Entrance Examination : M.Sc. Mathematics, 2012

Hall Ticket Number | | | | | | | |

Time : 2 hours
Max. Marks. 100

Part A : 25 marks
Part B : 75 marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR answer sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over both the question paper booklet and OMR answer sheet at the end of the examination.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 50 questions in Part A and Part B together.
8. **There is a negative marking in Part A. Each correct answer carries 1 mark and each wrong answer carries - 0.33 mark. Each question in Part A has only one correct option.**
9. **There is no negative marking in Part B. Each correct answer carries 3 marks. In Part B some questions have more than one correct option. All the correct options have to be marked in OMR sheet other wise zero marks will be credited.**
10. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
11. **THE MAXIMUM MARKS FOR THIS EXAMINATION IS 100 AND THERE WILL BE NO INTERVIEW.**

Y-6

Part-A

- Find the correct answer and mark it on the OMR sheet. Each correct answer carries 1 (one) mark. Each wrong answer carries **- 0.33 mark**

1. If α, β and γ are the roots of $x^3 + ax^2 + bx + c = 0$ then the value of $\alpha^2 + \beta^2 + \gamma^2$ is

   [A] $a^2 - 2b$. [B] $b^2 - 2c$. [C] $c^2 + 2a$. [D] $b^2 + 2c$.

2. Let $f : \mathbb{R} \to \mathbb{R}$ and $f(x) = |x-1| + |x-2|$. Let $S_1 = \{x \mid f \text{ is continuous at } x\}$ and $S_2 = \{x \mid f \text{ is differentiable at } x\}$. Then

   [A] $S_1 = \mathbb{R}, \ S_2 = \mathbb{R}$. [B] $S_1 = \mathbb{R}, \ S_2 = \mathbb{R} \setminus \{1,2\}$.

   [C] $S_1 = \mathbb{R} \setminus \{1,2\}, \ S_2 = \mathbb{R}$. [D] $S_1 = \mathbb{R} \setminus \{1,2\}, \ S_2 = \mathbb{R} \setminus \{1,2\}$.

3. Consider the following statements

   S_1: If f is Riemann integrable in $[0,1]$ then f^2 is Riemann integrable in $[0,1]$.

   S_2: If f^2 is Riemann integrable in $[0,1]$ then f is Riemann integrable in $[0,1]$.

   Then

   [A] S_1 is true but S_2 is false. [B] S_1 is false but S_2 is true.

   [C] both S_1 and S_2 are false. [D] both S_1 and S_2 are true.

4. The function $f(x) = \sin(x) + \cos(x)$ is

   [A] increasing in $[0, \pi/2]$.

   [B] decreasing in $[0, \pi/2]$.

   [C] increasing in $[0, \pi/4]$ and decreasing in $[\pi/4, \pi/2]$.

   [D] decreasing in $[0, \pi/4]$ and increasing in $[\pi/4, \pi/2]$.

5. Let G_1 and G_2 be two finite groups with $|G_1| = 100$ and $|G_2| = 25$. If $f : G_1 \longrightarrow G_2$ is a surjective group homomorphism, then

   [A] $|Ker(f)| = 2$. [B] $|Ker(f)| = 4$.

   [C] $|Ker(f)| = 5$. [D] $|Ker(f)| = 10$.

4-6

6. Let $\{p_n\}$ be a strictly increasing sequence of prime numbers and let $x_n = (-1)^{p_n+1}\left(1+\frac{1}{p_n}\right)$ then

[A] ; $\lim_{n\to\infty} x_n = -1/2$.
[B] $\lim_{n\to\infty} x_n = -1$.
[C] $\lim_{n\to\infty} x_n = 1$.
[D] $\lim_{n\to\infty} x_n$ does not exist.

7. Let V be a vector space of dimension n and $\{v_1, v_2, \cdots, v_n\}$ be a basis of V. Let $\sigma \in S_n$ and $T : V \longrightarrow V$ be a linear transformation defined by $T(v_i) = v_{\sigma(i)}$. Then

[A] T is nilpotent.
[B] T is one-one but not onto.
[C] T is onto but not one-one.
[D] T is an isomorphism.

8. Let G be a group and $a \in G$ be a unique element of order n where $n > 1$. Let $Z(G)$ denote the center of the group G. Then

[A] $O(G) = n$.
[B] $O(Z(G)) > 1$.
[C] $Z(G) = G$.
[D] $G = S_2$.

9. If the series $\sum_{n=0}^{\infty} (\sin x)^n$ converges to the value $(4 + 2\sqrt{3})$ for some value of x in $(0, \pi/2)$, then the value of x is

[A] $\pi/3$.
[B] $\pi/4$.
[C] $\pi/5$.
[D] $\pi/6$.

10. If m and M are respectively the greatest lower bound and the least upper bound of the set $S = \left\{\frac{2x+3}{x+2},\ x \geq 0\right\}$ then

[A] $m \in S, M \notin S$.
[B] $m \notin S$, $M \notin S$.
[C] $m \notin S$, $M \in S$.
[D] $m \in S$, $M \in S$.

11. The value of $\lim_{x\to 0} (\cos x)^{(1/\sin^2 x)}$ is

[A] $\exp(-1)$.
[B] $\exp(1)$.
[C] $\exp(-1/2)$.
[D] $\exp(1/2)$.

12. The graphs of the real valued functions $f(x) = 2\log(x)$ and $g(x) = \log(2x)$

[A] do not intersect.
[B] intersect at one point only.
[C] intersect at two points.
[D] intersect at more than two points.

Y-6

13. The points of continuity of the function $f : \mathbb{R} \to \mathbb{R}$ defined by
$$f(x) = \begin{cases} |x^2 - 1|, & \text{if } x \text{ is irrational} \\ 0, & \text{if } x \text{ is rational} \end{cases}$$
are

[A] $x = -1, x = 0, x = 1$.
[B] $x = -1, x = 1$.
[C] $x = -1, x = 0$.
[D] $x = 0, x = 1$.

14. The smallest positive integer n such that $5^n - 1$ is divisible by 36 is

[A] 2. [B] 3. [C] 5. [D] 6.

15. Let $f(x) = x^5 + a_1x^4 + a_2x^3 + a_3x^2$. Suppose $f(-1) > 0$ and $f(1) < 0$ then

[A] f has at least 3 real roots.
[B] f has at most 3 real roots.
[C] f has at most 1 real root.
[D] all roots of f are real.

16. Let $\{u, v\}$ be a linearly independent subset of a real vector space V. Then which of the following is **not** a linearly independent set?

[A] $\{u, u - v\}$. [B] $\{u + \sqrt{2}v, u - \sqrt{2}v\}$.
[C] $\{v, 2v - u/2\}$. [D] $\{2u + v, -4u - 2v\}$.

17. Let V be a vector space of 2×2 real matrices. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$ then the dimension of the subspace spanned by $\{A, A^2, A^3, A^4\}$ is

[A] 2. [B] 3. [C] 4. [D] 5.

18. Let $A \in M_3(\mathbb{Q})$. Consider the statements
P : Matrix A is nilpotent.
Q: $A^3 = 0$.
Pick up true statements from the following.

[A] $P \Rightarrow Q$.
[B] $Q \Rightarrow P$ and $P \not\Rightarrow Q$.
[C] $P \not\Rightarrow Q$ and $Q \Rightarrow P$.
[D] None of [A], [B], [C] is true.

4-6

19. Consider the statements

$S_1 : 1 - 1 + 1 - 1 + 1 - 1 + \cdots = \pm 1.$

S_2: $\frac{1}{1+2} = 1 - 2 + 2^2 - 2^3 + \cdots$. Then

[A] S_1 is true but S_2 is false.

[B] S_1 is false but S_2 is true.

[C] both S_1 and S_2 are true.

[D] both S_1 and S_2 are false.

20. Let $x_0 < x_1 < \cdots < x_n$ and $y_1, y_2, \ldots, y_n \in \mathbb{R}$. Then

[A] there exists a unique continuous function f such that $F(x_i) = y_i$ for all i.

[B] there exists a unique differentiable function f such that $F(x_i) = y_i$ for all i.

[C] there exists a unique n times differentiable function f such that $F(x_i) = y_i$ for all i.

[D] there exists a unique polynomial function f of degree n such that $F(x_i) = y_i$ for all i.

21. Solution of the differential equation $y'' - x\,(y')^2 = 0$, subject to the boundary conditions $y(0) = 0$, $y'(0) = -1$ is

[A] $y = \sqrt{\frac{-2}{a}} \tan^{-1}\left(\frac{x}{\sqrt{2a}}\right) + b$, where a and b are arbitrary constants.

[B] $y = -\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$.

[C] $y = \sqrt{\frac{2}{a}} \tan^{-1}\left(\frac{x}{\sqrt{2a}}\right) + b$, where a and b are arbitrary constants.

[D] $y = \frac{-1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2}x\right)$.

22. Let V be the vector space of all continuous functions on $\mathbb{R}$ over the field $\mathbb{R}$. Let $S = \{|x|, |x-1|, |x-2|\}$.

[A] S is linearly independent and does not span V.

[B] S is linearly independent and spans V.

[C] S is linearly dependent and does not span V.

[D] S is linearly dependent and spans V.

4-6

23. 10 red balls (all alike) and 10 blue balls (all alike) are to be arranged in a row. If every arrangement is equally likely, then the probability that the balls at two ends of the arrangement are of the same colour is

[A] equal to $\frac{1}{4}$. [B] equal to $\frac{1}{2}$. [C] less than $\frac{1}{2}$ [D] greater than $\frac{1}{2}$.

24. 3 students are to be selected to form a committee from a class of 100 students. The chances that the tallest student is one among them is

[A] less than 5%. [B] 6 to 10%. [C] 15%. [D] 50%.

25. Let $\vec{f}$ be a smooth vector valued function of a real variable. Consider the two statements
S_1 : div curl$\vec{f} = 0$.
S_1 : grad div $\vec{f} = 0$. Then

[A] both S_1 and S_2 are true. [B] both S_1 and S_2 are false.

[C] S_1 is true but S_2 is false. [D] S_1 is false but S_2 is true.

Part-B

- The following questions may have **more than** one correct answer.
- Find the correct answers and mark them on the OMR sheet. Correct answers (marked in OMR sheet) to a question get 3 marks and zero otherwise.
- For the answer to be right **all the correct options** have to be marked on the OMR sheet. **No credit** will be given for partially correct answers.

26. A sphere passing through the points $(1,0,0)$, $(0,1,0)$, $(0,0,2)$ that has the least radius is

[A] $18(x^2 + y^2 + z^2) - 16(x + y) - 35z = 2$.

[B] $9(x^2 + y^2 + z^2) - 5(x + y) - 16z = 4$.

[C] $9(x^2 + y^2 + z^2) - 7(x + y) - 17z = 2$.

[D] None of the above.

27. Let f be a function from $\mathbb{R} \to \mathbb{R}$. Consider the statement
P : There exists M in $\mathbb{R}$ such that $|f(x)| \leq M$ for all x in $\mathbb{R}$. Which of the following statements are equivalent to P.

[A] The range of f is a bounded set of $\mathbb{R}$

[B] $|f|$ is a bounded function.

[C] f is taking all values between $-M$ and M.

[D] $|f|$ is taking all values between 0 and $M/2$.

28. Let $\{x_n\}$ be a sequence of positive real numbers. Then which of the following is false?

[A] If $\sum_{n=1}^{\infty} x_n$ is convergent then $\sum_{n=1}^{\infty} \sqrt{x_n}$ is convergent.

[B] If $\sum_{n=1}^{\infty} x_n$ is convergent then $\sum_{n=1}^{\infty} {x_n}^2$ is convergent.

[C] If $\sum_{n=1}^{\infty} x_n^2$ is convergent then $\lim_{n\to\infty} x_n = 0$.

[D] If $\sum_{n=1}^{\infty} \sqrt{x_n}$ is convergent then $\lim_{n\to\infty} x_n = 0$.

29. Given S_1 and S_2, where

S_1: A series $\sum_{n=0}^{\infty} a_n$ converges if for a given $\epsilon > 0$ there exists $N_0 \in \mathbb{N}$ such that $|a_{n+1} - a_n| < \epsilon$ for all $n \geq N_0$.

S_2: A series $\sum_{n=0}^{\infty} a_n$ converges if $|a_{n+1} - a_n| < \alpha^n$ where α is a fixed real number in $(0, 1)$,

which of the following statements are true?

[A] S_1 is true but S_2 is false.

[B] S_1 is false but S_2 is true.

[C] Both S_1 and S_2 are true.

[D] Both S_1 and S_2 are false.

4-6

30. Let $x, y \in \mathbb{R}$. If $|x+y| = |x| + |y|$ then

[A] $|x-y| = |x| - |y|$.

[B] $|xy| = xy$.

[C] $|x^2+y| = |x^2| + |y|$.

[D] $|x+y| = x+y$.

31. Let $f : \mathbb{R} \to \mathbb{R}$ be a quadratic polynomial. Then which of the following is **impossible**?

[A] $f(x) < f'(0)$, for all $x \in \mathbb{R}$.

[B] $f'(x) > f(x)$, for all $x \geq 0$.

[C] $f'(0) = 0$ and $f(1) = f(4)$.

[D] $f'(0) = 0$ and $f(x) \neq 0$ for all $x \in \mathbb{R}$.

32. If α, β and γ are the roots of the polynomial $x^3 + x^2 + x + 1$, then the value of $\dfrac{1}{\alpha - 1} + \dfrac{1}{\beta - 1} + \dfrac{1}{\gamma - 1}$ is

[A] 1/2.

[B] −1/2.

[C] 3/2.

[D] −3/2.

33. Let V be the vector space of polynomials of degree less than or equal to 2. Let $S = \{x^2 + x + 1,\ x^2 + 2x + 2,\ x^2 + 3\}$. Then

[A] S is a linearly independent set.

[B] S does not span V.

[C] neither [A] nor [B] is false.

[D] None of [A], [B], [C] is false.

34. Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Which of these four statements mean that f is a constant function?

[A] For all $x, y \in \mathbb{R}$, $f(x) = f(y)$.

[B] There exists $x \in \mathbb{R}$ such that for all $y \in \mathbb{R}$, $f(x) = f(y)$.

[C] There exists $x \in \mathbb{R}$ and there exists $y \in \mathbb{R}$ such that $f(x) = f(y)$.

[D] For each $x \in \mathbb{R}$ there exists $y \in \mathbb{R}$ such that $f(x) = f(y)$.

35. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be a 2×2 real matrix. Then

[A] 1 is the only eigenvalue of A.

[B] A has two linearly independent eigenvectors.

[C] A satisfies a polynomial equation with real coefficients of degree 2.

[D] A is not invertible under multiplication.

4-6

36. Let M and N be two smooth functions from $\mathbb{R}^2$ to $\mathbb{R}$. The form $(M\ \mathrm{d}x + N\ \mathrm{d}y)$ is exact if and only if

[A] there exists a smooth function f such that $M\ \mathrm{d}x + N\ \mathrm{d}y = \mathrm{d}f$.

[B] $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ for all x and y.

[C] $\mathrm{Curl}(M\hat{\imath} + N\hat{\jmath}) = \hat{0}$.

[D] all the above statements are true.

37. The general solution of the differential equation $(D^2 - I)^2 y = 0$ is

[A] $(c_1 - c_2x)\exp(x) + (c_3 - c_4x)\exp(-x)$.

[B] $(c_1 + c_2x)\exp(ix) + (c_3 + c_4x)\exp(-ix)$.

[C] $(c_1 - c_2x)\sin(x) + (c_3 - c_4x)\cos(-x)$.

[D] $c_1\sinh(x) + c_2x\sinh(-x) + c_3\cosh(x) + c_4x\cosh(-x)$.

38. Let P be a polynomial of degree 5 having 5 distinct real roots. Then

[A] the roots of P and P' occur alternately.

[B] the roots of P' and P'' occur alternately.

[C] all the roots of P, P', P'', P''',P'''' are real.

[D] it is possible to have a repeated root for P''.

39. If each term of a 3×3 matrix A is constructed by selecting a number from the set $\{-1, 0, 1\}$ with the same probability $1/3$, then

[A] the probability that the trace of A is greater than 0 is more than $1/3$.

[B] the probability that A is a diagonal matrix is less than $1/81$.

[C] the probability that A is a non-singular lower triangle matrix is more than $1/81$.

[D] the probability that A is symmetric is less than $1/81$.

4-6

40. By revolving the curve $y = \sin(x)$ about the x-axis in the interval $[0, \pi]$, the surface area of the surface generated is

[A] $6\pi + 2\pi \log(1+\sqrt{2})$. [B] $2\sqrt{2}\pi + 2\pi \log(1+\sqrt{2})$.

[C] $2\pi \log(1+\sqrt{2})$. [D] $2\pi(1+\log(1+\sqrt{2}))$.

41. Let $A_i = \begin{bmatrix} \cos^2\theta_i & \cos\theta_i \sin\theta_i \\ \cos\theta_i \sin\theta_i & \sin^2\theta_i \end{bmatrix}$, $i = 1, 2$. Then $A_1A_2 = 0$ if

[A] $\theta_1 = \theta_2 + (2k+1)\pi/2,\ k = 0, 1, 2, \cdots$.

[B] $\theta_1 = \theta_2 + k\pi,\ k = 0, 1, 2, \cdots$.

[C] $\theta_1 = \theta_2 + 2k\pi,\ k = 0, 1, 2, \cdots$.

[D] $\theta_1 = \theta_2 + k\pi/2,\ k = 0, 1, 2, \cdots$.

42. Let $f : X \to Y$ and let A and B be subsets of X. Then

[A] $f(A \bigcup B) \subseteq f[A] \bigcup f[B]$. [B] $f[A] \bigcup f[B] \subseteq f(A \bigcup B)$.

[C] $f(A \bigcap B) \subseteq f[A] \bigcap f[B]$. [D] $f[A] \bigcap f[B] \subseteq f(A \bigcap B)$.

43. The value of the integral $\int_0^{10} (x - [x])dx$ is

[A] 2. [B] 3. [C] 4. [D] 5.

44. Let $f, g : (0,1) \to \mathbb{R}$. Let $f(x) = x\sin(1/x^2)$ and $g(x) = x^2$. Then

[A] both f and g are uniformly continuous.

[B] f is uniformly continuous but g is not uniformly continuous.

[C] f is not uniformly continuous but g is uniformly continuous.

[D] both f and g are not uniformly continuous.

4-6

45. Consider a linear transformation from $\mathbb{R}^4$ to $\mathbb{R}^4$ given by a matrix $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$.

Then the number of linearly independent vectors whose direction is invariant under this transformation is

[A] 0. [B] 1. [C] 2. [D] 4.

46. Let V be the vector space of polynomials of degree less than or equal to 2. Let $D : V \to V$ be defined as $Df = f'$. If $B_1 = \{1, x, x^2\}$, $B_2 = \{1, 1 + x^2, 1 + x + x^2\}$ be two ordered bases, then the matrix of linear transformation $[D]_{B_1, B_2}$ is

[A] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 2 \end{bmatrix}$. [B] $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. [C] $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 2 \end{bmatrix}$. [D] $\begin{bmatrix} 0 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$.

47. If α and β are the roots of $(7 + 4\sqrt{3})x^2 + (2 + \sqrt{3})x - 2 = 0$ then the value of $|\alpha - \beta|$ is

[A] $2 - \sqrt{3}$. [B] $2 + \sqrt{3}$. [C] $6 + 3\sqrt{3}$. [D] $6 - 3\sqrt{3}$.

48. Consider the following system of linear equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{15}x_5 &= b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{25}x_5 &= b_2, \\ &\vdots \\ a_{81}x_1 + a_{82}x_2 + \cdots + a_{85}x_5 &= b_8. \end{aligned}$$

A vector $(\lambda_1, \lambda_2, \ldots, \lambda_5) \in \mathbb{R}^5$ is said to be a solution of the system if $x_i = \lambda_i$, $i = 1, 2 \ldots, 5$ satisfies all the equations. Then

[A] If the system of equations has only finitely many solutions then it has exactly one solution.

[B] If all the b_i's are zero then the set of solutions of the system is a subspace of $\mathbb{R}^5$.

[C] A system of 8 equations in 5 unknowns is always consistent.

[D] If the system of equations has a unique solution then the rank of the matrix $[a_{ij}]$ must be 5.

4-6

49. What is the negation of the statement ' 'There is a town in which all horses are white"

[A] In every town some horse is non-white.

[B] There is a town in which no horse is white.

[C] There is a town in which some horse is non-white.

[D] There is no town without a non-white horse.

50. Let S be the surface of the cylinder $x^2 + y^2 = 4$ bounded by the planes $z = 0$ and $z = 1$. Then the surface integral $\int\int_S((x^2 - x)\hat{\imath} - 2xy\hat{\jmath} + z\hat{k}) \cdot \hat{n}\, \mathrm{d}S$

[A] -1. [B] 0. [C] 1. [D] None of [A], [B] [C].

4-6