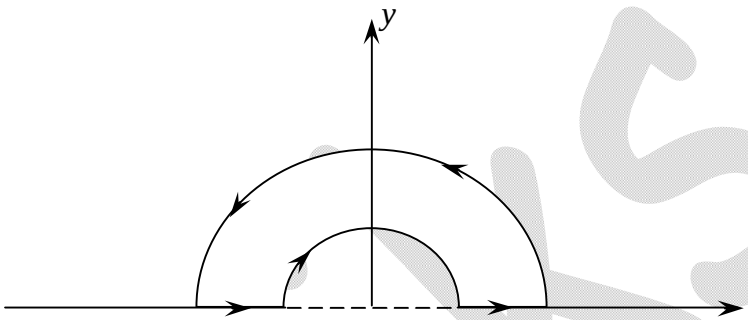
ENTRANCE EXAMINATION, 2013

Pre-Ph.D./Ph.D. PHYSICAL SCIENCES

PART-A

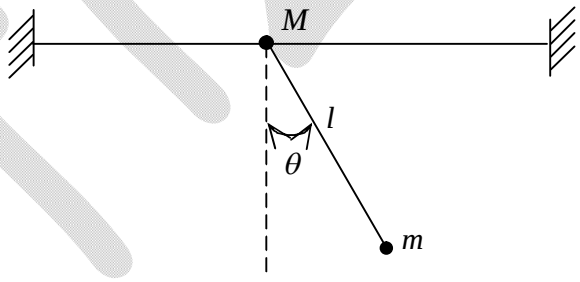
NOTE: Answer **all** questions. Each question carries 6 marks.

A1. Using the contour of integration in the complex plane (shown below)



Calculate the integral $\int_0^\infty \frac{\sin(2x)}{x}\,dx$.

A2. Consider a pendulum consisting of a point mass m and massless string of length l. The string is supported from a bead of mass M which slides freely (without friction) along a horizontal wire.



(a) Write the Lagrangian for the system.

(b) Find the frequency of the pendulum for the small-amplitude oscillations.

A3. The free energy of formation of a cluster consisting of *i* molecules has contributions from both volume and surface terms as follows:

$$\Delta G = \Delta_0 V + \gamma S$$

where Δ_0 is the free energy (per unit volume) cost for bringing the particles together and γ is the surface tension. V and S denote the volume and surface area of the cluster respectively. Assume that the cluster is spherical in shape and each molecule is associated with a volume v_m, so that $V = iv_m$,

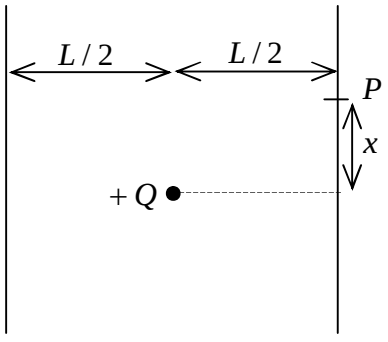
(*a*) Obtain how ΔG depends on *i*.

(*b*) Assuming $\Delta_0 < 0$, obtain the value of *i* for which ΔG becomes maximum.

(*c*) Compute the radius of the optimum cluster in (b).

A4. A free particle of mass *m* is moving inside a sphere of radius *R*. Assuming that the wall of the sphere is impenetrable, calculate the ground-state energy of the particle.

A5. A point charge $+Q$ is placed midway between the lines joining the centres of two large parallel conducting plates separated by a distance *L*. The plates are connected by a thin conducting wire. Using the method of images, obtain the surface charge density at a point *P* on the right plate at a distance *x* from its centre.



PART-B

NOTE: Answer **all** questions. Each question carries 4 marks.

B1. Consider the following two systems:

(*a*) A harmonic spring of spring constant *K* with a mass *m* attached to its one end while the other end is held fixed

(*b*) A harmonic spring of spring constant *K* with two identical masses (each *m*) attached to its respective ends

Write the Hamiltonian of the two systems. Which of these two Hamiltonians is invariant under translation in space?

B2. Write the ground-state wave function of the helium atom $\left(1s^2\right)$ in terms of hydrogen-like atomic orbitals (ignoring the effects of electron-electron repulsion). Denote the 1*s* orbital as $\Psi_{1s}(r)$ and the two spin states of electron as $\alpha\uparrow$ and $\alpha\downarrow$. What is the total spin of the *He* ground state?

B3. Write the first law of thermodynamics for a system whose thermodynamic variables are temperature *T*, magnetization *M* and number of particles *N*. Identify the Gibbs free energy *G* which becomes minimum at constant *T, H, N* (respectively denoting the temperature, magnetic field and number of particles).

B4. Solve the following non-linear equation by introducing a transformation which converts it to a linear equation:

$$2x\frac{du}{dx}+(x-1)u=\frac{x^2e^x}{u}$$

B5. A particle of mass m moves in a central force field given by the (repulsive) potential $V(r) = -V_0 r^n$ The particle is set on a radial trajectory at $r = r_0$ (directed to the centre of the force) with just enough energy (that is $E \to 0$) to reach the centre. What is the condition on the value of n that the particle reaches the centre in finite time?

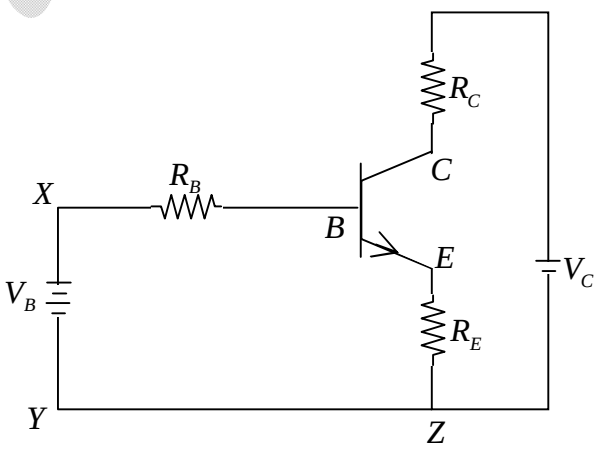
B6. Evaluate the electric and magnetic fields, and the charge and current densities corresponding to the potentials (q is the charge)

$$V(r, t) = 0; \qquad \vec{A}(r, t) = -\frac{1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$$

B7. Write down the expression for Maxwell's velocity distribution law. Obtain the most probable value of the velocity of the particle and its ratio with the root-mean-square speed of the particles.

B8. Consider a point charge Q that is moving in a circular orbit of radius a centred at the origin in the xy-plane. Find the Liénard-Wiechert potentials for points in the z-axis. Assume angular velocity ω of the particle to be a constant.

B9. Given below is a transistor circuit based on common-emitter configuration. The Q-point here drifts with temperature T as $V_{BE} = V_{BE}(T)$, where V_{BE} equals the junction voltage at the base-emitter p-n junction. Suggest a modification so as to make the circuit insensitive to temperature variations. What component (s) is (are) required to be incorporated and in which segment (s)? Why?



B10. A two-dimensional metallic system consists of a monovalent atom in a rectangular primitive cell with $a = 2\,\text{Å}$ and $b = 4\,\text{Å}$. Sketch the reciprocal lattice and draw the first Brillouin zone. Estimate the radius of free-electron. Fermi surface (circle in two dimensions) in cm^{-1}. Sketch the Fermi circle on top of the first Brillouin zone.