



ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE - 2007

MATHEMATICS

(SEMESTER - 4)

Time : 3 Hours]

[Full Marks : 70

Group - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following :

10 × 1 = 10

i) The number of binary relations on a set with n elements is

- a) n^2 b) 2^n
 c) 2^{n^2} d) none of these.

ii) The Boolean function $x'y' + xy + x'y$ is equivalent to

- a) $x' + y'$ b) $x + y$
 c) $x' + y$ d) none of these.

iii) A complete graph is

- a) regular b) connected
 c) simple d) circuit.

iv) The dual of a planar graph is dual.

- a) True b) False.

v) The generating function for the sequence $\left(1, 1, \frac{1}{2!}, \frac{1}{3!}, \dots \right)$ is

- a) e^x b) e^{-x}
 c) $\log(1+x)$ d) $(1-x)^{-1}$.

vi) The generating function for finite sequence $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \dots, \binom{n}{n}$ is

- a) $(1-x)^n$ b) $(1+x)^{-n}$
 c) $1+x$ d) $(1+x)^n$.

vii) The number of edges in a tree with 50 vertices is

- a) 48 b) 49
 c) 50 d) 51.



viii) If the cyclic group G contains 11 distinct elements, then it has

- a) 2 generators
- b) 7 generators
- c) 9 generators
- d) 10 generators.

ix) Solution of the recurrence relation $a_{n+1} - 2a_n = 5, n \geq 0, a_0 = 1$ is

- a) $6 \cdot 2^n - 5$
- b) $5 - 6 \cdot 2^n$
- c) $2^{n+1} - 1$
- d) none of these.

x) Which of the following set is closed under usual multiplication ?

- a) $\{ 1, -1, 0, 2 \}$
- b) $\{ 1, i \}$
- c) $\{ 1, \omega, \omega^2 \}$
- d) $\{ \omega, 1 \}$.

xi) $f: \left[0, \frac{\pi}{2} \right] \rightarrow R$ defined by $f(x) = \sin x$ is a one to one mapping.

- a) True
- b) False.

xii) The hamming distance between 11010 and 10101 is

- a) 2
- b) 3
- c) 4
- d) 0.

xiii) The sum of the degree of all vertices of a graph is 40. Then the number of edges is

- a) 10
- b) 20
- c) 25
- d) 40.

xiv) If a network contains 6 vertices then the number of cuts in the network is

- a) 14
- b) 15
- c) 16
- d) 32.

**Group - B****(Short Answer Type Questions)**Answer any *three* questions from the following. $3 \times 5 = 15$

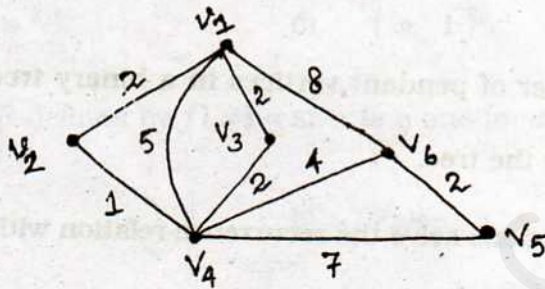
2. Show that all roots of the equation $x^4 = 1$ form an Abelian group under multiplication.
3. Give examples with reasons of relations which are
 - a) reflexive, symmetric but not transitive.
 - b) symmetric, transitive but not reflexive.
4. In a Boolean algebra $(B, +, \cdot, /, 0, 1)$ show that
 - a) $(a + b) + c = a + (b + c)$
 - b) $a + (a \cdot b) = a$
5. Show that the number of pendant vertices in a binary tree is $\frac{n+1}{2}$, where n is the number of vertices in the tree.
6. Using generating functions solve the recurrence relation with initial conditions :
 $a_n = 2a_{n-1}$ for $n \geq 1$, $a_0 = 3$.
7. Prove that every group of prime order is cyclic.

Group - C**(Long Answer Type Questions)**Answer any *three* questions. $3 \times 15 = 45$

8. a) Let $G = \{ (a, b) : a \neq 0, b \in R \}$ and $*$ be a binary composition defined on G by $(a, b) * (c, d) = (ac, bc + d)$.
 Show that $(G, *)$ is a non-Abelian group. 5
- b) Show that for any two subgroups H and K of a group G , $H \cap K$ is also a subgroup of G . 5
- c) Let G be a group. If $a, b \in G$ such that $a^4 = e$, the identity element of G and $ab = ba^2$, prove that $a = e$. 5



9. a) Prove that every cyclic group is an Abelian group. 4
- b) Show that the mapping $f: (Z, \cdot) \rightarrow (R, \cdot)$ defined by $f(x) = x^2, \forall x \in Z$ is a monomorphism but not isomorphism. 5
- c) If in a ring R with unity, $(xy)^2 = x^2y^2 \forall x, y \in R$, then show that R is commutative. 6
10. a) Prove that the number of vertices in a binary tree is always odd. 5
- b) Using Floyd Algorithm find the shortest path between v_1 and v_5 ; and v_2 and v_5 . 10

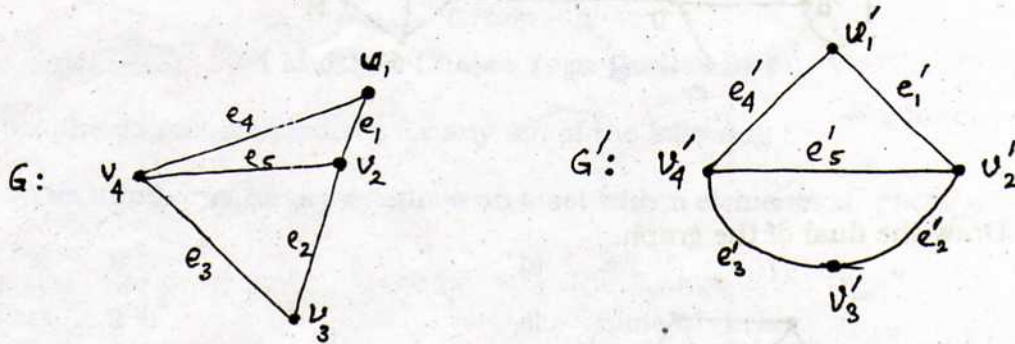


11. a) Prove that the set $\{\phi, \{a\}, \{a, c\}, \{c\}, \{a, b, c\}\}$ is a lattice with respect to the relations \cap and \cup . 5
- b) Construct a logic circuit that would satisfy the following truth table : 6

x	y	z	Output
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

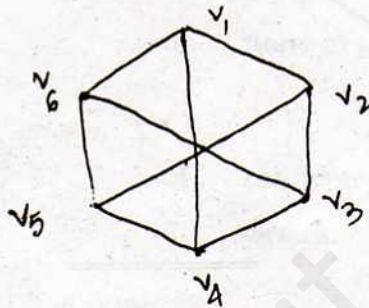
c) Examine whether the following two graphs are isomorphic.

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12. a) Define complement of a graph. Find the complement of the graph.

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b) Express the Boolean expression

$$E = \{ (xy)'z \}' \{ (x' + z)(y' + z') \}' \text{ in DNF.}$$

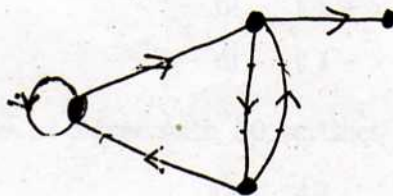
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c) Draw the Hasse diagram for the divisibility on the set $A = \{ 2, 3, 6, 12, 24, 36 \}$ and find the maximal and minimal elements.

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13. a) Construct the Adjacency matrix of the digraph.

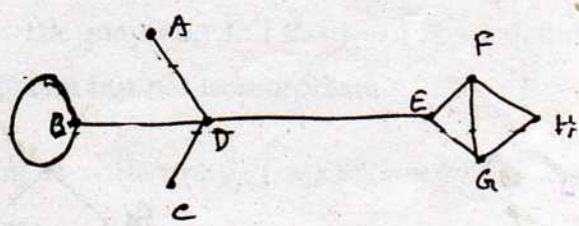
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b) Find all the spanning trees of the connected graph.

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c) Draw the dual of the graph.

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