MATHEMATICS

1. If $5f(x) + 3f(1/x) = x + 2$ and $y = xf(x)$, then $\left(\frac{dy}{dx}\right)_{x=1}$ is equal to
   a. 14
   b. $\frac{7}{8}$
   c. 1
   d. $\frac{8}{7}$

2. Let g(x) be the inverse of an invertible function f(x) and $f'(x) = \frac{1}{1+x^3}$. Then g′(x) is equal to
   a. $\frac{1}{1+[g(x)]^3}$
   b. $\frac{1}{1+[f(x)]^3}$
   c. $1+[g(x)]^3$
   d. $1+[f(x)]^3$

3. A function f from R to R(R being the set of real numbers) is defined by the following formula:
   $f(x) = 15 - |x - 10|$.
   The number of points at which the function $g(x) = f\{f(x)\}$ is not differentiable is
   a. 0
   b. 1
   c. 2
   d. 3

4. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ then $\frac{dy}{dx}$ is equal to
   a. 0
   b. x
   c. y
   d. [illegible]

5. For the function f (x) defined by $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1-x & \text{if } x \text{ is irrational} \end{cases}$
   Which one of the following statements is correct?
   a. f is continuous at all rational numbers
   b. f is continuous only at x = 0
   c. f is continuous only at x = 1/2
   d. f is not continuous at any point

6. Let f be defined for all x and suppose that $|f(x) - f(y)| \le (x - y)^2$ for all real x and y. Then
   a. f is strictly increasing
   b. f is strictly decreasing
   c. f is constant
   d. f is strictly increasing for x > 0

7. We define a function $f : R \to R$ as follows: $f(x) = 2x^2 + 3x + 4$ if $x \in (-\infty, 1)$ and $f(x) = kx + 9 - k$ if $x \in [1, \infty)$. If this function is differentiable on the whole real line, then the value of k must be
   a. 4
   b. 5
   c. 6
   d. 7

8. If $f(x) = x^3 - 2[illegible]x^2 + [illegible]40x$, then $f(x)$ is
   a. Monotonically decreasing everywhere
   b. Monotonically decreasing only in $(0, \infty)$
   c. Monotonically increasing everywhere
   d. Monotonically increasing only in $(-\infty, 0)$

9. The maximum value of $\frac{\log x}{x}$ is
   a. 1
   b. E
   c. $\frac{2}{e}$
   d. $\frac{1}{e}$

10. For the curve $y = be^{x/a}$, which one of the following is true?
    a. The subtangent is of constant length, and the subnormal varies as the square of the ordinate
    b. The subtangent varies as the square of the ordinate, and subnormal is of constant length
    c. The subtangent is of constant length, and subnormal varies as the ordinate
    d. The subtangent varies as the ordinate, and subnormal is of constant length

11. If the function defined by $f(x) = 2x^2 + 3x - m \log x$ is a monotonic decreasing function on the open interval (0, 1), then the least possible value of the parameter m is
    a. 7
    b. $\frac{15}{2}$

c. $\frac{31}{4}$

d. 8

12. $\int [f(x)g''(x) - f''(x)g(x)]dx$ is equal to

a. $\frac{f(x)}{g'(x)}$

b. $f(x)g'(x) - f'(x)g(x)$

c. $f(x)g'(x) + f'(x)g(x)$

d. $f'(x)g(x) - f(x)g'(x)$

13. The image of the open interval (0, 1) under the continuous mapping $y = x - x^2$ is

a. The open interval (0, ¼)

b. The semi-closed interval (0, ¼]

c. The semi-closed interval [0, ¼]

d. The closed interval [0, ¼]

14. If $f(a+b-x) = f(x)$, then $\int_a^b x f(x)dx$ is equal to

a. $\frac{a+b}{2}\int_a^b f(b-x)dx$

b. $\frac{a+b}{2}\int_a^b f(x)dx$

c. $\frac{b-a}{2}\int_a^b f(x)dx$

d. $\frac{a-b}{2}\int_a^b f(x)dx$

15. The slope of the tangent to the curve $y = \int_0^{x^2} \frac{dt}{1+t^3}$ at the point where x = 1 is

a. 2

b. 1

c. 1/2

d. 1/4

16. If a < b; then $\int_a^b (|x-a| + |x-b|)dx$ is equal to

a. $\frac{(b-a)^2}{2}$

b. $\frac{b^2 - a^2}{2}$

c. [illegible]

d. $(b-a)^2$

17. $\int_0 [x^2]\,dx$, where [ ] denotes the greatest integer function, equals

a. $2+\sqrt{2}$

b. $2-\sqrt{2}$

c. $-\sqrt{2}-2$

d. $-2+\sqrt{2}$

18. The value of $\lim_{x \to 0} \frac{x e^{x^2}}{\int_0^x e^{t^2} dt}$

a. is 0

b. is 1

c. does not exist

d. is -1

19. $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$ is equal to

a. $x \log(\log x) + \frac{x}{\log x} + c$

b. $x \log(\log x) - \frac{x}{\log x} + c$

c. $x \log(\log x) + \frac{\log x}{x} + c$

d. $x \log(\log x) - \frac{\log x}{x} + c$

20. The number of asymptotes of the curve $x(x^2+y^2) - a(x^2+y^2) = 0$ is

a. [illegible]

b. 1

c. [illegible]

d. more than 2

21. The area common to curves $y^2 = x$ and $x^2 = y$ is equal to

a. 1

b. 2/3

c. 0

d. 1/3

22. The whole length of the curve r = 2a sin θ, is equal to

a. πa

b. 2πa

c. 3πa

d. 4πa

23. The orthogonal trajectory of the cardioid r = a(1+cos θ), a being the parameter, is

a. r = a(1- cos θ)

b. r = a cos θ

c. r = a(1 + cos θ)

d. r = a(1+ sin θ)

24. The differential equation corresponding to the family of curves $y = c(x-c)^2$, where c is a constant is

a. $4y^2 = 8xyy' - (y')^2$

b. $8y^2 = 4xyy' - (y')^3$

c. $8y^2 = 8xy' - (y')^3$

d. $y^2 = xyy' + (y')^2$

25. The zero divisors in Z_8 are

a. $\bar{3}, \bar{4}, \bar{5}$

b. $\overline{1}, \overline{2}, \overline{4}$
c. $\overline{2}, \overline{4}, \overline{7}$
d. $\overline{2}, \overline{4}, \overline{6}$

26. The characteristic of the ring $Z_4 \times Z_6$ is
a. 0
b. 6
c. 12
d. 24

27. Which one of the following statements is not correct?
a. The set of rationals is a field
b. Z_{31}, the ring of integers modulo 31 is a field
c. R[x], the set of polynomials over the set of real numbers is an integral domain but not a field
d. R[x] is not an integral domain

28. Let R be the set of the real numbers and $R^2=\{(x_1,x_2): x_1 \in R, x_2 \in R\}$. Then which one of the following is a subspace of R^2 over R?
a. $\{(x_1,x_2): x_1>0, x_2>0\}$
b. $\{(x_1,z_2): x_1 \in R, x_2>0\}$
c. $\{(x_1,x_2): x_1<0, x_2<0\}$
d. $\{(x_1,0): x_1 \in R\}$

29. Define T on R^4 into R^4 by $T(x_1,x_2,x_3,x_4)=(x_1+x_2+x_3, x_2+x_3+x_4, x_3+x_4, x_2)$. Then rank of T is equal to (R being the set of real numbers)
a. 1
b. 2
c. 3
d. 4

30. If $W_1=\{(0, x_2, x_3, x_4, x_5): x_2, x_3, x_4, x_5 \in R\}$ and $W_2=\{(x_1, 0, x_3, x_4, x_5): x_1, x_3, x_4, x_5 \in R\}$ be subspaces of R^5, then dim $(W_1 \cap W_2)$ is equal to
a. 5
b. 4
c. 3
d. 2

31. Let T be linear transformation on R^2 into itself such that T(1, 0)=(1, 2) and T(1, 1)=(0, 2). Then T (a, b) is equal to
a. (a, 2b)
b. (2a , b)
c. (a- b, 2a)
d. (a- b, 2b)

32. If T: V → V is a linear transformation of vector space V and the composite ToT = 0 then
a. Kernel of T ⊆ image of T
b. Image of T ⊆ kernel of T
c. T is 0 linear transformation
d. T is a non-singular linear transformation

33. Define T on R^2 into itself by $T(x_1, x_2)=(x_1+x_2, x_1-x_2)$. Then matrix of T^{-1} relative to the standard basis for R^2 is
a. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
b. $\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$
c. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$
d. $\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

34. If A be a non-zero square matrix of order n, then
a. The matrix A+ A′ is anti-symmetric, but the matrix A- A′ is symmetric
b. The matrix A+ A′ is symmetric, but matrix A- A′is anti-symmetric
c. Both A+ A′ and A- A′ are symmetric
d. Both A+ A′ and A- A′ are anti-symmetric

35. If $U_n=\begin{vmatrix} n & 1 & 5 \\ n^2 & 2p+1 & 2p+1 \\ n^3 & 3p^2 & 3p \end{vmatrix}$ then $\sum_{n=1}^{p} U_n$ equals
a. 1
b. $25p^3$
c. 0
d. $15p^2(2p+1)$

36. If x, y, z are in A.P. with common difference d and the rank of the matrix $\begin{bmatrix} 4 & 5 & x \\ 5 & 6 & y \\ 6 & k & z \end{bmatrix}$ is 2 , the values of d and k are
a. d = x/ 2; k is an arbitrary number
b. d an arbitrary number ; k = 7
c. d = x; k = 5
d. d = x/2 ; k = 6

37. The point (4, 1) undergoes the following three transformations successively
1. Reflection about the line y = x
2. Translation through a distance 2 units along the positive direction of x-axis

3. Rotation through an angle $\pi/4$ about the origin in the anticlockwise direction

The final position of the point is

a. $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

b. $\left(-2, \frac{7}{\sqrt{2}}\right)$

c. $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

d. $\left(\sqrt{2}, 7\sqrt{2}\right)$

38. A particle can descend along a straight smooth tube from a point A to a point B where AB makes an angle 60^0 with the horizontal. There is a smooth semicircular tube (whose diameter is AB) from A to B. If V_1, V_2 be the velocities of the particle having reached at B in the two cases- (i) while descending along the straight path, and (ii) while descending along the semicircular path, and if AB=l, then

a. $V_1^2 = \sqrt{3}\ gl = V_2^2$

b. $V_1^2 = \sqrt{3}\ gl,\ V_2^2 = \pi gl/2$

c. $V_1^2 = 2gl,\ \ V_2^2 = \pi gl/2$

d. $V_1^2 = V_2^2 = gl$

39. A particle is at rest at the origin. It moves along the axis with an acceleration [illegible] where x is the distance of the particle at time t. The particle next comes to rest after has covered a distance

a. 1

b. 1/2

c. 3/2

d. 2

40. α and β ($\alpha \neq \beta$) be the two angles of projection of projectile to reach a point distance R on the horizontal through the point of projection, u being the speed of projection and $u^2 > gR$. If the greatest heights attained by the projectile in the two trajectories be h_1 and h_2 respectively, then

a. $h_1 + h_2 = u^2/2g$

b. $h_1 + h_2 = u^2/4g$

c. $h_1+h_2=u^2/g\ (\sin^2\alpha + \sin^2\beta)$

d. $h_1+h_2=u^2/4g\ (\cos^2\alpha + \cos^2\beta)$

41. Two motor cars are moving along two roads perpendicular to each other, towards point of intersection of the two roads. Their velocities at a particular time are v_1, v_2 respectively while their distances from the crossing are s_1 and s_2 respectively. If the accelerations of the two cars be f_1 and f_2 respectively, then they shall avoid collision. If

a. $(s_1f_2-s_2f_1)^2=2(v_2f_1-v_1f_2)(v_2s_1-v_1s_2)$

b. $(s_1v_2-s_2v_1)^2= 2(f_1s_2-f_2s_1)(v_2f_1-v_1f_2)$

c. $(v_2f_1-f_1f_2)^2= 2(s_1v_2-s_2v_1)(f_1s_2-f_2s_1)$

d. $(s_1v_1-s_2v_2)(f_1s_2-f_2s_1)\ (v_2f_1-v_1f_2)=$ [illegible]

42. If the Moon's radius 1/4 of Earth's radius, Moon's mass is 1/81 of the mass of Earth, and if V_M, V_E be respectively the escape velocities on the surface of Moon and on the surface of Earth, then

a. $V_E/V_M=2.25$

b. $V_E/V_M=4.5$

c. $V_E/V_M=$ [illegible]

d. $V_E/V_M=2/$[illegible]

43. What is the decimal equivalent of the hexadecimal number $(1\ 0\ \underbrace{0 \ldots 0}_{23\text{ zeros}}\ 0\ 1)_{16}$?

a. $2^{[illegible]}+1$

b. $2^{24}+1$

c. $2^{92}+1$

d. $2^{96}+1$

44. If the decimal number 2^{111} is written in the octal system, then what is its unit place digit?

a. 0

b. 1

c. 2

d. 3

45. Match List-I (Binary) with List –II (Octal) and select the correct answer using the codes given below:

List-I

A. 101110

B. 1101110

C. 1011101

D. 1111110

List-II

1. 135
2. 56
3. 176
4. 156

	A	B	C	D
a.	1	3	2	4
b.	2	4	1	3
c.	1	4	2	3
d.	2	3	1	4

46. Consider the following statements regarding algorithm of a problem:
1. It begins with instructions to accept inputs
2. The processing rules specified in the algorithms must be precise and unambiguous
3. Total time to carry out all the steps must be indefinite
4. It must produce one or more outputs

Which of the statements given above are correct?
a. 1 and 4
b. 1 and 2
c. 2 and 3
d. 1, 2 and 4

47. Which one of the following is not a merit of a flow chart?
a. It aids in communicating the facts of a problem due to pictorial representation
b. It begins at the inter relationship of different steps involved
c. Larger number of decision paths make the system analysis simple
d. With the help of flow charts all steps can be checked

48. A hemispherical bowl of radius r is filled with water upto a depth equal to half of the radius. The volume of water in the bowl is
a. $2/3\ \pi r^3$
b. $5/24\ \pi r^2$
c. $5/12\ \pi r^3$
d. $1/3\ \pi r^2$

49. Assertion (A): A relation R on the set of complex numbers defined by $z_1 R z_2 \Leftrightarrow z_1 - z_2$ is real, is an equivalence relation.
Reason (R): Reflexive and symmetric properties may not imply transitivity.
a. Both A and B are true and R is the correct explanation of A
b. Both A and R are true but R is not a correct explanation of A.
c. A is true but R is false
d. A is false but R is true

50. Assertion (A): 32-bit product must be stored in two memory words
Reason (R): A number of stored in two memory words is said to have single precision
a. Both A and B are true and R is the correct explanation of A
b. Both A and R are true but R is not a correct explanation of A.
c. A is true but R is false
d. A is false but R is true

51. Assertion (A): The order of a finite group is divisible by the order of its group.
Reason (R): Every finite group contains an element of every order that divides the order of the group.
a. Both A and B are true and R is the correct explanation of A
b. Both A and R are true but R is not a correct explanation of A.
c. A is true but R is false
d. A is false but R is true

52. Assertion (A): e^x cannot be expressed as sum of even and odd functions
Reason (R): e^x is neither even nor odd function
a. Both A and B are true and R is the correct explanation of A
b. Both A and R are true but R is not a correct explanation of A.
c. A is true but R is false
d. A is false but R is true

53. Assertion (A): A finite integral domain is a field.
Reason (R): In a finite integral domain D, there exists an element e such that ae=a, $\forall a \in D$ and for each element. $a \neq 0 \in D, \exists$ an element be $\in D$ such that ab=e
a. Both A and B are true and R is the correct explanation of A
b. Both A and R are true but R is not a correct explanation of A.
c. A is true but R is false
d. A is false but R is true

54. Let $A = \{(x, y): y = 1/x, 0 \neq x \in R\}$
$B = \{(x, y): y = -x, x \in R\}$, R being the set of reals, then which one of the following is true?
a. $A \cap B = A$
b. $A \cap B = B$
c. $A \cap B = A \cup B$
d. $A \cap B = \phi$

55. In a city, three daily newspapers A, B, C are published. 42% of the people in that city read A, 54% read B and 68% read C, 30% read A and B, 28% read B and C; 36% read A and C, 8% do not read any of the three newspapers. The percentage of persons who read all the three papers is

a. 20%
b. 25%
c. 18%
d. 30%

56. If $\cos\frac{\pi}{3^r} + i\sin\frac{\pi}{3^r} = x_r$, r= 1, 2, 3…… then $x_1\, x_2\, x_3$…… (upto infinity) equals
a. 1+ i
b. 1
c. i
d. $\frac{1}{2}+\frac{i\sqrt{3}}{2}$

57. The equation whose roots are the n^{th} powers of the roots of the equation $x^2-2x\cos\theta + 1=0$ is
a. $x^2 - 2x\cos n\theta + 1=0$
b. $x^2 + 2x\cos n\theta+1=0$
c. $x^2 - 2x\cos n\theta + 1=0$
d. $x^2 + 2x\cos n\theta - 1= 0$

58. The equation $|z-1|^2 + |z+1|^2 = 4$ represents on the Aragand plane
a. A straight line
b. A circle with centre at origin and centre 2
c. An ellipse
d. A circle with centre at origin and radius unity

59. If $(a_1 + ib_1)(a_2 + ib_2) \ldots (a_n + ib_n) = A + iB$; then $\tan^{-1}\left(\frac{b_1}{a_1}\right) + \tan^{-1}\left(\frac{b_2}{a_2}\right) + \ldots \tan^{-1}\left(\frac{b_n}{a_n}\right)$ will be equal to
a. $\tan^{-1}\left(\frac{A}{B}\right)$
b. $\tan^{-1}\left(\frac{B}{A}\right)$
c. $\tan^{-1}\left(\frac{A-B}{1+AB}\right)$
d. $\tan^{-1}\left(\frac{1+AB}{A-B}\right)$

60. Two candidates attempt to solve the equation $x^2+px+q=0$. In solving, one commits a mistake in writing the value of q and find the roots to be 8 and 2. The other commits a mistake in writing the value of p and finds the roots to be -9 and -1. The correct roots are
a. 9 and 1
b. -8 and -2
c. 8 and -9
d. 2 and –1

61. If α and β are the roots of the equation $x^2+x+1=0$, then $\alpha^{2001} + \beta^{2001}$ is equal to
a. -2
b. -1
c. 0
d. 2

62. If α, β and γ are the roots of the equation $x^3 + mx + n=0$, then $\sum\frac{\alpha}{\beta+\gamma}$ is equal to
a. m + n
b. m/n
c. 3
d. -3

63. The roots of the equation $x^4 - 6x^3 + 18x^2 - 30x + 25 = 0$ are
a. $-1 \pm 2i, 2 \pm i$
b. $1 \pm 2i, -2 \pm i$
c. $1 \pm 2i, 2 \pm i$
d. $-1 \pm 2i, -2 \pm i$

64. In the group of non-zero rational numbers under the binary operation * given by a * b=ab/5, the identity element and the inverse of 8 are respectively.
a. 5 and 8/25
b. 5 and 25/8
c. 1/5 and 1/40
d. 1 and 1/8

65. Let x_1, x_2 and x_3 be three distinct points and φ be the permutation $x_1 \to x_2$, $x_2 \to x_3$ and $x_3 \to x_1$ in S_3, then order of φ is
a. 1
b. 2
c. 3
d. 4

66. If < σ > and < τ > are the cyclic subgroups of S_4, the symmetric group on four letter generated by
$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$
respectively, then $<\sigma> \cap <\tau>$ is a subgroup of order
a. 0
b. 1
c. 2
d. 4

67. A ring (R, +, *) whose all elements are idempotent is

a. always abelian
b. an integral domain
c. an interval ring
d. a field

68. The ring of integers (mod 6)is
a. a finite integral domain
b. an infinite integral domain
c. a field
d. not an integral domain

69. If n donates the number of elements in a field, then n must be
a. a prime
b. a prime of the form 4k + 1
c. a product of distinct primes
d. a power of a prime

70. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then
a. $p + q = 0$
b. $pq = -1$
c. $p^2 + q^2 = 1$
d. $\frac{1}{p} + \frac{1}{p} = 1$

71. The centre of the conic $x^2 + 24xy - 6y^2 + 28x + 36y + 16 = 0$ is
a. (-1, -2)
b. (-2 , -1)
c. (0, 0)
d. (1, 1)

72. Straight lines are drawn joining the origin to the points where the straight line $x + y = 1$ meets the curve $x^2 + 3y^2 - kx + 1 = 0$. These straight lines will be at right angles provided
a. k = 7
b. k = 6
c. k = [illegible]
d. k = 4

73. The line $\frac{l}{r} = A \cos\theta + B \sin\theta$ will touch the conic $\frac{l}{r} = 1 + e\cos\theta$ if
a. $(A - e)^2 + B^2 = 1$
b. $(A + e)^2 + B^2 = 1$
c. $A + e + B = 1$
d. $A - e - B = 0$

74. The lines $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and $\frac{x-4}{2} = \frac{y-6}{3} = \frac{z-7}{3}$ are coplanar. Their point of intersection is
a. (4, 6, 7)
b. (2, 3, 4)
c. (1, 1, 1)
d. (4, 7, 10)

75. The plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if
a. $a + b + c = 0$
b. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$
c. $a + b + c = 1$
d. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$

76. Equation of a right circular cylinder is $x^2 + y^2 + z^2 - xy + yz - zx - 9 = 0$. Its axis is $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$. The radius of the cylinder is
a. $\sqrt{3}$
b. $\sqrt{5}$
c. $\sqrt{6}$
d. [illegible]

77. The value of p for which the four points with position vectors $4\vec{i} + p\vec{j} + 12\vec{k}$, $2\vec{i} + 4\vec{j} + 6\vec{k}$, $5\vec{i} + 8\vec{j} + 5\vec{k}$ are coplanar is
a. 6
b. 7
c. 8
d. 9

78. Let $\vec{u}$, $\vec{v}$, $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{0}$. If $|\vec{u}| = 3$, $|\vec{v}| = 4$ $|\vec{w}| = 5$, then the value of $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$ is
a. 47
b. -25
c. 0
d. 3

79. Unit vector equally inclined to $\vec{i} - \vec{j} + \vec{k}$ and $\vec{i} + \vec{j} - \vec{k}$ and ling in the plane containing them is
a. $\frac{\vec{i} + \vec{j} + \vec{k}}{\sqrt{3}}$
b. $\vec{i}$
c. $\frac{\vec{j} + \vec{k}}{\sqrt{2}}$
d. $\frac{\vec{j} - \vec{k}}{\sqrt{2}}$

80. Angle between the line $\vec{r} = (2\vec{i} - \vec{j} + \vec{k}) + \lambda(-\vec{i} + \vec{j} + \vec{k})$ and the plane $\vec{r}.(3\vec{i} + 2\vec{j} - \vec{k}) = 4$ is

a. $\cos^{-1}\left(\frac{2}{\sqrt{84}}\right)$

b. $\cos^{-1}\left(-\frac{2}{\sqrt{84}}\right)$

c. $\sin^{-1}\left(-\frac{2}{\sqrt{42}}\right)$

d. $\sin^{-1}\left(\frac{2}{\sqrt{14}}\right)$

81. The equation of the plane passing through three points A,B,C with position vectors $\vec{i}+\vec{j}$, $\vec{j}+\vec{k}$, $\vec{k}+\vec{i}$ respectively is

a. $\vec{r}.(\vec{i}+\vec{j}+\vec{k})=0$

b. $\vec{r}.(\vec{i}+\vec{j}+\vec{k})=1$

c. $\vec{r}.(\vec{i}+\vec{j}+\vec{k})=2$

d. $\vec{r}.(\vec{i}+\vec{j}+\vec{k})=3$

82. The value of $1+(1+2)+(1+2+3)+\ldots\ldots(1+2+3+\ldots\ldots\ldots+n)$ is

a. $\frac{n(n+1)(2n+1)}{6}$

b. $\frac{n(n+1)(n+2)}{6}$

c. $\frac{n(n+1)(2n-1)}{6}$

d. $\left[\frac{n(n+1)}{6}\right]^2$[^2]

83. If f and g are twice differentiable function and $f(p)=3, f'(p)=2, g(p)=$ [illegible], $g'(p)=4$, then $\lim_{x\to p}\frac{g(x)f(p)-\text{[illegible]}}{x-p}$ is equal to

a. -5

b. 10

c. -10

d. 5

84. An integrating factor of the differential equation sinh y dx + cosh y dy = 0 is

a. e

b. x

c. y

d. xy

85. The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3}+\left(\frac{d^3y}{dx^3}\right)^{3/2}=0$ is

a. 3

b. 5

c. 4

d. 9

86. The curves in which the tangent of the angle between the tangent and the radius vector at any point is equal to the tangent of the vectorial angle are

a. System of straight lines

b. System of circles

c. Systems of parabolas

d. System of ellipses

87. If c be an arbitrary constant, the general solution of the differential equation $x(x^2+3y^2)dx+y(y^2+3x^2)dy=0$; is

a. $(x^2-y^2)^2-4x^2y^2=c$

b. $(x^2+y^2)^2+4x^2y^2=c^2$

c. $(x^2+y^2)^2-x^2y^2=c$

d. $(x^2-y^2)^2+x^2y^2=c$

88. If c be an arbitrary constant, the general solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=y^2$ for all x is

a. xy (c + log |x|) = 1

b. xy (c - log|x|) = 1

c. xy (c- 2 log x) = 1

d. xy(c + log x^2) = 1

89. The general solution of the differential equation $\frac{d^2y}{dx^2}+9y=\sin^3 x$, is

a. $y=A\cos(3x+B)+\frac{1}{24}\sin x-\sin 3x$

b. $y=Ae^{3x}+Be^{-3x}+\frac{1}{32}\sin x+\frac{1}{2}\cos 3x$

c. $y=A+Bxe^{3x}+2\sin x-\frac{5}{13}\sin 3x$

d. $y=A\sin(3x+B)+\frac{3}{32}\sin x+\frac{x}{24}\cos 3x$

90. The semi-vertical angle of a right circular cone having sets of three mutually perpendicular generators is given by

a. $\tan^{-1}(1/\sqrt{2})$

b. $\tan^{-1}(\sqrt{2})$

c. $\pi/4$

d. $\pi/2$

91. By means of a suitable transform of the independent variable, the differential equation $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=6x+\frac{1}{x}$ reduces to the form

a. $\frac{d^2y}{dt^2}+2\frac{dy}{dt}=6e^{2t}+1$

b. $\frac{d^2y}{dt^2}+\frac{dy}{dt}=6e^{3t}+1$

c. $\frac{d^2y}{dt^2}=6e^{3t}+\log t$

d. $\frac{d^2y}{dt^2}=6e^{t}+t$

92. The singular solution of the differential equation $xyp^2-(x^2+y^2+1)p+xy=0$, where $p=\frac{dy}{dx}$

a. Is $y=0$
b. Is $y^2=(x-1)^2$
c. Does not exist
d. Is none of the above

93. A uniform straight beam AB of eight W and length 2 l stands with the end A fixed to the ground. AB is inclined at an angle 30^0 to the vertical. The end A is subjected to a

a. vertical upward force W
b. vertical upward force W and a horizontal force W/2
c. vertical upward force W and a couple of moment Wl/2
d. couple of moment Wl

94. A uniform heavy plank AB rest horizontally on two supports at C,D where AB= 2l, AC=BO=l/2. The weight of the plank is W and a man of weight w starts walking from end to the other. The plank does not overturn if

a. W < w < 2W
b. w< W
c. w > W
d. w > W/2

95. Two forces P and Q have result R. If P be increased, the new resultant bisects the angle between $\vec{Q}$ and $\vec{P}$. Then increase in P is given by

a. 3R
b. R
c. 2R
d. R

96. A system of forces of magnitudes 2P, Q, P, Q acts along the sides AB, BC, CD , DA respectively of the square ABCDA. If the side of the square is a; then the system of forces is equivalent to

a. P and AB
b. P along a line parallel to AB, at a distance –a $\frac{(P+Q)}{P}$ away form it
c. Q along BC
d. Q along a line parallel to BC, at a distance a $\frac{(P-Q)}{P}$ away from it

97. A rod of length l and weight W is suspended by two equal threads attached to the two ends of the rod, the other ends of the thread being attached to two points A, B on the same horizontal line. If AB = a, the tension of the threads is least when

a. a = l
b. $a=\frac{3l}{2}$
c. a= 2l
d. a = 3l

98. Forces P, Q act at O at an angle α; forces R, S act at O at an angle β. P, Q, R, S are coplanar and the resultants of P, Q and of R, S are at right angles. The resultant force T of forces P, Q, R and S is given by

a. $T^2=P^2+Q^2+R^2+S^2-2PQ\cos\alpha-2RS\cos\beta$
b. $T^2=P^2+Q^2+R^2+S^2+2PQ\cos\alpha-2RS\cos\beta$
c. $T^2=P^2+Q^2+R^2+S^2-2PQ\cos\alpha+2RS\cos\beta$
d. $T^2=P^2+Q^2+R^2+S^2+2PQ\cos\alpha+2RS\cos\beta$

99. A balance with unequal arms balances weights W_1,W_2 in the two pans. If W_2 is now transferred to the other pan , then weight W placed on the empty pan balances weights W_1 and W_2 in the other pan. W is given by

a. $\frac{(W_1+W_2)W_2}{W_1}$
b. $\frac{(W_1+W_2)W_1}{W_2}$
c. $\frac{W_1+W_2}{W_1+W_2}$
d. $\frac{(W_1+W_2)^2}{W_2}$

100. Distance x covered by a particle in time t is given by $x=2\cos\alpha^2t-\sin(2\alpha-1)t$, ($\alpha\neq1$). If motion is required to be simple harmonic, then α should be

a. $1\pm\sqrt{3}$
b. $1\pm\sqrt{2}$
c. $1\pm\sqrt{5}$
d. $-1\pm\sqrt{5}$