$$

$$=\frac{10}{3} \text{ cm}^2$$

Area of quad.BCED = Area of ΔABC – Area of ΔADE

$$=\frac{1}{2}\times BC\times AB-\frac{10}{3}$$

$$=\frac{1}{2}\times 5\times 12-\frac{10}{3}$$

$$=30-\frac{10}{3}$$

$$=\frac{90-10}{3}$$

$$=\frac{80}{3}\ cm^2$$

Thus ratio of areas of ΔADE to quadrilateral BCED= $\dfrac{\frac{10}{3}}{\frac{80}{3}}=\dfrac{1}{8}$

11.

(a) Let x and y be two numbers

Given that

x + y = 8 ...(1)

and

$\frac{1}{x}-\frac{1}{y}=\frac{2}{15}$...(2)

From equation (1), we have, y = 8 - x

Substituting the value of y in equation (2), we have,

$$\frac{1}{x}-\frac{1}{8-x}=\frac{2}{15}$$

$$\Rightarrow\frac{8-x-x}{x(8-x)}=\frac{2}{15}$$

$$\Rightarrow\frac{8-2x}{x(8-x)}=\frac{2}{15}$$

$$\Rightarrow\frac{4-x}{x(8-x)}=\frac{1}{15}$$

$$\Rightarrow 15(4-x)=x(8-x)$$

$$\Rightarrow 60-15x=8x-x^2$$

$$\Rightarrow x^2-15x-8x+60=0$$

$$\Rightarrow x^2-23x+60=0$$

$$\Rightarrow x^2-20x-3x+60=0$$

$$\Rightarrow x(x-20)-3(x-20)=0$$

$$\Rightarrow (x-3)(x-20)=0$$

$\Rightarrow (x-3)=0$ or $\Rightarrow (x-20)=0$

$\Rightarrow x=3$ or $x=20$

Since sum of two natural numbers is 8, x cannot be equal to 20

Thus x = 3

From equation (1), y = $8-x=8-3=5$

Thus the values of x and y are 3 and 5 respectively.

(b) Given that

$$\frac{x^3+12x}{6x^2+8}=\frac{y^3+27y}{9y^2+27}$$

Using componendo-dividendo, we have,

$$\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8}=\frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$$

$$\Rightarrow \frac{(x+2)^3}{(x-2)^3}=\frac{(y+3)^3}{(y-3)^3}$$

$$\Rightarrow \left(\frac{x+2}{x-2}\right)^3=\left(\frac{y+3}{y-3}\right)^3$$

$$\Rightarrow \frac{x+2}{x-2}=\frac{y+3}{y-3}$$

Again using componendo-dividendo, we have,

$$\frac{x}{2}=\frac{y}{3}$$

$$\frac{x}{y}=\frac{2}{3}$$

Thus the required ratio is x : y = 2 : 3.

(c)

1. Draw a line segment AB of length 5.5 cm.
2. Make an angle m∠BAX = 105° using a protractor.
3. Draw an arc AC with radius AC = 6 cm on AX with centre at A.
4. Join BC.

Thus ΔABC is the required triangle.

(i) We know that the locus of a point equidistant from two intersecting lines is the bisector of the angle between the lines.

$\therefore$ By considering BA and BC as arms, we need to find the angle bisector of the vertex B.

Step1: Draw an arc with centre at B and with any radius.

Step 2: The arc intersects the arms BA and BC at P and Q respectively. Mark the points of intersection.

Step 3: Keeping the same radius, now draw arcs with centres P and Q.
Step 4: Let R be the point of intersection of the arcs. Now Join B and R.
Thus, BR is equidistant from BA and BC.

(ii) We know that the locus of a point equidistant from the given points is the perpendicular bisector of the line joining the two points.
$\therefore$ We need to find the perpendicular bisector of the line segment BC.
Step 1: Take more than half of length BC.
Step 2: Draw arcs with centres B and C on the two sides of the segment BC.
Step 3: Mark the points of intersection as M and N respectively.
Step 4: Join M and N.
MN is the perpendicular bisector of the line segment BC. MN is equidistant from both the points B and C.

(iii) Here, the angle bisector and perpendicular bisector meet at S.
Thus, S is the required point.
Length of CS is 4.8 cm