2011

MATHEMATICS

Paper : 1.5

(Tensors and Mechanics)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

PART—A

(Objective-type Questions)

(*Marks* : 32)

1. Answer the following questions :

(a) State the law of transformation of a covariant tensor. 2

(b) How many components does a tensor of rank m have in a space of n dimensions? 2

(c) Show that

$$[ij, k] + [jk, i] = \frac{\partial g_{ki}}{\partial x_j}$$ 4

(d) State the mathematical condition for which a system of forces acting on a rigid body can reduce to a single force. 2

(e) Define wrench. 2

(f) If a horizontal beam is subjected to any vertical loading, show that the vertical shear S is equal to $\frac{dM}{dx}$, where M is the bending moment of the point considered. 4

(g) If a particle of mass m moves in xy-plane so that its position vector is

$$\bar{r} = (a\cos\omega t)\,i + (b\sin\omega t)\,j$$

show that the force acting on the particle is always directed towards the origin. 4

(h) A particle falls under gravity in a resisting medium, whose resistance varies as the square of the velocity. Show that the differential equation of motion is

$$v\frac{dv}{dx} = \frac{g}{V^2}(V^2 - v^2)$$

where V is the terminal velocity. 2

(i) Prove that if a particle is moving in a central force field, the angular momentum is conserved. 2

(j) Equal forces are acting along the coordinate axes and along the lines

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

where l, m and n are the direction cosines. Find the dyname $(X, Y, Z; L, M, N)$. 4

(k) Deduce the equation of motion of a compound pendulum. 4

PART—B

(Subjective-type Questions)

(*Marks* : 48)

UNIT—1

2. Answer any *one* part :

(a) Define contravariant and covariant tensors of rank one. Show that the metric function g^{ij} is a symmetric contravariant tensor of rank two. 6

(b) Show that a vector $\vec{u}(u^i)$ of constant magnitude is orthogonal to its intrinsic derivative. 6

(c) Show that the 'cosine' of the angle between the two parametric curves x^i = constant and x^j = constant is given by

$$\cos\theta = \frac{g^{ij}}{\sqrt{g^{ii}}\sqrt{g^{jj}}}$$ 6

UNIT—2

3. Answer any *one* part :

(a) Obtain the covariant differentiation of A_{ij} as $A_{ij,k} = \frac{\partial A_{ij}}{\partial x^k} - A_{\alpha j}\Gamma^{\alpha}_{ik} - A_{i\alpha}\Gamma^{\alpha}_{jk}$. 6

Or

Prove that $g_{ij,k} = 0 = g^{ij}_{,k}$.

(b) If two vectors undergo parallel displacement, show that they incline at a constant angle. 6

UNIT—3

4. Derive the equation of Poinsot's central axis for a system of forces in three dimensions. Hence, show that it is unique for every system. 6

UNIT—4

5. Answer any *one* part :

(a) A heavy elastic string of natural length $2\pi a$ is placed round a smooth cone whose axis is vertical and semi-vertical angle is α. If w be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium when in the form of a circle of radius $a[l + (w/2\pi\lambda)\cos\alpha]$. 6

(b) A heavy rod AB rests on three horizontal supports A, B, C; C being midway between A and B; ABC horizontal. If the rod be slightly deflected, find its form. 6

UNIT—5

6. Answer any *one* part :

(a) Show that the change in the KE per unit space is equal to the acting force whether it is constant or variable. 6

(b) Show that the force field $\overline{F}$ defined by

$$\overline{F} = (y^2z^3 - 6xz^2)i + 2xyz^3 j + (3xy^2z^2 - 6x^2z)k$$

is a conservative force field. Hence find the potential of the field. 6

UNIT—6

7. Answer any *one* part :

(a) A particle subject to the central acceleration $\left(\frac{\mu}{r^3}+f\right)$ is projected from an apse at a distance a with a velocity $\sqrt{\frac{\mu}{a}}$. Prove that at any subsequent time t, $r = a-\frac{1}{2}ft^2$. 6

(b) A particle falls vertically under gravity (suppose constant) in a resisting medium whose resistance varies as the square of the velocity. If the particle starts from rest, show that the distance of the particle at any time is

$$x = \frac{w^2}{g}\log\left\{\cosh\left(\frac{gt}{w}\right)\right\} \quad \left(w^2 = \frac{g}{k}\right)$$ 6

(c) A satellite moving in an elliptic orbit has its greatest and least orbital speeds given by U and V respectively. Prove that the eccentricity of the orbit is $\frac{U-V}{U+V}$. Also prove that the major axis of the orbit is of the length $\frac{T}{\pi}\sqrt{UV}$, where T is the periodic time of the satellite. 6

UNIT—7

8. Stating clearly the variables assumed, write down the expressions of velocity and acceleration of a particle in cylindrical polar coordinates. A particle moves on a smooth sphere under no forces except the pressure of the surface; show that its path is given by the equation $\cot\theta = \cot\beta\cos\phi$, where θ and ϕ are its angular coordinates and $\theta = \beta$ initially.

1+5=6

UNIT—8

9. Answer any *one* part :

(a) A uniform solid sphere rolls down an inclined plane, rough enough to prevent any sliding; show that for pure rolling $\mu > \frac{2}{7}\sin\alpha$, where μ is the coefficient of friction. 6

(b) *AB* and *BC* are two equal similar rods freely hinged at *B* and lie in a straight line on a smooth table. The end *A* is struck by a blow perpendicular to *AB*; show that the velocity of *A* is $3\frac{1}{2}$ times that of *B*. 6

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