MATHEMATICS

1. If the equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is $(2, -1)$,then the length of its side is
   a. $\frac{1}{\sqrt{3}}$
   b. $\sqrt{\frac{3}{2}}$
   c. $\sqrt{\frac{2}{3}}$
   d. None of the above

2. The line(s) passing through the intersection of $4x- 3y- 1=0$ and $2x- 5y + 3 =0$ and equally inclined to the axes will have
   a. $y = \pm\ x$ as the equations
   b. $y = x,\ x + y = 2$ as the equations
   c. $(y - x)^2 = 0$ as the equations
   d. an indefinite number of straight lines

3. If the three vertices of a parallelogram ABCD are A(1, 0), B(2, 3), C(3, 2), then the coordinates of the fourth vertex D will be
   a. (2, 1)
   b. (2, -1)
   c. (-1, 2)
   d. none of the above

4. Consider the following statements
   Assertion (A): The tangent and normal at any point P on an ellipse bisect the external and internal angles between the focal distances of P.
   Reason(R): The straight line joining the foci of the ellipse subtends a right angle at P.
   Of these statements
   a. Both A and R are true and R is the correct explanation of A
   b. Both A and R are true but R is not a correct explanation of A
   c. A is true but R is false
   d. A is false but R is true

5. The condition that the straight line $\frac{1}{r} = a \cos\theta + \sin\theta$ may touch the circle $r = 2c \cos\theta$
   a. $b^2c^2 + 2ac = 1$
   b. $a^2 + b^2 = c^2$
   c. $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$
   d. $a^2c^2 + b^2c^2 = 1$

6. The lines $3x - 4y + 4 = 0$ and $6x - [illegible]y - [illegible] = 0$ are tangents to the same circle. The radius of the circle is
   a. $\frac{1}{4}$
   b. $\frac{3}{4}$
   c. $\frac{4}{3}$
   d. None of the above

7. Two circles $x^2 + y^2 + 2ax + c = 0$ and $x^2+y^2 + 2by + c = 0$ touch if
   a. $a^2 + b^2 = c^2$
   b. [illegible] $= \frac{1}{a^2} + \frac{1}{b^2}$
   c. $\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$
   d. $c^2 = 4b^2(a^2 - c)$

8. The general equation of a circle $x^2 + y^2 + 2gx + 2fy + d = 0$ will cut the given circle $x^2+y^2=c^2$, orthogonally
   a. if $g = f = 0$
   b. if $d = c^2$
   c. if $d = - c^2$
   d. under none of the above conditions

9. The distances from the major axis of any point on an ellipse
   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
   and its corresponding point on the auxiliary circle are in the ratio
   a. $\frac{a}{b}$
   b. $\frac{b}{a}$
   c. $\frac{a^2}{b^2}$
   d. $\frac{a^2}{b}$

10. The equation $\sqrt{ax} + \sqrt{by} = 1$ represents
   a. a parabola
   b. a hyperbola
   c. an ellipse

d. none of the above

11. The line lx +my =n is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if

a. $a^2l^2+b^2m^2=(a^2-b^2)^2n^2$

b. $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$

c. $\frac{l^2}{a^2}+\frac{m^2}{b^2}=\frac{n^2}{(a^2-b^2)^2}$

d. $b^2l^2-a^2m^2=(a^2-b^2)^2n^2$

12. In the equation $ax^2+by^2+cz^2+2hxy+2gzx+2fyz+2ux+2vy+2wz+d=0$ if a = b = c = k (≠0) constant and f = g = h = 0, then the above equation would represent

a. a pair of straight lines

b. a plane

c. a circle

d. a sphere

13. The equation of the right circular cone whose axis is x = y = z, vertex is the origin and the semi-vertical angle is 45^0 is given as

a. $x^2 + y^2 + z^2 = 0$

b. $2(x^2+y^2+z^2) = 3(x+y+z)^2$

c. $3(x^2+y^2+z^2) = 2(x+y+z)^2$

d. $x^2+y^2+z^2+xy+yz+zx=0$

14. The general equation of the cone which passes through the coordinate axes is

a. $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$

b. $ax^2+by^2+cz^2=0$

c. $fyz + gzx + hxy = 0$

d. $y^2z+zx + xy = 0$

15. The equation of the right circular cylinder whose radius is r and axis is the z-axis is

a. $x^2+y^2=r^2$

b. $x^2+y^2+z^2=r^2$

c. $xy + yz + zx = r^2$

d. $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=r^2$

16. Forces of magnitudes 5, 1, 1, 3 act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$ respectively of a square ABCD of side q. If AB and AD are along the x – and the y-axis respectively, then the equation of the line of action along which the single resultant acts is

a. 2(x+y) = q

b. (x+y) = 2q

c. 2(x- y) = q

d. (x- y) = 2q

17. A person weighing 80 kg is standing on a lift. The lift moves upwards with a uniform acceleration of 4.9 m/s^2. The apparent weight (in kg). of the person is

a. 160

b. 120

c. 80

d. 40

18. A particle at P of unit mass describes an ellipse under an attraction f ' to the focus S and an attraction f to the focus S'. If SP = r, S'P = r' and the angle which SP and S'P make with the tangent at P is ϕ, then the equation of motion in the direction of normal to the curve is

a. $v^2/\rho = f\sin\phi + f'\sin\phi$

b. $v^2 = (f + f')\rho\sin\phi$

c. $v^2 = (f + f')\rho\cos\phi$

d. $\frac{dv}{ds} = -\left[f\frac{dr}{ds}+f'\frac{dr'}{ds}\right]$

19. The equation of the path of a particle moving in a central orbit is

a. $F= h^2u^2\left[u+\left(\frac{du}{d\theta}\right)^2\right]$

b. $F=h^2u^2\left[u^2+\frac{d^2u}{d\theta^2}\right]$

c. $F=h^2u^2\left[u+\frac{d^2u}{d\theta^2}\right]$

d. None of the above

20. The periodic time of the motion described by the differential equation $\frac{d^2x}{dt^2}+\mu x=0$ is

a. $2\pi/\mu$

b. $4\pi/\mu$

c. $2\pi/\sqrt{\mu}$

d. $4\pi/\sqrt{\mu}$

21. A particle is projected with a velocity u at an angle θ to the horizontal. The time of flight of the projectile is equal to

a. $\frac{u\sin\theta}{g}$

b. $\frac{u\sin\theta}{g}$

c. $\frac{2u\cos\theta}{g}$

d. $\frac{u\cos\theta}{g}$

22. A particle projected from the lowest point with velocity u moves along the inside of the arc of the smooth vertical circle of radius r. It will oscillate about the lowest point if

a. $u^2 > 2gr$

b. $u^2 < 2gr$

c. $u^2 > 5gr$

d. $u^2 = 5gr$

23. The earth's escape velocity (where R is the radius of the earth) is equal to

a. $\sqrt{2g}$

b. $\sqrt{2gR}$

c. $\sqrt{2g/R}$

d. None of the above

24. A body of 6.5 kg is suspended by two strings of length 5 and 12 metres attached to two points in the same horizontal line whose distance apart is 13 metres. The tension of the strings are

a. 2.0 kg and 6. 5 kg

b. 2.5 kg and 6.0 kg

c. 2.25 kg and 6.26 kg

d. 3.0 kg and 5.5 kg

25. A light L-shaped strip ABC is hinged smoothly at A and is kept in equilibrium by a force P at A and Q at C. If Q = 100 kg, AB = 12 cm , and BC = 9 cm, then the force with which hinge B is pressed, is

a. 75 kg

b. 100 kg

c. 125 kg

d. 150 kg

26. Three forces P, Q, R are acting at a point in plane. If the angle between P and Q and Q and R are 150^0 and 120^0 respectively, then for equilibrium , forces P, Q, R will be in the ratio

a. 1:2:3

b. 1:2: $\sqrt{3}$

c. 3:2:1

d. $\sqrt{3}$:2:1

27. Three vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ are coplanar if the value of the scalar triple product is

a. 0

b. 1

c. 2

d. 3

28. If θ is the angle between the vector $\vec{a}$ and $\vec{b}$, such that $|\vec{a}\cdot\vec{b}| = |\vec{a}\cdot\vec{b}|$, then θ is

a. 0

b. 45^0

c. 120^0

d. 180^0

29. Two non-zero vectors $\vec{A}$ and $\vec{B}$ are parallel if

a. $\vec{A}\times\vec{B} = \vec{0}$

b. $|\vec{A}\times\vec{B}| = 1$

c. $\vec{A}\cdot\vec{B} = 0$

d. $|\vec{A}| = |\vec{B}|$

30. The displacement of a point moving in a straight line is $s = 8t^2 + 3t - 5$

s being measured in meters and t in seconds. The velocity when the displacement is zero, is

a. 3m / sec

b. 13m /sec

c. 16m /sec

d. 12m /sec

31. Let x, y, z ∈I, the set of integers. Consider the following statement with regard to some properties associated with the set I:

1. either x = y or x< y or x > y

2. $x < y \Rightarrow x + z < y + z$

3. $x < y \Rightarrow xy < yz$

Of these statements

a. 1 and 2 are correct

b. 1 and 3 are correct

c. 2 and 3 are correct

d. 1, 2 and 3 are correct

32. Which one of the following is true with regard to a positive real number given by r = 3.12753753753.....?

a. r is the irrational number π

b. r is neither a rational number nor an irrational number

c. r is the rational number $\frac{104147}{3300}$

d. r is the irrational number nearest to 3.12753

33. If p is a irrational number such that 0<p<1 and x and y are real numbers with x < y, then

a. $p^x < p^y$

b. $p^y > y^x > 1$

c. $p^y > 1 > p^x$

d. $p^x > p^y$

34. Let z be a complex number satisfying $z^2 + z + 1 = 0$. If n is not a multiple of 3, then the value of $z^n + z^{2n}$ is

a. 2

b. –2

c. 0

d. –1

35. If 1, ω, ω^2 are the cube roots of unity, then the value of $(1 + \omega - \omega^2)^6$ is

a. 12

b. 32

c. 64

d. 128

36. The number of real solution(s) of the equation $x^2 + 3|x| + 2 = 0$ is

a. 1

b. 2

c. 3

d. none of these

37. Consider the following statements:

Assertion (A): If a positive integer is divisible by 2 and 3, then it is divisible by 6

Reason(R): If a positive integer is divisible by two positive integers, then it is divisible by their product.

Of these statements

a. Both A and R are true but R is not a correct explanation of A

b. Both A and R are true but R is not a correct explanation of A

c. A is true but R is false

d. A is false but R is true

38. If in a seminar the number of participants in Hindi, English and Mathematics is 60, 84 and 108 respectively, then the minimum number of rooms required, if in each room the same number of participants is to be seated and all of the being in the same subject is

a. 12

b. 21

c. 63

d. 24

39. Let $f(x) = \sqrt{2}x^2 + 3x - \sqrt{3}$ and $g(x) = x - \sqrt{2}$ are polynomials in x with real numbers as coefficients, when f(x) is divided by g(x), the remainder is $5\sqrt{2} - \sqrt{3}$, quotient is given by

a. $\sqrt{2}x - 5$

b. $\sqrt{2}x + 5$

c. $\sqrt{2}x + \sqrt{3}$

d. none of the above

40. If $f(x) = x^4 - 2x^3 - 3$ and $g(x) = x$ [illegible] the polynomials with real numbers as coefficients, then which one of the following is false?

a. $f(-1) = 0$

b. g(x) divides f(x)

c. f(x)= 0 has at least two real roots

d. g(x) does not divide f(x)

41. If α, β, γ and δ are the roots of $x^4 + 6x^2 - 5x + 4 = 0$ then $(\alpha+\beta+\gamma)(\beta+\gamma+\delta)(\gamma+\delta+\alpha)(\delta+\alpha+\beta)$ is

a. [illegible]

b. [illegible]

c. 4

d. none of the above

42. If one of the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ is 2, then the other two roots are

a. 1 and 3

b. 0 and 4

c. –1 and 5

d. –2 and 6

43. If 1 and 2 are two roots of the equation $x^4 - x^3 - 19x^2 + 49x - 30 = 0$, then the remaining two roots are

a. –3 and 5

b. 3 and –5

c. –6 and 5

d. 6 and –5

44. The equation whose roots are the reciprocals of the roots of $x^3 + px^2 + qx + r = 0$ is

a. $x^3 + \frac{1}{p}x^2 + \frac{1}{q}x + \frac{1}{r} = 0$

b. $\frac{1}{r}x^3 + \frac{1}{q}x^2 + \frac{1}{p}x + 1 = 0$

c. $rx^3 + px^2 + qx + 1 = 0$

d. $rx^3 + qx^2 + px + 1 = 0$

45. The number of non-empty subsets of a set consisting of 8 elements is

a. 256

b. 255

c. 128

d. none of the above

46. Let A and B be two sets having 5 common elements. The number of elements common to A×B and B×A is

a. 0

b. 5^2

c. 2^5

d. none of the above

47. If X_n be a set with n elements , where n is any finite cardinal number, then the cardinal number of its power set $P(X_n)$ is

a. 2^{n-1}

b. 2^n

c. 2^{n+1}

d. none of the above

48. Let R be a relation in the set of integers I , defined by a R b iff a and b both are neither even nor odd. Then R is

a. reflexive and symmetric

b. symmetric and transitive

c. symmetric but neither reflexive nor transitive

d. an equivalence relation

49. The set R={0,1,2,3} ,under addition and multiplication modulo 4 is

a. a field

b. a ring with zero divisors

c. a ring without zero divisors

d. a division ring

50. If S_3 denotes the group of permutations on three symbols then which one of the following would be false?

a. S_3 is of order 6

b. S_3 is not abelian

c. S_3 contains an element which generates the whole group

d. S_3 contains an element of order 2

51. Which one of the following is not a group?

a. The set of rotations about a point O in the plane with binary operation as the resultant of two rotations.

b. The set of complex numbers whose modulus is 1 with respect to multiplication of complex number

c. The set of non-zero residue classes modulo a composite positive integer m with respect to multiplication of residue classes

d. The power set of a non-empty set X with symmetric difference of sets as binary operation

52. If 1, $Z_1, Z_2, \ldots Z_{11}$ are the 12 roots of unity forming the cyclic group under multiplication, then Z_9 generates a cyclic subgroup of the above containing

a. 12 elements

b. 9 elements

c. 8 elements

d. 4 elements

53. Which one of the following statements is false?

a. Every permutation is a cycle

b. Every cycle is a permutation

c. S_n is not cyclic for all n

d. Every permutation $\sigma \in S_n$ can be written as product of(n-1) transposition

54. If $\begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, then X equals

a. $\begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$

b. $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 3 & -14 \\ 4 & -17 \end{bmatrix}$

55. If $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$, then a, b, c, are in

a. arithmetical progression

b. geometrical progression , $a\alpha^2+2\alpha\beta+c=0$

c. Harmonical progression

d. None of these

56. Let $A= \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ where a, b are non-zero real numbers. If $MA= A^{2m}$, for m, a positive integer, then which one the following is true?

a. $M= \begin{bmatrix} a^{2m} & b^{2m} \\ b^{2m} & -a^{2m} \end{bmatrix}$

b. $M=(a^{2m}+b^{2m})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c. $M=(a^{m}+b^{m})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d. $M= (a^2+b^2)^m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

57. Let A be an n×n matrix from the set of real numbers and $A^2-3A^2+4A-6I = 0$
Where I is n ×n unit matrix
If A^{-1} exists , then
a. $A^{-1}= A-I$
b. $A^{-1}= A+6I$
c. $A^{-1}= 3A-6I$
d. $A^{-1}=\frac{1}{6}(A^2-3A+4I)$

58. Let $A= \begin{bmatrix} 5 & 0 & 2 \\ 0 & 1 & 0 \\ -4 & 0 & -1 \end{bmatrix}$ and I be 3 ×3 unit matrix
If M= I-A, then rank of I-A is
a. 0
b. 1
c. 2
d. 3

59. If $A= \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ then A(adj A) equals
a. $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$
b. $\begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$
c. $\begin{bmatrix} 10 & 1 \\ 1 & 10 \end{bmatrix}$
d. None of the above

60. If $3x+ 2y + z =0$
$x + 4y + z = 0$
$2x + y + 4z = 0$ be a system of equations, then
a. it is inconsistent
b. it has only the trivial solution x = 0 , y= 0, z = 0
c. it can be reduced to a single equation and so solution does not exist
d. the determinant of the matrix of coefficients is zero

61. If x and y are real numbers, then which one of the following is always true?
a. $|x- y| \le |x| - |y|$
b. $|x- y| \ge |x|+ |y|$
c. $|x-y| \ge |x| - |y|$
d. $|x- y| = |x|- |y|$

62. Which one of the following is the correct solution of the inequality
$|x + 3| > 1$?
a. $-4 < x < -2$
b. $-3 < x < -1$
c. $-3 < x < -2$
d. $x < -4$ or $x > -2$

63. If we draw the graph of the function y = log x and take its reflection in the straight line x + y = 0, then we shall get the graph of the function
a. $x = \log(-y)$
b. $y = -e^{-x}$
c. $y = e^{-x}$
d. $y = -e^{x}$

64. The range of the function $y = f(x) = x^2- 4x + 7$, $2 \le x \le 3$ is
a. $0 \le y \le 3$
b. $2 \le y \le 7$
c. $3 \le y \le 7$
d. $3 \le y \le 4$

65. If $f(x+1) - 2f(x)+f(x-1)=2$ for all x then
a. $f(x) = \sqrt{x}$
b. $f(x) = x$
c. $f(x) =x^2$
d. $f(x) =x^3$

66. If $\lim_{x\to 0} (1+x)^{1/x} = e$, then $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{2n}$ is
a. e
b. 2e
c. $\frac{1}{e}$
d. $\frac{1}{e^2}$

67. The correct value of $\lim_{x\to 0}\frac{x}{\sqrt{1-\cos x}}$
a. does not exist
b. is $\sqrt{2}$
c. is $-\sqrt{2}$
d. is $1/\sqrt{2}$

68. $\lim_{n\to\infty} e^{1/n \log n}$ is
a. 1
b. 0
c. ∞
d. does not exist

69. If $y = x^{\log x}$, then $\frac{dy}{dx}$ equals
a. $\log x \,.\, x^{\log x-1}$
b. $x^{(\log x-1)}.\ 2 \log x$
c. $x \log (\log x)$

d. $\frac{1}{x}\log x \,.\, x^{\log x-1}$

70. A rod 26 meters long leans against a vertical wall. The foot of the rod is drawn away from the wall at a rate of 24 meters per sec. When the foot of the rod is 10 meters from the wall, the velocity of the middle point of the rod sliding down vertically, will be

a. 10 m /sec
b. 8m/sec
c. 6m /sec
d. 5m / sec

71. The length of the arc of the parabola $x^2 = 4ay$ from the vertex to the extremity of the latus rectum is given by

a. $\int_0^{2a}\sqrt{1+\frac{x^2}{4a^2}}\,dx$
b. $\int_0^{2a}\sqrt{1+\frac{4a^2}{x^2}}\,dx$
c. $\int_0^{a}\sqrt{\frac{1+y}{a}}\,dx$
d. $\int_0^{a}\sqrt{1+\frac{x^2}{4a^2}}\,dx$

72. If S_n denoted the sum of n terms of the series $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}+\ldots$ then

a. $S_n > n^2$
b. $S_n > n$
c. $S_n \geq \sqrt{n}$
d. $S_n = \infty$

73. If a curve is defined by the parametric co-ordinates $x = l\cos t$, $y = m\sin t$, then the radius of curvature at any point t is given by :

a. $l^2\sin^2 t + m^2\cos^2 t$
b. $\frac{1}{lm}(l^2\cos^2 t + m^2\sin^2 t)^{3/2}$
c. $\frac{1}{lm}(l^2\sin^2 t + m^2\cos^2 t)^{3/2}$
d. $\sin^2 t\cos^2 t$

74. If z = uv,
$u^2 + v^2 - x - y = 0$,
$u^2 - v^2 + 3x + y = 0$
then $\frac{\partial z}{\partial x}$ is equal to

a. u + v
b. $\frac{2u^2 - v^2}{2uv}$
c. $\frac{3u^2 + v^2}{2uv}$
d. $\frac{u^2 - 3v^2}{2uv}$

75. If $u = f(y + dx) + \phi(y - dx)$, then $\frac{\partial^2 u}{\partial x^2} - d^2\frac{\partial^2 u}{\partial y^2}$ is

a. 0
b. d^2
c. $d^2(f'' - \phi'')$
d. $d^2(f'' + \phi'')$

76. If $f = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is

a. f
b. 2f
c. tan f
d. sin f

77. Consider the following statements with regard to the curve $y - 3 = (x-2)^5$

Assertion (A): (2, 3) is a point of inflexion.

Reason (R): $\frac{d^2y}{dx^2} = 0$ at (2, 3)

Of these statements

a. Both A and R are true and R is the correct explanation of A
b. Both A and R are true but R is not a correct explanation of A
c. A is true but R is false
d. A is false but R is true

78. $\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\ldots+\frac{n}{n^2+(n-1)^2}\right]$ is equal to

a. $-\frac{\pi}{4}$
b. 0
c. $\frac{\pi}{4}$
d. $\frac{\pi}{3}$

79. If $y = \int_{\cos x}^{x}(u+u^2)\,du$, then $\frac{dy}{dx}$ is

a. $x + x^2 + \sin x\cos x + \cos^2 x\sin x$
b. $x + x^2 + 2\sin x\cos x$
c. $\frac{x^2}{2}+\frac{x^3}{3}-\frac{\cos^2 x}{2}-\frac{\cos^3 x}{3}$
d. None of the above

80. $\int_0^{\pi/2} \frac{e^x}{2}\left(\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right)dx$ is equal to

a. e^{π}

b. $e^{\pi/2}$

c. e

d. $e^{\pi/4}$

81. $\int_0^{\pi/2} \frac{\cos 2x}{\cos x+\sin x}dx$ equals

a. –1

b. 0

c. 1

d. 2

82. The area of the loop of the curve $py^2=x^2(p-x)$ is

a. $2/15p^2$

b. $4/15p^2$

c. $8/15p^2$

d. $15/8p^2$

83. The length of the arc of the equiangular spiral $r = ae^{\theta\cot\alpha}$ between the points for which the radii vectors are r_1 and r_2 is

a. $(r_2-r_1)\operatorname{cosec}\alpha$

b. $(r_2-r_1)\cos\alpha$

c. $(r_2-r_1)\sin\alpha$

d. $(r_2-r_1)\sec\alpha$

84. The series $\frac{3}{5}x^4+\frac{8}{10}x^6+\frac{15}{17}x^8+\ldots+\frac{n^2-1}{n^2+1}x^{2n}+\ldots$ is

a. convergent if $x^2 \geq 1$ and divergent if $x^2 <$

b. convergent if $x^2 \leq 1$ and divergent if $x^2>1$

c. convergent if $x^2 < 1$ and divergent if $x^2 \geq 1$

d. convergent if $x^2 > 1$ and divergent if $x^2 \leq 1$

85. The series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\ldots$ is convergent if

a. $0<x<1/e$

b. $x>1/e$

c. $2/e<x<3/e$

d. $3/e<x<4/e$

86. The sum of the alternating harmonic series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ is

a. Zero

b. Infinite

c. log 2

d. Not defined as the series is not convergent

87. The primitive of the differential equation $(2xy^4e^y+2xy^3+y)dx+(x^2y^4e^y-x^2y^2-3x)dy=0$ is

a. $x^2e^y+\frac{x^2}{y}+\frac{x}{y^2}=C$

b. $x^2e^y-\frac{x^2}{y}+\frac{x}{y^3}=C$

c. $x^2e^y+\frac{x^2}{y}-\frac{x}{y^3}=C$

d. $x^2e^y-\frac{x^2}{y}-\frac{x}{y^3}=C$

88. The solution of the equation $\frac{dy}{dx}+2xy=2xy^2$ is

a. $y=\frac{C}{1+e}-x^2$

b. $y=\frac{1}{1-Ce^x}$

c. $y=\frac{1}{1+Ce}x^2$

d. $y=\frac{Cx}{1+e}x^2$

89. The solution of the differential equation $(x+y)^2\frac{dy}{dx}=a^2$ is given by

a. $(y+x)=a\tan\left(\frac{y-C}{a}\right)$

b. $(y-x)=a\tan(y-C)$

c. $(y-x)=\tan\left(\frac{y-C}{a}\right)$

d. $a(y-x)=\tan\left(\frac{y-C}{a}\right)$

90. The general and singular solutions of $\left(\frac{dy}{dx}\right)+x\frac{dy}{dx}-y=0$ are

a. $(y-C_1x)(y-x^2/4-C_2)=0$; $x^2+4y=0$

b. $y=Cx+C^2$; $x^2+4y=0$

c. $(y-2x)^2=Cx$; $x^2+y^2-xy=0$

d. $(x^2+y^2)=Cxy+C^2$; $(xy)^2-4(x^2+y^2)=0$

91. The singular solution /solutions of $x\left(\frac{dy}{dx}\right)^2-2y\frac{dy}{dx}+4x=0$ $(x>0)$ is/are

a. $y = \pm x^2$
b. $y = 2x + 3$
c. $y = x^2 - 2x$
d. $y = \pm 2x$

92. The singular solution of p=log(px-y),is
a. $y = x(\log x - 1)$
b. $y = x \log x - 1$
c. $y = \log x - 1$
d. $y = x \log x$

93. The differential equation of the family of parabola with foci at the origin and axis along the x-axis is
a. $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$
b. $y^2\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$
c. $y\left(\frac{dy}{dx}\right)^2 + 2x^2\frac{dy}{dx} - y = 0$
d. $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} + y = 0$

94. Let $(y-C)^2 = Cx$ be the primitive of the differential equation $4x\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$ The number of integral curves which will pass through ([illegible], 2) is
a. One
b. Two
c. Three
d. Four

95. The set of orthogonal trajectories to a family of curves whose differential equation $\phi\left(r,\theta,\frac{dr}{d\theta}\right) = 0$ is found by the differential equation
a. $\phi\left(r,\theta,r\frac{dr}{d\theta}\right) = 0$
b. $\phi\left(r,\theta,r\frac{d\theta}{dr}\right) = 0$
c. $\phi\left(r,\theta,-r^2\frac{d\theta}{dr}\right) = 0$
d. $\phi\left(r,\theta,-\frac{1}{r}\frac{dr}{d\theta}\right) = 0$

96. The solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$ satisfying the conditions y(0) = 1, $y\left(\frac{\pi}{2}\right) = 2$ is
a. $\cos x + 2\sin x$
b. $\cos x + \sin x$
c. $2\cos x + \sin x$
d. $2(\cos x + \sin x)$

97. $e^{-x}(C_1 \cos\sqrt{3}x + C_2 \sin\sqrt{3}x) + C_3e^{2x}$ is the general solution of
a. $\frac{d^3y}{dx^3} + 4y = 0$
b. $\frac{d^3y}{dx^3} + 8y = 0$
c. $\frac{d^3y}{dx^3} - 8y = 0$
d. $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2 = 0$

98. The solution of the differential equation $(D^4 + 8D^2 + 16)y = 0$ is given by
a. $C_1e^{2x} + C_2e^{-2x} + C_3e^{x} + C_4e^{-x}$
b. $(C_1 + C_2)e^{2x} + (C_3 + C_4)e^{-2x}$
c. $(C_1 + C_2x)\cos 2x + (C_3 + C_4x)\sin 2x$
d. $(C_1 + C_2x)\cosh 2x + (C_3 + C_4)\sinh 2x$

99. The general solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 10\cos x$ is
a. $y = C_1e^{-x} + C_2e^{2x} - 3\cos x - \sin x$
b. $y = C_1e^{x} + C_2e^{2x} - 3\cos x$
c. $y = C_1e^{-x} + C_2e^{2x} - 3x + \sin x$
d. $x = C_1e^{x} + C_2e^{-2x} - 3\cos x - \sin x$

100. The solution of the equation $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$ is
a. $y = (C_1 + C_2x)e^{2x}$
b. $y = (C_1 + C_2x)e^{x}$
c. $y = (C_1 + C_2x)\log x$
d. $y = (C_1 + C_2\log x)x^2$