# Syllabus for M. Sc Mathematics Entrance 2015

**Unit-I**

ε-δ definition of the limit of a function. Basic properties of limits. Infinitesimals; Definition with examples. Theorems on infinitesimals. Comparing infinitesimals, Definition with examples and related theorems. Principal part of an infinitesimal and related theorems. Continuity and basic properties of continuous functions on closed intervals. If a function is continuous in a closed interval, then it is bounded therein. If a function is continuous in a closed interval [a, b], then it attains its bounds at least once in [a,b]. Differentiation, Rolle's theorem with proof and its applications. Lagrange's Mean value theorem and Cauchy's Mean value theorem with their applications.

**Unit-II**

Taylor's and Maclaurin's theorem with their applications. Intermediate forms. Successive differentiation with Leibnitz theorem.

Tangents and normals (polar co-ordinates only). Pedal equations, length of arcs. Partial differentiation of functions of two and three variables. Euler's theorem on homogeneous functions. Curvature, radius of curvature for Cartesian and polar coordinates, double points, Asymptotes, Cartesian and polar coordinates, envelopes, involutes and evolutes, tracing of curves( Cartesian coordinates only).

**Unit-III**

Review of complex number system, triangle inequality and its generalization. Equation of circle (Apollonius circle), Geometrical representation of complex numbers. De Moivere's theorem for rational index and its application. Expansion of Sin nθ, Cos nθ etc. in terms of powers of Sin θ, Cos θ and expansion of $\text{Sin}^n\,\theta$ and $\text{Cos}^n\,\theta$ in terms of multiple angles of Sinθ and Cosθ

Functions of complex variable. Exponential, circular, Hyperbolic, Inverse hyperbolic and Logarithmic functions of a complex variable and their properties. Summation of trigonometric series, Difference method, C + iS

**Unit-IV**

Parabola: Equation of tangent and normal, pole and polar, pair of tangents from a point, equation of a chord of a parabola in terms of its middle point, parametric equations of a parabola. Ellipse; Tangents and Normals, pole and polar, parametric equations of ellipse, Diameters, conjugate diameters and their properties.

Hyperbola: Equations of tangents and normals, equation of hyperbola referred to asymptotes as axes, Rectangular and conjugate diameters and their properties. Tracing of conics (Cartesian co-ordinates only).

The plane, Every first degree equation in X,Y,Z represents a plane, Equation of plane in normal and intercepts forms , and through points. Systems of planes, Two sides of a plane. Bisectors of angles between two planes, joint equation of two planes, Volume of a tetrahedron in terms of the co-ordinates of its vertices. Straight line. Equation in symmetrical and unsymmetrical form. Equation of a straight line through two points.

Transformation of the equation of a line to the symmetrical form. The condition that two given lines may intersect.

**Unit-V**

Sphere; Definition and equation of a Sphere, condition for two spheres to be orthogonal. Radical plane. Coaxial system. Simplified form of the equation of two spheres.

Definition of Cone, Vertex, guiding curve, generator, equation of cone with vertex as origin or a given vertex and guiding curve, condition that the general equation of the second degree should represent a cone. Angle between generators of section of a cone and plane through vertex. Necessary and sufficient conditions for a cone to have three mutually perpendicular generators. Definition of a cylinder, equation of the cylinder whose generators intersect a given conic and are parallel to given line enveloping cylinder of a sphere. Central conicoids. Tangent lines and tangent planes. Normal to conicoid at a point on it. Normal from a point to a conicoid, polar plane. Shapes and features of the three central coincides. Diametric planes. Generating lines of ruled surfaces.

**Unit VI**

Review of the methods of integration, integration by substitution and by parts, integration of algebraic rational functions; case of non-repeated or repeated linear factors. Case of linear or quadratic non-repeated factors. Integration of algebraic rational functions by substitution, integration of irrational functions, Reduction formulae.

Review of the definite integral as the limit of a sum. Summation of series with the help of definite integrals. Quaderature. Area of a region bounded by a curve, X-axis (y-axis) and two ordinates (abscissa), Sectorial areas bounded by a closed curve. Lengths of plane curves. Volumes and surfaces of revolution.

Vector Analysis: Scalar and vector product of three and four vectors. Reciprocal vectors. Vector functions of a single scalar variable, limit of a vector function, continuity. Vector Differentiation, Gradient, Divergence and curl. Vector integration. Theorems of Gauss, Green, Stoke's and problems based on these.

**Unit-VII**

Degree and order of a differential equations. Equations of first order and first degree. Equations in which the variables are separable. Homogeneous equations. Linear equations and equations reducible to linear form. Bernoulli's equations, Exact differential equations, Symbolic operators. Linear differential equations with constant coefficients. Differential equations of the forms f (D) y = Sin ax, $e^{ax}V$, where V is any function of x. Homogeneous linear equations.

Miscellaneous form of differential equations. First order higher degree equations solvable for x,y,z,p. Equations from which one variable is explicitly absent, Clairut's form, equations reducible to Clairut's form. Legendre polynomials. Recurrence relation and differential equation satisfied by it. Bessel functions, recurrence relation and differential equation.

**Unit-VIII**

Symmetric, Skew-symmetric, Hermition and skew-Hermition matrices, Diagonal, scalar and triangular matrices, sum of matrices and properties of the addition composition. Representation of a square matrix as a sum of a symmetric (Hermition) and a skew-symmetric (Skew-Hermition) matrix. Representation of a square matrix in the form of P + iQ, where P and Q are both Hermition.

Product of matrices. Transpose of a product of two matrices and its generalization to several matrices. Associative law for the product and Distributive law of matrices. Adjoint of a square matrix A and relation A(adj.A) = (adj.A)A = |A|I, Inverse of a square matrix. Reversal law for the inverse of a product of two matrices and its generalization to several matrices. A square matrix A possess an inverse if and only if it is nonsingular. The operation of transposing and inverting are commutative.

Trace of a matrix, trace of AB = trace of BA, Inverse of partitioned matrices. Inverse of a lower triangular matrices is lower triangular.

**Unit-IX**

Matrix polynomials, Characteristic and minimal equations of a matrix. Cayley Hamilton theorem. Rank of a matrix. Elementary row and column transformations of a matrix do not alter its rank. Finding the inverse and rank of matrix by elementary transformations. Reduction of matrix to normal form. Elementary matrices. Every non-singular matrix is a product of elementary matrices. Employment of only row(column) transformations. The rank of a product of two matrices. Linear dependence and linear independence of column (row) vectors.

Linear combination: the columns of a matrix A are linearly dependent iff there exists vector $X \neq 0$ such that $AX = 0$. The columns of a matrix A of order mxn are linearly dependent iff rank of A is less than n. The matrix A has rank r iff it has r linearly independent columns where as any s columns, $s>r$ are linearly dependent. Analogous results for rows. Linear homogeneous and non-homogeneous equations. The equation $AX = 0$ has a non-zero solution iff rank of A is less than n, the number of its columns. The number of linearly independent solutions of the equation $AX = 0$ is $(n-r)$ where r is the rank of mxn matrix A. The equation $AX=B$ is consistent iff the two matrices A and [A:B] are of the same rank.

**Unit-X**

General properties of equations, synthetic division, Relation between the roots and the coefficients of an equation, Transformation of equations, Diminishing the roots of an equation by a given number, Removal of terms of an equation, Formation of equations whose roots are functions of the roots of a given equation, Equations of squared differences.

Symmetric functions, Newton's method of finding the sum of powers of the roots of an equation. Cardan's solution of the cubic, nature of the roots of a cubic, Descarts solution of a biquaderatic. Descart's rule of signs. Rational roots of integral polynomial. Location of roots of an equation (simple cases).

**Unit-XI**

Real numbers: Bounded and unbounded sets. L.u.b. (suprimum) and g.l.b.( infimum) of a set. Completeness in the set of real numbers and statement of least upper bound property. The set of rational numbers is not of order complete.

Definition of a Sequence; Theorems on limit of sequences.Bounded and monotonic sequences. Cauchy’s convergence criterian for sequences.Nested intervals theorem. Balzano-Weirstrass theorem Limit inferior and limit superior of a sequence.

Series: Definition of convergence, divergence, finite and infinite oscillation, Cauchy’s general principle of convergence for series. Series of positive terms. Comparison test, Integral test, Cauchy’s root test, D-Alembert’s ratio test, Rabee’s test and Gauss test, Notions of absolute and conditional convergence. Some theorem on continuity viz. If a function f is continuous on a closed interval [a,b] then the closed interval [a,b] can be divided into a finite number of subintervals such that oscillation of f in each of the subintervals can be made arbitrarily small. If a function is continuous on [a,b], and f(a) and f(b) are of opposite in sign, then there is at least one point c in (a,b) such that f(c) = 0. The intermediate value theorem. Darboux intermediate value theorem for derivative.

**Unit-XII**

Riemann- Integration: The Riemann- Integral ,Definition and existence of the Riemann integral. Upper and lower sums. Refinement of a partition. Under a refinement , the lower sums do not decrease and upper sums do not increase. The necessary and sufficient condition for integrability of bounded functions. Integrability of sum, difference, product and quotient of two functions If f is bounded and integrable on [a,b], then so is |f| and

$$\left| \int_a^b f(x)dx \right| \leq \int_a^b |f(x)|dx \leq M\,(b-a).$$

A function f having a finite number of points of discontinuity is Riemann- integrable . Integrability of continuous and monotone functions. Fundamental theorem of integral calculus, Mean Value theorem for integrals.

**Unit-XIII**

Advanced Calculus: Limit, continuity and differentiability of functions of two or more variables. Total and second derivatives. Sufficient conditions for validity of reversal in the order of derivation. Schwartz’s theorem, Young’s theorem. Change of variables. Extreme values of functions of two or more variables. Restricted maxima and minima, Curve linear integrals. Green’s theorem, Beta, Gamma functions and relation between them. Multiple integrals: Integral over the plane areas in xy-plane, double integrals, evaluation, change of order of integration for two variables, double integrals in polar co-ordinates, integral over regions in xyz-space, triple integrals, evaluation, triple integrals in cylindrical and spherical polar co-ordinates, change of variables, Jacobian.

**Unit-XIV**

Brief resume of sets and mappings. Semi groups, subgroups and criteria for a subset to be a subgroup. Cyclic groups and their subgroups. Cosets and Lagrange’s theorem. Product of two subgroups. Counting principle for the number of elements in HK. Normilizer and center.

Normal subgroups and its various criteria; quotient groups. Homomorphism and isomorphism. Fundamental theorem on homomorphism. Correspondence theorem, second and third theorems of isomorphism for groups. Permutation groups , Even and odd permutations, symmetric groups of degree n, Alternating group; simple groups. Cayley’s theorem.

**Unit-XV**

Rings and fields: Definition and examples of rings, Subrings and fields and subfields. Ring homomorphism , Ideals and Quotient rings. The field of quotients and integral domain. Polynomial rings. Characterization of a ring. Prime and maximal ideal and their characterization in terms of the associated quotient ring.

Vector spaces and their examples, subspace, criteria for a vector space to be subspace, intersection and sum of subspaces of a vector space. Quotient spaces.Homomorphisms and Isomorphisms. Notion of linear independence and basis of a vector space.

Dimension of a vector space. Linear transformation. Algebra of linear transformations. Dimension of the space of linear transformations. Matrix of linear transformation.

Similarity of matrix corresponding to a linear transformation with respect to different basis.