Compound Interest: 10 Important Shortcuts & Tricks explained with Examples

Majority of business operations and goes by the name of Compound Interest. The basic concept operating behind compound interest is very simple.

For example, Sham borrows a sum of Rs. 100 from Ghansham for a period of two years. The rate of interest for this loan is 10%. At the end of year one, the amount due is the principal and 10% interest on it, that is a total of Rs. 110. Now, effectively the principal value of the loan for the second year is no longer Rs. 100, it is in fact Rs. 110. That is what Ghansham would say and believe. According to him, for the 2nd year, he has lent Rs. 110 as that was the amount he would have had if he taken back the money at the end of year 1. Now for the 2nd year, the interest becomes Rs. 11 (10% of Rs. 110) and the total amount Ghansham would get would be Rs. 121.

If the same calculation was done using the logic of simple interest, you would see that the interest due for two years would be Rs. 20 (10% of Rs. 100 for two years). Thus, replace a S with a C and there is such a big difference in the calculations carried out.

Effectively, for compound interest, the 2nd term of interest is actually the sum total of the principal and the interest for the first term.

Compound Interest Tool tip 1: The Definitions

Principal (P): The original sum of money loaned/deposited. Also known as capital.

Interest (I): The amount of money that you pay to borrow money or the amount of money that you earn on a deposit.

Time (T): The duration for which the money is borrowed. The duration does not necessarily have to be years. The duration can be semi-annual, quarterly or any which way deemed fit.

Rate of Interest (R): The percent of interest that you pay for money borrowed, or earn for money deposited.

Compound Interest Tool tip 2: The Basic Formula

Amount Due at the end of the time period, $A = P (1+r/100)t$

Where:

P: Principal (original amount)

R: Rate of Interest (in %)

T: Time period (yearly, half-yearly etc.)
Compound Interest (CI) = A- P = P (1+r/100)t -P
= P {(1+r/100)t -1}

Compound Interest Tool tip 3: Basic Problems to explain the concept
Example 1: Maninder took a loan of Rs. 10000 from Prashant . If the rate of interest is 5% per annum compounded annually, find the amount received by Prashant by the end of three years
Solution:
The following is the data given:
Principal, P= 10000
Rate = 5%
Time =3 years
Using the formula for Compound Interest:
A = P(1+R/100)t
So A= 10000(1+5/100)3
A = 10000(1+1/20)3
A = 10000 x 21/20 x 21/20 x21/20 =11576.25
So the total amount paid by Maninder at the end of third year is Rs.11576.25
Example 2: Richa gave Rs. 8100 to Bharat at a rate of 9% for 2 years compounded annually. Find the amount of money which she gained as a compound interest from Bharat at the end of second year.
Solution:
Principal value = 8100
Rate = 9%
Time = 2 years
So the total amount paid by Bharat
= 8100(1+9/100)2
=Rs. 9623.61
The question does not probe the amount, rather, it wants to know the CI paid, that the difference between the total amount and original principal.
The Compound Interest = 9623.61 – 8100 = 1523.61

Compound Interest Tooltip 4: Multiple Compounding in a year
Amount Due at the end of the time period
formula for compound interest
Where:
A = future value

P = principal amount (initial investment)
r = annual nominal interest rate
n = number of times the interest is compounded per year
t = number of years money borrowed
Amount for Half Yearly Compounding, A = P {1+(R/2)/ 100}2T
(compound interest applied two times an year).
Like Half Yearly Compound Interest, we can calculate the amount for Quarterly Compounding:
A = P {1+(R/4)/ 100}4T
Example 3: Sona deposited Rs. 4000 in a bank for 2 years at 5% rate. Find the amount received at the end of year by her from the bank when compounded half yearly.
Solution:
Principal value = Rs. 4000
Rate = 5%
Time = 2 years
Since the interest is compounded half yearly so 2 years = 4 times in two years
So we have = A = P {1+(R/2)/ 100}2T
A= 4000{1+ (5/2)/100}4
A = 4000 x 41/40 x 41/40 x 41/40 x 41/40
A = Rs. 4415.2
So, Sona received Rs. 4415.2 from the bank after two years
Example 4: Manpreet lent Rs 5000 to Richa at 10% rate for 1 year. But she told her that she will take her money on compound interest. So find the amount of interest received by Manpreet when compounded quarterly?
Solution:
Principal value = Rs. 5000
Rate = 10%
Time = 1 year
Since the interest is compounded quarterly, that is 4 times in 1 year
Using the formula= A = P {1+(R/4)/ 100}4T
A= 5000{1+ (10/4)/100}4
A = 5000 x 41/40 x 41/40 x 41/40 x 41/40
A = Rs. 5519.064
So, Manpreet received Rs. 4415.2 from bank after two years
And the total amount of interest received by her is 5519.064 – 5000 = Rs. 519.06

Compound Interest Tooltip 5: Difference between Simple Interest and Compound Interest

In case the same principle P is invested in two schemes, at the same rate of interest r and for the same time period t, then in that case:

Simple Interest = (P x R x T)/100

Compound Interest = P [(1+R/100)T – 1] So, the difference between them is

= PRT/100 – P[(1+R/100)T -1] = P [(1+r/100)T -1-RT/100]

Two shortcuts which we can use:

Difference between CI and SI when time given is 2 years = P(R/100)2

Difference between CI and SI when time given is 3 years = P[(R/100)3 + 3(R/100)2]

Example 5: The difference between compound interest and simple interest is 2500 for two years at 2% rate, then find the original sum.

Solution: Given Interest is = 2500

So, Simple Interest = (P X R X T)/100

Compound Interest = P [(1+r/100)t – 1] So the difference between both of them is

= PRT/100 – P [(1+R/100)T -1] = P [(1+r/100)T -1-RT/100] So the sum is 2500

= P [{(1+2/100)2-1}-4/100] On simplification this equation the sum will be = Rs. 6250000

We can check it by our shortcut method

When time given is 2 years = P(R/100)2

Since we are given by the difference so

2500 = P (2/100)2

=> 2500 = P (1/50)2

=> 2500= P (1/2500)

=> 6250000=P

So the sum is Rs.6250000.

Similarly we can conclude the sum when time given is 3 years.

In the first article of our series, we covered the basics of Compound interest, including what does it stand-for. We gave a few tooltips and formulas that are important for the concept and help you grasp the topic better. In the second article of the series, we help you master the topic by providing a series of tooltips providing various formulae and tools that you can use to solve Compound Interest questions.

Compound Interest Tool tip 6: Formula for compound interest when compounding in a year but time is in fraction

The formula to calculate compound interest when the time given is in fractions is as follows:

A =P[(1+R/100)real part{1+(Fraction part x R/100)}]

Where

A: Amount at the end of the time period

P: Principal amount

R: Rate of interest

Real Part and Fraction part: For example, the time given is two and half years, then real part would be two and half years.

Example 1: Manpreet gave Rs. 1000 to Richa for 1 year 6 months. Then find the amount paid by Richa to Manpreet after this duration if rate of interest is 5% per annum compounded annually?

Solution :

We have a time of 1 years and 6 months = (1 year + 1/2 years)

So to find the amount we will use

A =P[(1+R/100)real part{1+(Fraction part x R/100)}] A = 1000[(1+5/100)1{1+{(6/12 x 5)/100] A= P[(21/20) x (41/40)] A = Rs. 1076.25

Compound Interest Tool tip 7: Concept of Equal Installments

How does the concept of equal installments work for Compound interests? Well, in this case, the problem basically tells us that a certain sum of money is borrowed on compound interest for a certain period and it is returned with the help of equal installments. Lets us derive a formula for these installments.

Let us derive a formula where the amount is returned in two equal installments for a time period of two years.

Assume P to be the principal and r the rate of interest.

Step 1: P(1+r/100)=P1 (Amount for one year)

Step 2: New Principal

Now let X be the first installment. After giving the first installment, the principal value will change and the new principal will be = P1 – X

P2 = P1 –X (1+R/100)

Step 3: Amount and Interest for the second year

Now the interest charged will be charged on this amount.

Amount at the end of second year: [P(1+r/100)-X][1+r/100] Step 4: Since the installments are equal, this new amount has to equal X.

Hence,

[P(1+r/100)-X][1+r/100]=X

On solving, we have

P [(1+R/100)2-X (1+R/100)] = X
P [(1+R/100)2] = X+X (1+R/100)
Divide both sides by (1+r/100)2
So we left with
P= X/(1+r/100)2 + X/(1+r/100)
Generalizing the formula for EQUAL INSTALLMENTS
P= X/(1+r/100)n + ...X/(1+r/100)
Where x is the installment and n is number of installment

Example 2: Richa borrowed a sum of Rs. 4800 from Ankita as a loan . She promised Ankita that she will pay it back in two equal installments .If the rate of Interest be 5% per annum compounded annually, find the amount of each installment
Solution:
Given that principal value = 4800
Rate =5%
Two equal installments annually = 2 years
Appling the formula = X/(1+r/100)n.....X/(1+r/100)
So we have here two equal installments so
P= X/(1+r/100)2 + X/(1+r/100)
4800=X/(1+5/100)2 + X/(1+5/100)
On simplifying
We have x= Rs. 2581.46
So the amount of each installment is 2581.46

Compound Interest Tooltip 8: Application of Compound Interest for concepts of population growth.
Case 1: When population growing in a constant rate
If the rate growth of population increased with a constant, rate then the population after T years will be = P (1+R/100)t
In fact, this is nothing else but an application of the fundamentals of compound interest.
It is actually similar to finding the compound amount after time T years
Net population after T years = = P (1+R/100)t
Net population increase = P [(1+R/100)t– 1]
Example 3:
The population of Chandigarh is increasing at a rate of 4% per annum. The population of Chandigarh is 450000, find the population in 3 years hence.
Solution:

P = 450000
Rate of increasing = 4%
Time =3 years
Therefore, the total population will be
=> T = P (1+R/100)3
=> T = 450000(1+4/100)3
=> T = 506188
Case 2: When the population growing with different rates and for different intervals of time
If the rate growth of population increased with different, rate and for different intervals of time then the population after T years will be =
P (1+R1/100)t1 x (1+R2/100)t2..... (1+RN/100)tn
Let us take an example for this concept.

Example 4: The population of Chandigarh increased at a rate of 1% for first year, the rate for second year is 2%, and for third year, it is 3%. Then what will be the population after 3 years if present population of Chandigarh is 45000?
Solution:
Since the rate growth of population increased with different rates for the three difference years, the population after T years will be =
P (1+R1/100)t1 x (1+R2/100)t x (1+R/100)tn
45000(1+1/100)1 x (1+2/100)1 x (1+3/100)1=47749.77
Case 3: When the population is decreasing with rate R
Population after a time period of T years=P (1-R/100)t
Where T is the total population
R is rate at which the population is decreasing
Example 5: The population of Chandigarh is increases at a rate of 1% for first year, it decreases at the rate of 2% for the second year and for third year it again increases at the rate of 3%. Then what will be the population after 3 years if present population of Chandigarh is 45000.
Solution:
Since the rate growth of population is increasing first and then decreasing for the second year and again it increases for third year, then the population after T years will be =
Present Population: P (1+1/100)t1 x (1-2/100)t x (1+3/100)t3
Present Population: 45000(1+1/100)1 x (1-2/100)1 x (1+3/100)1=45877.23

Compound Interest Tooltip 9: Negative Compound Interest

As we can see from the last case above, it is not necessary that there is always an increase in any quantity or amount. There can also be a reduction in the amount of something. This reduction is actually called the rate of depreciation, especially in the financial world. In this case, we do nothing else but take the interest rate to be negative. The formula for this is as follows:

Let P be the original amount.

Let P1 be the new amount at the end of t years.

P1 = P (1-R/100)t

Here R is the rate of interest (negative rate).

Always remember, the rate of depreciation is nothing else but negative rate of interest.

Example 6: Manpreet bought a new car. The value of the car is Rs. 45000. If rate of depreciation is 10% per annum then what will be the value of the car after 2 years

Solution:

P = 45000

Rate of depreciation = 10%

T = 2 years

Therefore, the value will be after 2 years

= P (1 – R/100)t

= 45000(1-10/100)2

= 36450

Aplications

Case-i: City's Population

i. Decline/Decrease

A city has 10,000 residents. Its population declines at the rate of 10% per annum, what'll be its total population after 5 years?

Sol: After 5 years population

= P*(1-R/100)^5

= 10000*(9/10)^5

= 5904

ii. Incline/Increases

A city has 10,000 residents. Its population increases at the rate of 10% per annum, what'll be its total population after 5 years?

Sol: After 5 years population

= P*(1+R/100)^5

= 10000*(11/10)^5

= 16105

Case-ii: Wine Replacement (Adulteration)

i.Remaining volume

A butler steals 10 ml of whiskey from 100 ml bottle and replaces it with water. He repeats this process 5 more times, how much % whisky is left in the bottle?

Sol: After 5 times remaining whisky

= P*(1-R/100)^5

= 100*(9/10)^5

= 59.05 ml

ii. Adulteration→Finding Original volume

A tanker is full of milk, 25% of the liquid is stolen and replaced with water. If this process is repeated 4 times and ultimate mixture contains 810 litres of milk, what is the total capacity of this tanker?

Sol: Original volume

= P*(1-R/100)^5

810 = O.V*(75/100)^4

= 2560 ml

Trick-10:

Pascal's Triangle for computing Compound Interest (CI)

The traditional formula for computing CI is

CI = P*(1+R/100)^N –P

Using Pascal's Triangle, Number of Years (N) ----------

1 1

2 1 2 1

3 1 3 3 1

4 1 4 6 4 1

.....

Eg: P = 7000, R=5 %, and N=4 years. What is CI & Amount?

Sol: Time is 4 years and so we will focus on 4 th line of the Pascal triangle: i.e. 1 4 6 4 1

Amount = 7000 x [ist pascal number on 4 th line i.e. "1"] + (7000 x 5% =350 x [IInd number on 4 th line i.e. "4"] +

(350 x 5 %= 17.5 x [III rd number on 4 th line i.e. "6"] +

(17.5 x 5% = 0.875 x

[IV th number on 4 th line i.e. "4"] + (0.875 x 5% =0.044 x

[V th number on 4 th line i.e. "1"] implying compounded amount is : (7000 x 1 + 350 x 4 + 17.5 x 6 + 0.875 x 4 + 0.044 x 1) = 8508.55 Using traditional formula we get (7000*(1+5%)^4) = 8508.55 Compound interest = 8508.55-7000 = 1508.55

* If compounded half yearly. Then,make R half and T double.
* If compounded quarterly. Then,make R one by fourth and T four times.