AIEEE – 2011 TEST PAPER WITH ANSWER
(HELD ON SUNDAY 01ST MAY, 2011)

PART A - CHEMISTRY

1. Silver Mirror test is given by which one of the following compounds ?

(1) Formaldehyde (2) Benzophenone

(3) Acetaldehyde (4) Acetone

Ans. (1,3)

Sol. Aldehyde shows reducing property.

2. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution ?

(1) 0.086 (2) 0.050 (3) 0.100 (4) 0.190

Ans. (1)

Sol. $m = \frac{X_A \times 1000}{(1-X_A)\times M_{solvent}}$

X_A = mole fraction of solute

$m = 5.2$

$5.2 = \frac{X_A \times 1000}{(1-X_A)\times 18}$

$X_A = 0.0859$

3. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is :-

(1) 2,2,2–Trichloropropanol

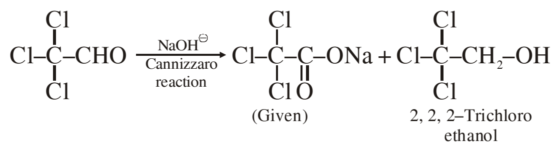
(2) Chloroform

(3) 2,2,2–Trichloroethanol

(4) Trichloromethanol

Ans. (3)

Sol.



4. The rate of a chemical reaction doubles for every 10°C rise of temperature. If the temperature is raised by 50°C, the rate of the reaction increases by about :-

(1) 32 times (2) 64 times

(3) 10 times (4) 24 times

Ans. (1)

Sol. $\frac{R_2}{R_1} = 2^{\frac{(T_2-T_1)}{10}}$ $T_1 = 0°C$

$\frac{R_2}{R_1} = 2^{\frac{(50-0)}{10}}$ $T_2 = 50°C$

$\frac{R_2}{R_1} = 2^5 = 32$

$\boxed{R_2 = 32R_1}$

5. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because :-

(1) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6

(2) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

(3) a and b for Cl_2 > a and b for C_2H_6

(4) a and b for Cl_2 < a and b for C_2H_6

Ans. (2)

Sol. for easily liquification

a for Cl_2 > a for C_2H_6

but b for Cl_2 < b for C_2H_6

6. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm^3 to a volume of 100 dm^3 at 27°C is :-

(1) 32.3 J mol^{-1} K^{-1} (2) 42.3 J mol^{-1} K^{-1}

(3) 38.3 J mol^{-1} K^{-1} (4) 35.8 J mol^{-1} K^{-1}

Ans. (3)

Sol. $\Delta S = 2.303\ nR \log \frac{V_2}{V_1}$

$V_1 = 10\ dm^3$

$V_2 = 100\ dm^3$

$n = 2$

$T = 300\ K$

$\Delta S = 2.303 \times 2 \times 8.314 \times \log \frac{100}{10}$

$\Delta S = 2.303 \times 2 \times 8.314 = 38.29\ JK^{-1}\ mol^{-1}$

7. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is :-

(1) 0.3 atm (2) 0.18 atm

(3) 1.8 atm (4) 3 atm

Ans. **(3)**

Sol. $C_{(graphite)} + CO_2 \rightleftharpoons 2CO$

Initial pressure 0.5 0

0.5 – x 2x

total pressure = 0.5 – x + 2x = 0.8

x = 0.3

$$K_p = \frac{(P_{CO})^2}{(P_{CO_2})} = \frac{(0.6)^2}{0.2} = \frac{0.36}{0.2} = 1.8$$

8. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at :-

(1) 743 nm (2) 518 nm

(3) 1035 nm (4) 325 nm

Ans. **(1)**

Sol. $\Delta E_1 = \Delta E_2 + \Delta E_3$

$$\frac{hc}{\lambda_1} = \frac{hc}{\lambda_2} + \frac{hc}{\lambda_3}$$

$$\frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$$

$$\frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_3} \Rightarrow \frac{1}{\lambda_3} = \frac{1}{355} - \frac{1}{680}$$

λ_3 = 742.76 nm

9. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is :-

(1) A_2B_3 (2) A_2B_5 (3) A_2B (4) AB_2

Ans. **(2)**

Sol. A = 8 × 1/8 = 1 (corner)

B = 5 × 1/2 = 5/2 (face centre)

$\Rightarrow$ A : B $\Rightarrow$ A_2B_5

2 : 5

10. Among the following the maximum covalent character is shown by the compound :-

(1) $AlCl_3$ (2) $MgCl_2$

(3) $FeCl_2$ (4) $SnCl_2$

Ans. **(1)**

Sol. According to Fajan's rule

11. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides ?

(1) $Na_2O < K_2O < MgO < Al_2O_3$

(2) $K_2O < Na_2O < Al_2O_3 < MgO$

(3) $Al_2O_3 < MgO < Na_2O < K_2O$

(4) $MgO < K_2O < Al_2O_3 < Na_2O$

Ans. **(3)**

Sol. Basic nature of oxide & hydroxide along the period decreaes & down the group increses.

12. The structure of IF_7 is :-

(1) octahedral

(2) pentagonal bipyramid

(3) square pyramid

(4) trigonal bipyramid

Ans. **(2)**

Sol. $sp^3 d^3$ hybridisation having no. L.P. i.e. why have regular geometry - i.e. Pentagonal bipyramidal.

13. Identify the compound that exhibits tautomerism :-

(1) 2-Pentanone (2) Phenol

(3) 2-Butene (4) Lactic acid

Ans. **(1)**

Sol. $CH_3-\underset{\underset{O}{\|}}{C}-CH_2-CH_2-CH_3 \rightleftharpoons CH_3-\underset{\underset{O-H}{|}}{C}=CH-CH_2-CH_3$ (enol)

14. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is :-

(1) 4-Bromophenol

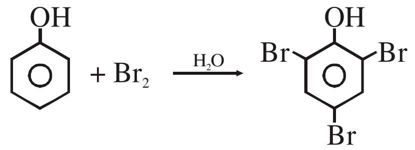
(2) 2,4,6-Tribromophenol

(3) 2-Bromophenol

(4) 3-Bromophenol

Ans. **(2)**

Sol. $KBrO_3 + 5KBr + 6H^+ \longrightarrow 3Br_2 + 6K^+ + 3H_2O$



2,4,6-Tribromo phenol

15. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at – 6°C will be :

(K_f for water = 1.86 K kgmol^{-1}, and molar mass of ethylene glycol = 62 gmol^{-1})

(1) 400.00 g (2) 304.60 g

(3) 804.32 g (4) 204.30 g

Ans. **(3)**

Sol. $\Delta T_f = \dfrac{1000 \times K_f \times w}{m_w \times W_{H_2O}}$ $\Delta T_f = 6$

$6 = \dfrac{1000 \times 1.86 \times w}{62 \times 4000}$ $W_{H_2O} = 4 \times 1000$ g

$w = \dfrac{6 \times 62 \times 4000}{1000 \times 1.86}$ $K_f = 1.86$

$m_w = 62$

w = 800 g (Nearly to 804.32g)

16. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression :-

(1) $\alpha = \dfrac{x+y-1}{i-1}$ (2) $\alpha = \dfrac{x+y+1}{i-1}$

(3) $\alpha = \dfrac{i-1}{(x+y-1)}$ (4) $\alpha = \dfrac{i-1}{x+y+1}$

Ans. **(3)**

Sol. $A_x B_y \rightleftharpoons x A^{-y} + yB^{-x}$

$1 \quad\quad 0 \quad\quad 0$

$1-\alpha \quad\quad x\alpha \quad\quad y\alpha$

$i = \dfrac{1-\alpha+x\alpha+y\alpha}{1} = \dfrac{\text{mole after dissociation}}{\text{mole before dissociation}}$

$i = 1 + \alpha(x + y - 1)$

$\alpha = \left(\dfrac{i-1}{x+y-1}\right)$

17. Boron cannot form which one of the following anions ?

(1) $B(OH)_4^-$ (2) BO_2^-

(3) BF_6^{3-} (4) BH_4^-

Ans. **(3)**

Sol. due to absence of vacant d-orbital boron can't expand its co-ordination no. by 4.

18. The strongest acid amongst the following compounds is ?

(1) $CH_3CH_2CH(Cl)CO_2H$

(2) $ClCH_2CH_2CH_2COOH$

(3) CH_3COOH

(4) HCOOH

Ans. **(1)**

Sol. Order of acidic strength is

$CH_3CH_2CHClCOOH > HCOOH > Cl{-}CH_2CH_2CH_2COOH > CH_3COOH$

pKa value = 2.84 3.74 4.52 4.74

19. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is :-

(1) Ethyl chloride (2) Ethyl ethanoate

(3) Diethyl ether (4) 2–Butanone

Ans. **(2)**

Sol. $CH_3{-}\underset{\displaystyle O}{\underset{\|}{C}}{-}Cl + C_2H_5ONa \xrightarrow{NSR} CH_3{-}\underset{\displaystyle O}{\underset{\|}{C}}{-}O{-}C_2H_5 + NaCl$

Ethyl ethanoate

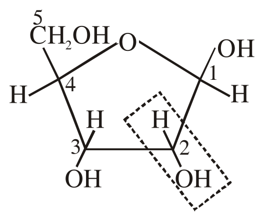
20. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA ?

(1) 3rd (2) 4th

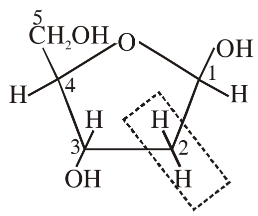
(3) 1st (4) 2nd

Ans. **(4)**

Sol.



D-ribose
(found in RNA)



D–2–deoxy ribose
(found in DNA)

21. The outer electron configuration of Gd (Atomic No. : 64) is :-

(1) $4f^4\ 5d^4\ 6s^2$ (2) $4f^7\ 5d^1\ 6s^2$

(3) $4f^3\ 5d^5\ 6s^2$ (4) $4f^8\ 5d^0\ 6s^2$

Ans. (2)

22. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is :-

(1) 2.82 BM (2) 1.41 BM

(3) 1.82 BM (4) 5.46 BM

Ans. (1)

Sol. $Ni^{+2} \rightarrow 3d^8$ [↿⇂|↿⇂|↿⇂|↿|↿] due to presence of weak ligand pairing of el⁻ does not takes place. i.e. why contain 2 unpaired el⁻

दुर्बल लिगेण्ड की उपस्थिति के कारण el⁻ का युग्मन नही हो पाता अत: 2 अयुग्मित el⁻ होते है।

23. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively :-

(1) sp, sp^3, sp^2 (2) sp^2, sp^3, sp

(3) sp, sp^2, sp^3 (4) sp^2, sp, sp^3

Ans. (4)

24. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of :-

(1) an isopropyl group

(2) an acetylenic triple bond

(3) two ethylenic double bonds

(4) a vinyl group

Ans. (4)

Sol. $\underset{\text{Vinyl group}}{-CH=CH_2} \xrightarrow{\text{Ozonolysis}} -\underset{\|}{C}-H + \underset{\text{product given in questions}}{H-\underset{\|}{C}-H}$ (each C=O)

25. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong ?

(1) The complex is an outer orbital complex

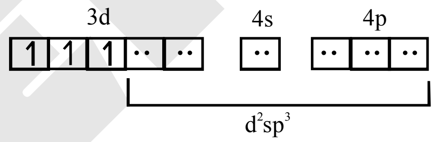
(2) The complex gives white precipitate with silver nitrate solution

(3) The complex involves d^2sp^3 hybridisation and is octahedral in shape

(4) The complex is paramagnetic

Ans. (1)

Sol. $Cr^{+3} \rightarrow 3d^3$



i.e. why it is a inner orbital complex. not the outer orbital complex.

26. The reduction potential of hydrogen half-cell will be negative if :-

(1) $p(H_2)$ = 2 atm $[H^+]$ = 1.0 M

(2) $p(H_2)$ = 2 atm and $[H^+]$ = 2.0 M

(3) $p(H_2)$ = 1 atm and $[H^+]$ = 2.0 M

(2) $p(H_2)$ = 1 atm and $[H^+]$ = 1.0 M

Ans. (1)

Sol. $2H^+_{(aq.)} + 2e^- \rightarrow H_2\ ^{(gas)}$

$$E_{Red} = E^{\circ}_{Red} - \frac{0.0591}{2} \log \frac{(P_{H_2})}{[H^+]^2}$$

$$E^{\circ}_{Red} = 0,\ P_{H_2} = 2,\ H^+ = 1M$$

$$E_{Red} = -\frac{0.0591}{2} \log \frac{(2)}{(1)^2}$$

$$\boxed{E_{Red} = -ve}$$

27. In content of the lanthanoids, which of the following statements is not correct ?

(1) Because of similar properties the separation of langhanoids is not easy

(2) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series

(3) There is a gradual decrease in the radii of the members with increasing atomic number in the series

(4) All the members exhibit +3 oxidation state

Ans. (2)

Sol. General oxidation state of Lanthanoid = + 3

28. Which of the following reagents may be used to distinguish between phenol and benzoic acid ?

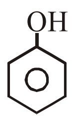
(1) Molisch reagent

(2) Neutral $FeCl_3$

(3) Aqueous NaOH

(4) Tollen's reagent

Ans. (2)

Sol. 

$+ \underset{\text{Neutral}}{FeCl_3} \longrightarrow$ Violet colour

(Test of phenolic –OH group)

29. Which of the following statements regarding sulphur is incorrect ?

(1) At 600°C the gas mainly consists of S_2 molceules

(2) The oxidation state of sulphur is never less than +4 in its compounds

(3) S_2 molecule is paramagnetic

(4) The vapour at 200°C consists mostly of S_8 rings

Ans. (2)

Sol. Sulphur shows minimum oxidation state – 2 in H_2S.

30. Which of the following statement is wrong ?

(1) Single N–N bond is weaker than the single P–P bond

(2) N_2O_4 has two resonance structures

(3) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table

(4) Nitrogen cannot form $d\pi$-$p\pi$ bond

Ans. (3)

Sol. On moving from top to bottom in group 15 bond length of hydride increases. i.e. why stability decreases.

PART B - MATHEMATICS

31. If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $(2\vec{a} - \vec{b}).[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$ is :-

(1) 5 (2) 3

(3) – 5 (4) – 3

Ans. (3)

Sol. $\vec{a}.\vec{b} = 0$ and $|a| = |b| = 1$

$(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b}) = (\vec{a} \times \vec{b}) \times \vec{a} + (\vec{a} \times \vec{b}) \times 2\vec{b}$

$= -\left[\vec{a} \times (\vec{a} \times \vec{b}) + 2\vec{b} \times (\vec{a} \times \vec{b})\right]$

$= -\left[(\vec{a}.\vec{b})\vec{a} - (\vec{a}\,.\,\vec{a})\vec{b} + 2(\vec{b}\,.\vec{b})\vec{a} - 2(\vec{b}\,.\,\vec{a})\vec{b}\right]$

$= -\left[0 - \vec{b} + 2\vec{a} + 0\right] = \left[\vec{b} - 2\vec{a}\right]$

$\therefore (2\vec{a} - \vec{b}) \cdot \left[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})\right] = (2\vec{a} - \vec{b})\,.\,(\vec{b} - 2\vec{a})$

$= -4a^2 - b^2 + 4\,\vec{a}\,.\,\vec{b} = -5$

32. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is :-

(1) – 144 (2) 132

(3) 144 (4) – 132

Ans. (1)

Sol. $(1 - x - x^2 + x^3)^6$

$\Rightarrow [(1 - x) - x^2 (1 - x)]$

$\Rightarrow (1 - x)^6 (1 - x^2)^6$

$\Rightarrow (1 - {}^6C_1 x + {}^6C_2x^2 - {}^6C_3x^3 + {}^6C_4 x^4 - {}^6C_5 + {}^6C_6 x^6) \times (1 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + ...)$

$\Rightarrow$ cofficient of x^7 is $\rightarrow$

${}^6C_1 . {}^6C_3 - {}^6C_3 . {}^6C_2 + {}^6C_5 . {}^6C_1$

$\Rightarrow \frac{6.5.4}{3.2} - \frac{6.5.4}{3.2.1} + 6.6$

$= 120 - 300 + 36$

$\Rightarrow -144$

33. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that :-

(1) $|\beta| = 1$ (2) $\beta \in (1, \infty)$

(3) $\beta \in (0,1)$ (4) $\beta \in (-1,0)$

Ans. (2)

Sol. Let $z^2 + \alpha z + \beta = 0$ has $(1 + iy_1)$ and $(1 + iy_2)$

so $z_1z_2 = \beta$

$(1 + iy_1)(1 + iy_2) = \beta$

$\beta = 1 - y_1y_2 + i(y_1 + y_2)$ ($\because$ β is purely real)

here $y_1 + y_2 = 0$

$y_1 = -y_2$

$\beta = 1 - y_1y_2$

$\beta = 1 + y_1^2$

$\beta > 1$

$\Rightarrow \beta \in (1, \infty)$

34. Consider the following statements

P : Suman is brilliant

Q : Suman is rich

R : Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as :-

(1) $\sim Q \leftrightarrow \sim P \wedge R$

(2) $\sim (P \wedge \sim R) \leftrightarrow Q$

(3) $\sim P \wedge (Q \leftrightarrow \sim R)$

(4) $\sim (Q \leftrightarrow (P \wedge \sim R))$

Ans. (2,4)

Sol. Given Statement :

$(P \wedge \sim R) \Leftrightarrow Q$

Negations of $A \Leftrightarrow B$ are $\sim (A \Leftrightarrow B)$, $\sim (B \Leftrightarrow A)$, $\sim A \Leftrightarrow B$ and $\sim B \Leftrightarrow A$

Hence negations of given statement are $\sim(Q \Leftrightarrow (P \wedge \sim R))$ and $\sim (P \wedge \sim R) \Leftrightarrow Q$

35. $\frac{d^2x}{dy^2}$ equals :-

(1) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (2) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(3) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (4) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

Ans. (2)

Sol. $\frac{d}{dy}\left(\left(\frac{dy}{dx}\right)^{-1}\right)$

$= \frac{d}{dy}\left(\left(\frac{dy}{dx}\right)^{-1}\right).\frac{dx}{dy}$

$= -\left(\frac{dy}{dx}\right)^{-2}.\frac{d^2x}{dx^2}.\frac{dx}{dy}$

$= -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

36. **Statement-1:**

The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Statement-2:

The line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A (1, 0, 7) and B(1, 6, 3).

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

Ans. (4)

Sol. $1(1-1) + 2\,(0-6) + 3\,(7-3)$

$= 0 - 12 + 12 = 0$

mid point AB (1, 3, 5)

lies on $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

37. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is :-

(1) $P(C|D) < P(C)$

(2) $P(C|D) = \frac{P(D)}{P(C)}$

(3) $P(C|D) = P(C)$

(4) $P(C|D) \geq P(C)$

Ans. (4)

Sol. $P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$

$P(D) = \frac{P(C)}{P\left(\frac{C}{D}\right)} \leq 1$

$P(C) \leq P\left(\frac{C}{D}\right)$

$P\left(\frac{C}{D}\right) \geq P(C)$

38. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval :-

(1) $\left[0, \frac{1}{2}\right]$ (2) $\left(\frac{11}{12}, 1\right]$

(3) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (4) $\left(\frac{3}{4}, \frac{11}{12}\right]$

Ans. (1)

Sol. at least one failure = 1 – all sucess

$$1 \geq 1 - p^5 \geq \frac{31}{32}$$

$$0 \leq p^5 \leq \frac{1}{32}$$

$$0 \leq p \leq \frac{1}{2}$$

$$p \in \left[0, \frac{1}{2}\right]$$

39. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is :-

(1) $\frac{\pi}{2}\log 2$ (2) log 2

(3) π log 2 (4) $\frac{\pi}{8}\log 2$

Ans. (3)

Sol. put x = tan θ then

$$I = \int_0^{\pi/4} \frac{8\log(1+\tan\theta)}{\sec^2\theta}\sec^2\theta d\theta \quad ....(i)$$

$$I = 8\int_0^{\pi/4} \log(1+\tan\theta)d\theta$$

$$= 8 \times \frac{\pi}{8} \log 2$$

$$I = \pi \log 2$$

40. For $x \in \left(0, \frac{5\pi}{2}\right)$, define

$$f(x) = \int_0^x \sqrt{t}\sin t\,dt$$

Then f has :-

(1) local minimum at π and local maximum at 2π

(2) local maximum at π and local minimum at 2π

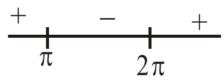
(3) local maximum at π and 2π

(4) local minimum at π and 2π

Ans. (2)

Sol. $f'(x) = \sqrt{x}\sin x$

f'(π) & f'(2π) are 0.

f '(x) 

⇒ local maximum at x = π and local minimum at x = 2π

41. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}.\vec{d} = 0$. Then the vector $\vec{d}$ is equal to :-

(1) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (2) $\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

(3) $\vec{b}-\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (4) $\vec{c}+\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

Ans. (2)

Sol. **a**.**b**. ≠ 0

a.**d** = 0

b × **c** = **b** × **d**

a × (**b** × **c**) = **a** × (**b** × **d**)

(**a**.**c**)**b** – (**a**.**b**)**c** – (**a**.**c**)**b** – (**a**.**b**)**d** {**a**.**d**=0}

⇒ (**a**.**b**)**d** = (**a**.**b**)**c** (**a**.**c**)**b** (divide by **a**.**b**)

$$\boxed{\mathbf{d} = \mathbf{c} - \frac{(\mathbf{a.c})}{(\mathbf{a.b})}\mathbf{b}}$$

42. Let R be the set of real numbers.

Statement-1:

A = {(x, y) ∈ R × R : y – x is an integer} is an equivalence relation on R.

Statement-2:

B = {(x, y)∈ R × R : x = αy for some rational number α} is an equivalence relation on R.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

Ans. (1)

Sol. St. 1 Given A = {(x, y) R × R : y – x (Integer}

For Reflexive xRx ⇒ x – x = 0 (integer)

for symmetric xRy ⇒ y – x = I (integer)

$\Rightarrow$ yRx $\Rightarrow$ x – y = –I (integer)

so xRy $\Rightarrow$ yRx.

for transitive let xRy and yRx

$y - x = I_1$ and $z - y = I_2$

xRz $\Rightarrow z - x = I_1 + I_2$ (integer)

so it is equivalente relation.

St.-2 B = (x, y) $\in$ RxR : x = 29 (α is relational)

for transitive xRy and yRz

$x = \alpha y$ and $y = \alpha z$

$\Rightarrow \alpha = \frac{x}{y}$ and $\alpha = \frac{y}{z}$

Multiply

$\alpha^2 = \frac{x}{z}$ (rational)

but α may be rational or may be not so it is not transitive.

Ans. (1) St.1 is true and 2 is false.

43. Let A and B be two symmetric matrices of order 3.

Statement-1:

A(BA) and (AB)A are symmetric matrices.

Statement-2:

AB is symmetric matrix if matrix multiplication of A with B is commutative.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

Ans. (4)

Sol. $A^T = A$

$B^T = B$

St-1 : $(A(BA))^T = (BA)^TA^T$

$= A^TB^TA^T = A(BA) \rightarrow$ symetric

$((AB)A))^T = A^T B^T A^T = (AB) A \rightarrow$ symetric

Statement - 1 is true

St-2 : $(AB)^T = B^TA^T = BA$

if AB = BA then

$(AB)^T = BA = AB$

St.- 2 is true

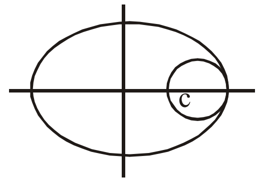
but Not a correct expalnation.

44. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ (c > 0) touch each other if :-

(1) a = 2c (2) |a| = 2c (3) 2|a| = c (4) |a| = c

Ans. (4)

Sol.



$\left|\frac{a}{2}\right| = c - \left|\frac{a}{2}\right|$

|a| = C

45. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(1) equals $-\sqrt{2}$ (2) equals $\frac{1}{\sqrt{2}}$

(3) does not exist (4) equals $\sqrt{2}$

Ans. (3)

46. If $A = \sin^2x + \cos^4x$, then for all real x :-

(1) $1 \le A \le 2$ (2) $\frac{3}{4} \le A \le \frac{13}{16}$

(3) $\frac{3}{4} \le A \le 1$ (4) $\frac{13}{16} \le A \le 1$

Ans. (3)

Sol. $A = \sin^2x + \cos^4x$

$= \cos^4x - \cos^2x + 1$

$= (\cos^2x)^2 - 2.\cos^2x.\frac{1}{2} + \frac{1}{4} - \frac{1}{4} + 1$

$= (\cos^2x - \frac{1}{2})^2 + \frac{3}{4}$

Again put cosx = 0 $\Rightarrow \frac{1}{4}+\frac{3}{4}=1$

cosx = 1 $\Rightarrow \frac{1}{4}+\frac{3}{4}=1$

$\frac{3}{4} \le A \le 1$ Ans.

47. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement - 1 :

The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement - 2 :

In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement-1 is true, Statement-2 is false.
(2) Statement-1 is false, Statement-2 is true
(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

Ans. (1)

48. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is:-

(1) $(-\infty, 0)$ (2) $(-\infty, \infty) - \{0\}$
(3) $(-\infty, \infty)$ (4) $(0, \infty)$

Ans. (1)

Sol. $f(x) = \frac{1}{\sqrt{|x| - x}}$

For domain of real function
$|x| - x > 0$
$|x| > x$
$\boxed{x \in (-\infty, 0)}$

49. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals :-

(1) $\frac{2}{5}$ (2) $\frac{5}{3}$ (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

Ans. (3)

Sol. $\frac{x}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$ equation of line

equation of plane $x + 2y + 3z = 4$

$\sin\theta = \frac{1 + 4 + 3\lambda}{\sqrt{14}\sqrt{1+4+\lambda^2}}$

$\Rightarrow \lambda = \frac{2}{3}$

50. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is :-

(1) $\frac{8}{3\sqrt{2}}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{\sqrt{3}}{4}$ (4) $\frac{3\sqrt{2}}{8}$

Ans. (4)

51. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after :-

(1) 20 months (2) 21 months
(3) 18 months (4) 19 months

Ans. (2)

Sol. Saving after first 3 month = 600

$600 + \left\{\frac{240 + 280 + ...}{\text{let n month}}\right\} = 11040$

$|240 + 280 + ... \text{ n terms}| = 10440$
$n/2\ [480 + (n - 1)40] = 10440$
$n\ \{440 + 40n\} = 20880$
$n^2 + 11n - 522 = 0$
$n = 18, -29$ (-29 rejected)
Total months = n + 3
18 + 3 = 21 Months

52. If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then $|a|$ equals:-

(1) 4 (2) 5 (3) 2 (4) 3

Ans. (1)

Sol. Mediam = 25.5 a

x_i	:	a	2a	3a	50 a
$\|x_i - M\|$	:	24.5a	23.5a	....	24.5a

$\sum|x_i - M| = [24.5a + 23.5\ a + ... + 0.5\ a + 0.5\ a + ... + 24.5a]$
$= 2a\ [0.5 + 1.5 + ... + 24.5]$

$\sum|x_i - M| = 25 \times 25\ a$

M. D. $= \dfrac{25 \times 25a}{50}$

$50 = \dfrac{25 \times 25a}{50}$

$\Rightarrow a = 4$

53. If $\omega(\neq 1)$ is a cube root of unity, and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals :-

(1) (1, 0) (2) (–1, 1)

(3) (0, 1) (4) (1, 1)

Ans. (4)

Sol. $(1 + \omega)^7 = A + B\omega$

$(-\omega^2)^7 = A + B\omega$

$-\omega^2 = A + B\omega$

$1 + \omega = A + B\omega$

$A = 1$

$B = 1$ (1, 1)

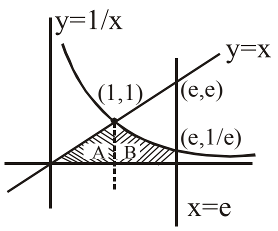
54. The area of the region enclosed by the curves $y = x$, $x = e$, $y = \dfrac{1}{x}$ and the positive x-axis is:-

(1) $\dfrac{3}{2}$ square units (2) $\dfrac{5}{2}$ square units

(3) $\dfrac{1}{2}$ square units (4) 1 square units

Ans. (1)

Sol. $y = x$, $x = e$, $y = \dfrac{1}{x}$



Area of part A $= \dfrac{1}{2}$

Area of part B $= \int_1^e \dfrac{1}{x}dx = \log e = 1$

Total area $= \dfrac{1}{2} + 1 = \dfrac{3}{2}$

55. The number of values of k for which the linear equations

$4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

possess a non-zero solution is :-

(1) 1 (2) zero (3) 3 (4) 2

Ans. (4)

Sol. $\Delta = 0$ (For Non zero solution)

$$\begin{vmatrix} 4 & K & 2 \\ K & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$8 - K(K - 2) + 2(2K - 8) = 0$

$8 - K^2 + 2K + 4K - 16 = 0$

$-K^2 + 6K - 8 = 0$

$K^2 - 6K + 8 = 0$

$(K - 4)(K - 2) = 0$

$K = 2, 4$ Two solution

56. The values of p and q for which the function

$$f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{\frac{3}{2}}}, & x > 0 \end{cases}$$

is continuous for all x in R, are :-

(1) $p = -\dfrac{3}{2}, q = \dfrac{1}{2}$

(2) $p = \dfrac{1}{2}, q = \dfrac{3}{2}$

(3) $p = \dfrac{1}{2}, q = -\dfrac{3}{2}$

(4) $p = \dfrac{5}{2}, q = \dfrac{1}{2}$

Ans. (1)

Sol. LHL = $\lim_{x\to 0}\frac{\sin(p+1)x}{x}+\frac{\sin x}{x}$

$= (p + 1) + 1 = p + 2$

LHL = f(0) $\Rightarrow$ $\boxed{p+2=q}$ ...(1)

RHL = $\lim_{x\to 0}\frac{x^2}{x^{3/2}(\sqrt{x+x^2}+\sqrt{x})}=\frac{1}{2}$

$p + 2 = q = \frac{1}{2} \Rightarrow q = \frac{1}{2}, p = \frac{-3}{2}$

57. **Statement - 1 :**

The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement - 2 :

The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

Ans. (3)

Sol. $B_1 + B_2 + B_3 + B_4 = 10$

St - 1 : $B_1 \geq 1, B_2 \geq 1, B_3 \geq 1, B_4 \geq 1$

so no. of negative integers solution of equation

$x_1 + x_2 + x_3 + x_4 = 10 - 4 = 6$

${}^{6+4-1}C_{4-1} = {}^9C_3$

St - 2 : selction of 3 places from out of 9 places = ${}^9C_3$

Both statements are true and correct explaination

58. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{2/5}$ is :-

(1) $3x^2 + 5y^2 - 15 = 0$

(2) $5x^2 + 3y^2 - 32 = 0$

(3) $3x^2 + 5y^2 - 32 = 0$

(4) $5x^2 + 3y^2 - 48 = 0$

Ans. (3)

59. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$, where k > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is :-

(1) $I - \frac{k(T-t)^2}{2}$ (2) e^{-kT}

(3) $T^2 - \frac{I}{k}$ (4) $I - \frac{kT^2}{2}$

Ans. (4)

Sol. $\frac{dV}{dt} = -k(T-t)$

$\int dV = \int -K(T-t)dt$

$V = -K\left[Tt-\frac{t^2}{2}\right]+C$

At t = 0 V = I $\Rightarrow$ C = I

$V = -Kt\left(T-\frac{t}{2}\right)+I$

$V(T) = -KT\left(T-\frac{T}{2}\right)+I$

$=\frac{-KT^2}{2}+I$

60. If $\frac{dy}{dx} = y + 3 > 0$ and y(0) = 2, then y(ln 2) is equal to :-

(1) 13 (2) –2 (3) 7 (4) 5

Ans. (3)

Sol. $\frac{dy}{dx} = y+3>0$ y (0) = 2, y(log 2) = ?

$\int\frac{dy}{y+3}=\int dx$

$\log|y + 3| = x + c$

$y(0) = 2$

$\log|2 + 3| = 0 + c \Rightarrow c = \log 5.$

y.(log 2) = ?

$\log|y + 3| = \log 2 + \log 5$

$\log|y + 3| = \log 10$

$y + 3 = 10$

$y = 7$

PART C - PHYSICS

61. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is :-

(1) 10 m/s (2) 15 m/s

(3) $\frac{1}{10}$ m/s (4) $\frac{1}{15}$ m/s

Ans. **(4)**

Sol. Velocity of image = $-m^2V_0 = -\left(\frac{f}{f-u}\right)^2 V_0$

$= -\left(\frac{20}{20-(-280)}\right)^2 (15)$

$= -\frac{1}{15}$ m/s

62. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is :-

(1) 20 min (2) 28 min

(3) 7 min (4) 14 min

Ans. **(1)**

Sol. $\frac{1}{3} = \left[\frac{1}{2}\right]^{\frac{t_2}{T}}$ $\quad \because \frac{N}{N_0} = \left[\frac{1}{2}\right]^{t/T}$

$\frac{2}{3} = \left[\frac{1}{2}\right]^{\frac{t_1}{T}}$ $\quad \frac{1}{2} = \left[\frac{1}{2}\right]^{(t_2-t_1)\frac{1}{T}}$

$1 = \frac{(t_2 - t_1)}{T}$ $\quad T = t_2 - t_1$

$t_2 - t_1 = 20$ min.

63. A boat is moving due east in a region where the earth's magnetic field is $5.0\times10^{-5} NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the indueced emf in the wire of aerial is :-

(1) 0.50 mV (2) 0.15 mV

(3) 1 mV (4) 0.75 mV

Ans. **(2)**

Sol. $e = B\ell v = (5 \times 10^{-5})(2)(1.50) = 0.15$ mV

64. The transverse displacement y(x, t) of a wave on a string is given by

$y(x,t) = e^{-\left(ax^2+bt^2+2\sqrt{ab}xt\right)}$

This represents a :-

(1) standing wave of frequency $\sqrt{b}$

(2) standing wave of frequency $\frac{1}{\sqrt{b}}$

(3) wave moving in +x directionwith speed $\sqrt{\frac{a}{b}}$

(4) wave moving in –x direction with speed $\sqrt{\frac{b}{a}}$

Ans. **(4)**

Sol. $y(x_1t) = e^{-[\sqrt{a}x+\sqrt{b}t]^2}$

$V = \omega/K = \frac{\sqrt{b}}{\sqrt{a}}$

$V = \sqrt{\frac{b}{a}}$ in –ve x direction.

65. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :-

(1) $\frac{\pi}{2}\frac{v^4}{g^2}$ (2) $\pi\frac{v^2}{g^2}$ (3) $\pi\frac{v^2}{g}$ (4) $\pi\frac{v^4}{g^2}$

Ans. **(4)**

Sol. $R_{max.} = \frac{u^2}{g}$

$A = \pi r^2 = \frac{\pi v^4}{g^2}$

66. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is :-

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

Ans. **(4)**

Sol. $X_1 = A \sin \omega t$

$X_2 = X_0 + A \sin(\omega t + \phi)$

$X_2 - X_1 = X_0 + A \sin(\omega t + \phi) - A \sin \omega t$

Become $|X_2 - X_1|_{max} = X_0 + A$

so $|A \sin(\omega t + \phi) - A \sin \omega t)|_{max} = A$

$\Rightarrow \phi = \frac{\pi}{3}$

67. This question has Statememtn-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement–1 :

A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also boubled.

Statement-2 :

The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(1) Statement–1 is true, Statement–2 is true, Statement–2 is not the correct explanationof Statement–1

(2) Statement–1 is false, Statement–2 is true

(3) Statement–1 is true, Statement–2 is false

(3) Statement–1 is true, Statement–2 is true, Statement–2 is the correct explanation of Statement–1

Ans. **(2)**

Sol. If ν is double k_{max} and v_0 become more than double so statement 1 is incorrect but 2 is correct according to einstein equation $K_{max} = h\nu - \phi_0$ or $v_0 = \frac{h}{e}\nu - \frac{\phi_0}{e}$

68. Let the x–z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is :-

(1) 60° (2) 75° (3) 30° (4) 45°

Ans. **(Bonus)**

Sol. If given plane is x – y then

incident angle $\theta_1 = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{k}}{A}\right)$

$= \cos^{-1}\left(\frac{-10}{20}\right) = \cos^{-1}\left(\frac{-1}{2}\right) = \left(\frac{\pi}{3}\right)$

Now by snells law $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$\sin\theta_2 = \left(\frac{n_1}{n_2}\right)\sin\theta_1 = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{\sqrt{2}}$

$= \theta_2 = 45°$

69. A Carnot engine operating between temperatures T_1 and T_2 has efficientcy $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively:-

(1) 330 K and 268 K (2) 310 K and 248 K

(3) 372 K and 310 K (4) 372 K and 330 K

Ans. **(3)**

Sol. $1 - \frac{T_2}{T_1} = \frac{1}{6} \Rightarrow \frac{T_2}{T_1} = \frac{5}{6}$ ... (1)

and $1 - \frac{T_2 - 62}{T_1} = \frac{1}{3} \Rightarrow \frac{T_2 - 62}{T_1} = \frac{2}{3}$ ... (2)

By solving equation (1) and (2)

$T_1 = 372$ K and $T_2 = 310$ K

70. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is :-

(1) 108.8 eV (2) 122.4 eV

(3) 12.1 eV (4) 36.3 eV

Ans. **(1)**

Sol. Rquired energy $= 13.6\,(Z)^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ eV

$= 13.6\,(3)^2 \left[\frac{1}{1^2} - \frac{1}{3^2}\right] = 108.8$ eV

71. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10} 2.5 = 0.4$)

(1) $2.7 \times 10^6\ \Omega$ (2) $3.3 \times 10^7\ \Omega$

(3) $1.3 \times 10^4\ \Omega$ (4) $1.7 \times 10^5\ \Omega$

Ans. **(1)**

Sol. $V = V_0\left(1 - e^{-t/RC}\right)$

$\Rightarrow 120 = 200\left(1 - e^{-\frac{5}{RC}}\right) \Rightarrow R = 2.7 \times 10^6\ \Omega$

72. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly broght to rest. Assuming no heat is lost to the surroundings, its temperature increases by :-

(1) $\frac{\gamma M v^2}{2R}K$ (2) $\frac{(\gamma-1)}{2R}Mv^2K$

(3) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2K$ (4) $\frac{(\gamma-1)}{2\gamma R}Mv^2K$

Ans. (2)

Sol. $\frac{1}{2}Mv^2 = \frac{f}{2}R\Delta T$ and $\gamma = 1 + \frac{2}{f}$

$\Rightarrow \Delta T = \frac{(\gamma-1)}{2R}Mv^2$

73. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1}) :-

(1) 2π mJ (2) 0.4 π mJ

(3) 4π mJ (4) 0.2 π mJ

Ans. (2)

Sol. $W = 8\pi T\ [(r_2^2) - r_1^2]$

$= 8 \times \pi \times 0.03\ [25 - 9] \times 10^{-4}$

$= \pi \times 0.24 \times 16 \times 10^{-4}$

$= 3.8 \times 10^{-4}\ \pi$

$= 0.384\ \pi$ mJ $\approx 0.4\ \pi$ mJ

74. A fully charged capacitor C with intial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is :-

(1) $2\pi\sqrt{LC}$ (2) $\sqrt{LC}$

(3) $\pi\sqrt{LC}$ (4) $\frac{\pi}{4}\sqrt{LC}$

Ans. (4)

Sol. Energy is shared equally between L and C at

$t = \frac{T}{8}, \frac{3T}{8}......$

where $T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$

so $t = \frac{T}{8} = \frac{2\pi\sqrt{LC}}{8} = \frac{\pi}{4}\sqrt{LC}$

75. Direction :

The question has a paragraph followed by two statement, Statement-1 and statement-2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film

Statement-1:

When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement-2:

The centre of the interference pattern is dark :-

(1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

Ans. (1)

Sol. From Newtons ring experiment. Statement-1 is true and statement-1 is the correct explanation of statement-2.

76. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading : 0 mm.

Circular scale reading : 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above date is :-

(1) 0.026 cm (2) 0.005 cm

(3) 0.52 cm (4) 0.052 cm

Ans. (4)

Sol. Reading = Main scale reading + L.C × Number of division on circular scale.

$= 0 + \frac{1}{100} \times 52$ mm

$= +\ 0.52$ mm

$= 0.052$ cm

77. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2, and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, then final temperature of the mixture is :-

(1) $\frac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

(2) $\frac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

(3) $\frac{T_1 + T_2 + T_3}{3}$

(4) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

Ans. (4)

Sol. $\left(n_1C_{v_1}T_1 + n_2C_{v_2}T_2 + n_3C_{v_3}T_3\right)$

$= (n_1 + n_2 + n_3)C_{V_{mix}}T$

$\Rightarrow T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

78. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constant. Then the charge density inside the ball is :-

(1) $-24\pi\ a\in_0$ (2) $-6\ a\in_0$

(3) $-24\pi\ a\in_0 r$ (4) $-6\ a\in_0 r$

Ans. (2)

Sol. $E_r = -\frac{\partial\phi}{\partial r} = -2ar$

By gauss theorem $4\pi r^2E_r = \frac{q}{\varepsilon_0}$

By differention

$4\pi d\ (r^2E_r) = \frac{dq}{\varepsilon_0} = \frac{(4\pi r^2 dr)\rho}{\varepsilon_0}$

$\Rightarrow r^2dE_r + 2rE_r\ dr = \frac{1}{\varepsilon_0}\rho r^2 dr$

$\Rightarrow \frac{\partial E_r}{\partial r} + \frac{2}{r}E_r = \frac{\rho}{\varepsilon_0}$

$\Rightarrow \rho = -6\varepsilon_0 a$

79. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1:

Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement-2 :

The state of ionosphere varies from hour to hour, day to day and season to season.

(1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

Ans. (4)

80. A mass m hangs with the help of a string wrapped around a pulley on a firctionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is :-

(1) $\frac{2}{3}g$ (2) $\frac{g}{3}$ (3) $\frac{3}{2}g$ (4) g

Ans. (1)

Sol. $\tau = T \times R$ $\tau = I\alpha$

$T \times R = I\alpha$

$T \times R = \frac{mR^2}{2} \times \frac{a}{R}$

$T = \frac{ma}{2}$

$mg - T = ma$

$mg - \frac{ma}{2} = ma$

$a = \frac{2g}{3}$

81. A current I flows in an infinity long wire with cross section in the from of a semicircular ring of radius R. the magnitude of the magnetic induction along its axis is :-

(1) $\frac{\mu_0 I}{2\pi R}$ (2) $\frac{\mu_0 I}{4\pi R}$ (3) $\frac{\mu_0 I}{\pi^2 R}$ (4) $\frac{\mu_0 I}{2\pi^2 R}$

Ans. (3)

Sol. Net megnet field due to both elements

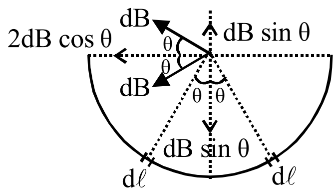
$= 2\,(dB)\cos\theta$

$= 2 \times \frac{\mu_0}{2\pi r} dI \cos\theta$ (here $dI = \frac{I}{\pi r} d\ell$)

$= 2 \times \frac{\mu_0}{2\pi r} \frac{I}{\pi r} d\ell \cos\theta$

$= 2 \times \frac{\mu_0}{2\pi r} \frac{I}{\pi r} r d\theta \cos\theta$

$B = \int_0^{\pi/2} \frac{\mu_0}{\pi^2 r} \cos\theta \, d\theta = \frac{\mu_0 I}{\pi^2 r}$



82. If a wire is stretched to make it 0.1 % longer its resistance will :-
(1) decrease by 0.2%
(2) decrease by 0.05%
(3) increase by 0.05%
(4) increase by 0.2%

Ans. **(4)**

Sol. $R = \rho \frac{\ell}{A}$

$R \propto \ell^2$

$\frac{\Delta R}{R} = \frac{\Delta R}{R} = \frac{2\Delta \ell}{\ell} = 2[0.1] = 0.2\%$ increase.

83. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. Euring the fjourney of the insect, then angular speed of the disc :-
(1) continuously increases
(2) first increases and then decreases
(3) remains unchanged
(4) continuously decreses

Ans. **(2)**

84. 100 g of water is heated from 30°C to 50°C Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K) :-
(1) 84 kJ (2) 2.1 kJ
(3) 4.2 kJ (4) 8.4 kJ

Ans. **(4)**

Sol. $Q = ms\Delta\theta = 0.1 \times 4184 \times 20 = 8.4$ KJ

85. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be :-
(1) 4 s (2) 8 s
(3) 1 s (4) 2 s

Ans. **(4)**

Sol. $\int_{6.25}^{0} \frac{dv}{\sqrt{v}} = -\int_0^t 2.5 dt$

$2\left[\sqrt{v}\right]_{6.25}^{0} = -2.5t$

$-2\sqrt{6.25} = -2.5t$

$-2 \times 2.5 = -2.5\, t$

$t = 2s$

86. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to :-
(1) 9.6×10^{-3} m (2) 3.6×10^{-3} m
(3) 5.0×10^{-3} m (4) 7.5×10^{-3} m

Ans. **(2)**

Sol. According to equation of continuity $A_1V_1 = A_2V_2$ or

$r_2 = \sqrt{\frac{r_1^2 v_1}{v_2}}$

Velocity of stream at 0.2 m below tap.

$V_2^2 = V_1^2 + 2as = 0.16 + 2 \times 10 \times 0.2 = 4.16$ m/s

$r_2 = \sqrt{\frac{r_1^2 v_1}{v_2}} = \sqrt{\frac{16 \times 10^{-6} \times 0.4}{2}} = \sqrt{3.2} \times 10^{-3}$ m

so diameter $= 2 \times \sqrt{3.2} \times 10^{-3}$ m

$= 2 \times 1.8 \times 10^{-3}$

$= 3.6 \times 10^{-3}$ m

87. Two identical charged spheres suspended from a common point by two massless string of length ℓ are initially a distance d(d << ℓ) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them :-

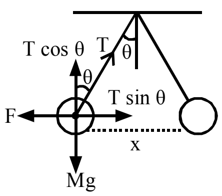
(1) $v \propto x^{1/2}$ (2) $v \propto x$

(3) $v \propto x^{-1/2}$ (4) $v \propto x^{-1}$

Ans. (3)

Sol. $\tan\theta = \dfrac{F}{Mg}$ (since θ small)

$\theta = \dfrac{F}{Mg} \Rightarrow F = Mg\theta$



$\dfrac{KQ^2}{x^2} = Mg\ \dfrac{x}{\ell}$

$Q^2 = \dfrac{Mg}{K\ell}x^3 \Rightarrow Q = \sqrt{\dfrac{Mg}{K\ell}}\,x^{3/2}$

$\left(\dfrac{dQ}{dt}\right) = \sqrt{\dfrac{Mg}{K\ell}}\left(\dfrac{3}{2}x^{1/2}\right)\left(\dfrac{dx}{dt}\right)$

$= \left(\dfrac{dQ}{dt}\right)$ is constant so $\dfrac{dx}{dt} \propto x^{-1/2}$

Or $V \propto x^{-1/2}$

88. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\dfrac{A_1}{A_2}\right)$ is :-

(1) $\left(\dfrac{M}{M+m}\right)^{1/2}$ (2) $\left(\dfrac{M+m}{M}\right)^{1/2}$

(3) $\dfrac{M}{M+m}$ (4) $\dfrac{M+m}{M}$

Ans. (2)

Sol. $n_1 = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{M}}$ ..... (1)

$n_2 = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{M+m}}$ ..... (2)

according to conservation of linear momentum

$MV_1 = (M + m)V_2$

$M_1A_1\omega_1 = (M + m)A_2\omega_2$

From equation (1) & (2)

$\dfrac{A_1}{A_2} = \left(\dfrac{M+m}{M}\right).\dfrac{\omega_2}{\omega_1} = \left(\dfrac{M+m}{M}\right)\sqrt{\dfrac{M}{M+m}}$

$\dfrac{A_1}{A_2} = \sqrt{\dfrac{M+m}{M}}$

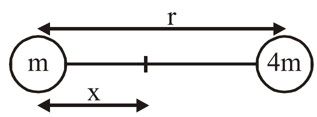
89. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is :-

(1) $-\dfrac{6Gm}{r}$ (2) $-\dfrac{9Gm}{r}$

(3) zero (4) $-\dfrac{4Gm}{r}$

Ans. (2)

Sol.



$\dfrac{G\times m}{x^2} = \dfrac{G\times 4m}{(r-x)^2} \Rightarrow x = \dfrac{r}{3}$

Potential at point the gravitational field is zero between the masses.

$V = -\dfrac{3Gm}{r} - \dfrac{3\times G\times 4m}{2r}$

$= -\dfrac{3Gm}{r}\,[1 + 2]$

$= -\dfrac{9GM}{r}$

90. A pulley of radius 2 m is rotated about its axis by a force F = $(20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion it reversed, is :-

(1) more than 6 but less than 9

(2) more than 9

(3) less than 3

(4) more than 3 but less than 6

Ans. **(4)**

Sol. $\tau = rF$

$\tau = 2(20t - 5t^2) = 40t - 10t^2$

$\because \tau = I\alpha = 40t - 10t^2$

$10 \times \alpha = 40t - 10t^2$

$\alpha = 4t - t^2$

$\therefore \frac{d^2\theta}{dt^2} = 4t - t^2$

$\omega = \frac{d\theta}{dt} = \frac{4t^2}{2} - \frac{t^3}{3}$ ....(i)

$\Rightarrow \frac{4t^2}{2} - \frac{t^3}{3} = 0$

ω will be zero at t = 6 sec.

from eq. (i) $\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$

$\theta = \frac{2t^3}{3} - \frac{t^4}{3 \times 4}$

$\theta = \frac{2}{3} \times 216 - \frac{36 \times 36}{12} = 36$ rad.

Number of turns $N = \frac{\theta}{2\pi} = \frac{18}{\pi} = 5.732$

so more than 3 but less than 6