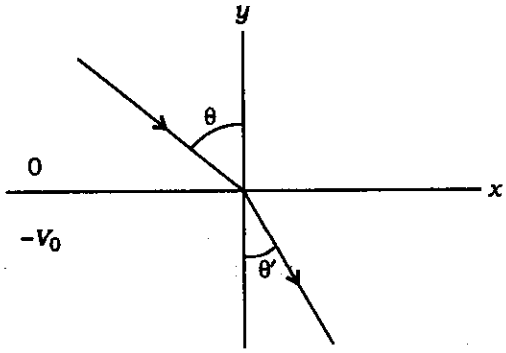
7. Consider the potential field $V(x, y)$ which is 0 and $-V_0$ $(V_0 > 0)$ respectively in the regions of y greater and less than zero. Let θ and θ' be the angles of incidence and refraction of the particle with the y-axis at the point of incidence as it crosses the x-axis. The ratio $\sin(\theta)/\sin(\theta')$ is given (in terms of $\Delta = V_0/E$) by



(a) $\sqrt{1+\frac{2V_0}{E}}$

(b) $\sqrt{1+\frac{V_0}{E}}$

(c) $1+\frac{V_0}{E}$

(d) $1+\frac{2V_0}{E}$

8. Consider the simple Bohr model of the H atom. Its ground state ($n = 1$) has an energy of $-13{\cdot}6$ eV. The wavelength of the electromagnetic radiation emitted when the atom makes a transition from the first excited ($n = 2$) state to the ground state is

(a) $1{\cdot}22 \times 10^{-5}$ cm

(b) $2{\cdot}44 \times 10^{-5}$ cm

(c) $6{\cdot}22 \times 10^{-5}$ cm

(d) $4{\cdot}32 \times 10^{-5}$ cm

9. Radon has a half-life of 3·8 days. If we start with 10·24 gm of radon, the amount of it which will be left after 38 days is

(a) 10^{-4} gm

(b) 10^{-2} gm

(c) 10^{-6} gm

(d) 10^{-3} gm

10. A capacitor with capacitance C is charged to a voltage V. If it is fully discharged by shorting through a resistor R, the total heat generated in the resistor is

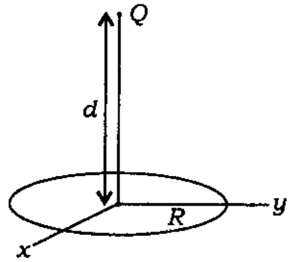
(a) $\frac{1}{2}CV^2$

(b) $\frac{1}{4}CV^2$

(c) CV^2

(d) dependent on R

11. The flux of electric field through a circle of radius R placed in the x-y plane with its centre at the origin due to a point charge Q placed at $(0, 0, d)$ is



(a) $\frac{Q}{2\varepsilon_0}\left[1-\frac{d}{(d^2+R^2)^{1/2}}\right]$

(b) $\frac{Q}{2\varepsilon_0}\frac{d^3}{(d^2+R^2)^{3/2}}$

(c) $\frac{Q}{4\pi\varepsilon_0}\frac{d}{R}$

(d) $\frac{Q}{4\varepsilon_0}\frac{R^2}{d^2}$

12. The electric field of an electromagnetic wave is given by

$$\vec{E} = \hat{i}E_0 \sin(ky - \omega t)$$

where k and ω respectively denote the wave vector and angular frequency of the wave. $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively denote the unit vectors in the x, y and z directions. The magnetic field $\vec{B}$ of the wave is given by

(a) $\hat{k}E_0 k \sin(ky - \omega t)$

(b) $\hat{j}E_0 \cos(ky - \omega t)$

(c) $-\hat{k}\frac{E_0\omega}{k}\cos(ky - \omega t)$

(d) $-\hat{k}\frac{E_0 k}{\omega}\sin(ky - \omega t)$

13. The contour integral

$$\oint_C \frac{z\,dz}{(z-1)^2}$$

where the contour C is a circle of radius 2, is obtained as

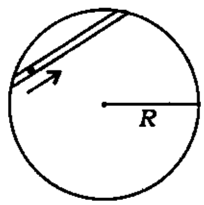
(a) $2\pi i$

(b) 0

(c) πi

(d) $4\pi i$

14. Consider the earth as a uniform (density) sphere of total mass M and radius R. A small object slides along a tunnel connecting two points on the surface of the earth and is acted upon only by the gravitational force due to the earth. Using the value of the acceleration due to gravity as 9·81 metre/sec^2 and the radius of the earth $R = 6 \cdot 37 \times 10^6$ m, the time to slide between the two points is



(a) 21·1 min

(b) 84·4 min

(c) 42·2 min

(d) 63·3 sec

15. What is the value of the definite integral $\int_0^1 (x \ln x)^4 \, dx$?

(a) $\frac{4!}{5^5}$

(b) $\frac{6!}{5^4}$

(c) $\frac{2!}{5^2}$

(d) $\frac{1}{5^3}$

16. Let λ_i $(i = 1, 2, 3)$ be the eigenvalues of the matrix

$$\begin{bmatrix} 2 & -1 & -3 \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{bmatrix}$$

The sum $\sum_{i=1}^{3} \lambda_i^2$ is equal to

(a) 14

(b) 42

(c) 6

(d) 0

17. Consider the magnetic field $\vec{B} = \alpha_0(\hat{i}y + \hat{j}x)$ and two vector potentials $\vec{A}_1 = \alpha_0(\hat{i}xz - \hat{j}yz)$ and $\vec{A}_2 = -\hat{k}\frac{\alpha_0}{2}(x^2 - y^2)$, where α_0 is a constant and $\hat{i}$, $\hat{j}$, $\hat{k}$ represent the unit vectors along the cartesian axes. Identify the correct statement.

(a) Only $\vec{A}_1$ produces the magnetic field $\vec{B}$ but not $\vec{A}_2$

(b) Only $\vec{A}_2$ produces the magnetic field $\vec{B}$ but not $\vec{A}_1$

(c) Neither $\vec{A}_1$ nor $\vec{A}_2$ produces the magnetic field $\vec{B}$

(d) Both $\vec{A}_1$ and $\vec{A}_2$ produce the magnetic field $\vec{B}$

18. The kinetic energy of a free relativistic particle is defined as $E - m_0c^2$, where E and m_0 are respectively its total energy and rest mass. Let v be the speed of the particle when its kinetic energy is half of its rest mass energy. Then the ratio v/c is

(a) $\frac{\sqrt{5}}{3}$

(b) $\frac{1}{\sqrt{2}}$

(c) 1

(d) $\frac{\sqrt{3}}{2}$

19. If the age of the universe is 10^{10} years, the humans have existed for 10^6 years. If we take the age of the universe to be a day, how many seconds have the humans existed?

(a) 2·40 sec

(b) 86·40 sec

(c) 8·64 sec

(d) 24 sec

20. The mass density and momentum density in a fluid at position $\vec{r}$ at time t, respectively denoted by $\rho(\vec{r}, t)$ and $\vec{g}(\vec{r}, t)$, are related by the continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{g} = 0$$

The above equation is a consequence of the law of conservation of

(a) linear momentum

(b) mass

(c) energy

(d) angular momentum

21. If the number of seconds in a year of 365 days is taken as $\pi \times 10^7$ seconds, the percentage error in this approximation would be close to

(a) 4·8

(b) 0·048

(c) 0·38

(d) 2·3

22. Consider a regular arrangement of identical spheres in a face-centred cubic (fcc) structure in which the centres of the respective spheres are located at each of the eight corners and the centres of the six surfaces of a unit cube. The fraction of each cubic unit cell occupied by the spheres in the close-pack configuration is

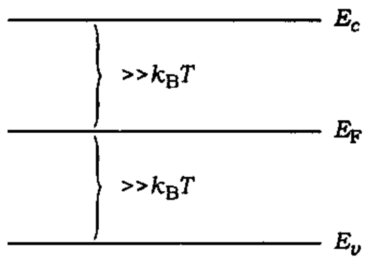
(a) 0·50

(b) 0·62

(c) 0·74

(d) 0·88

23. In a simple model of an intrinsic semiconductor without doping, we assume that there are N_v states (all having same energy E_v) in valence band and N_c states (all having same energy E_c) in the conduction band. The probability of occupying an electronic state of energy E at temperature T is given by the Fermi-Dirac distribution $1/\left(\exp\left(-\frac{E-E_F}{k_B T}\right)+1\right)$, where E_F is the Fermi energy. At equilibrium, the average number of electrons and holes in the conduction band and the valence band respectively are equal. Assuming that the Fermi level E_F lies in the gap between the valence and conduction band and is far from both compared to $k_B T$, it follows that the difference $E_F - (E_v + E_c)/2$ is given by



(a) $\frac{k_B T}{2} \ln \frac{N_v}{N_c}$

(b) $\frac{k_B T}{4} \ln \frac{N_v}{N_c}$

(c) $k_B T \frac{N_v}{N_c}$

(d) $2k_B T \frac{N_c}{N_v}$

24. For an ideal *p-n* junction diode with sharp boundary between two semiconducting materials, the current i is related to the potential difference V across the diode by the relation

$$i = i_0(e^{eV/k_B T} - 1)$$

where e is the electronic charge and T is the temperature; i_0 is a material-dependent constant. The junction is termed respectively as forward and reverse biased depending on whether V is positive or negative. At temperature T, the magnitude of the current changes by a factor f for changing from forward to backward bias of the voltage which is equal to

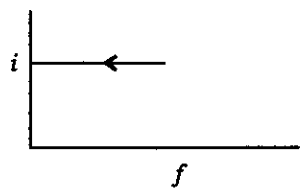
(a) $\dfrac{2k_B T}{e} e^{2f}$

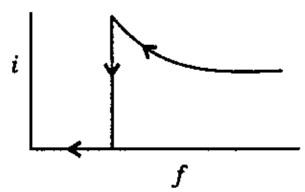
(b) $\dfrac{2k_B T}{e} \ln f$

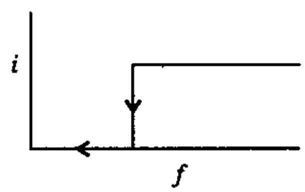
(c) $\dfrac{k_B T}{e} e^{f}$

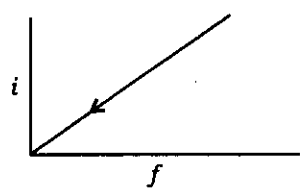
(d) $\dfrac{k_B T}{e} \ln f$

25. In a photoelectric effect experiment, a stream of photons of frequency f and intensity (energy per unit time) is incident on the photocathode to produce a photocurrent i. If the frequency f is steadily reduced without any change of intensity, the plot of current vs. frequency looks like

(a) 

(b) 

(c) 

(d) 

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SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

1200

Total Pages : 32

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ENTRANCE EXAMINATION, 2012

M.Sc. PHYSICS

[Field of Study Code : SPSM (226)]

Time Allowed : 3 hours

Maximum Marks : 99

Weightage : 100

INSTRUCTIONS FOR CANDIDATES

(i) All questions are compulsory.

(ii) For each question, one and only one of the four choices given is the correct answer.

(iii) For each question, **answer must be given in the space provided in the SHEET–I (Answer Table)**. Answer given in any other place will not be evaluated.

(iv) Each correct answer will be given +3 marks.

(v) Each wrong answer will be given –1 mark.

(vi) Use of calculators is permitted.

(vii) Extra pages are attached at the end of the Question Paper for Rough Work.

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1. The general solution of the differential equation

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$$

in terms of two arbitrary constants A and B, is

(a) $e^{1/x}(A\cos(\frac{1}{x})+B\sin(\frac{1}{x}))$

(b) $Ax+\frac{B}{x}$

(c) $Ax+Bxe^x$

(d) $Ax+Bx^2$

2. If a, b and c are non-zero real numbers not equal to 1, $\log_a c$ can be expressed as

(a) $\log_b c/\log_b a$

(b) $\log_b a/\log_b c$

(c) $\log_c a/\log_b a$

(d) $\log_c b/\log_a b$

3. A homogeneous linear transformation takes the point (1, 1) in the xy-plane to the point (3, 3) and keeps the point (1, −1) fixed (i.e., it remains (1, −1) after the transformation). The matrix corresponding to this transformation is

(a) $\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$

(b) $\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix}$

(c) $\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$

(d) $\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}$

4. The function $\frac{1}{\cosh x}$ may be expressed around the point $x = 0$ as a power series as

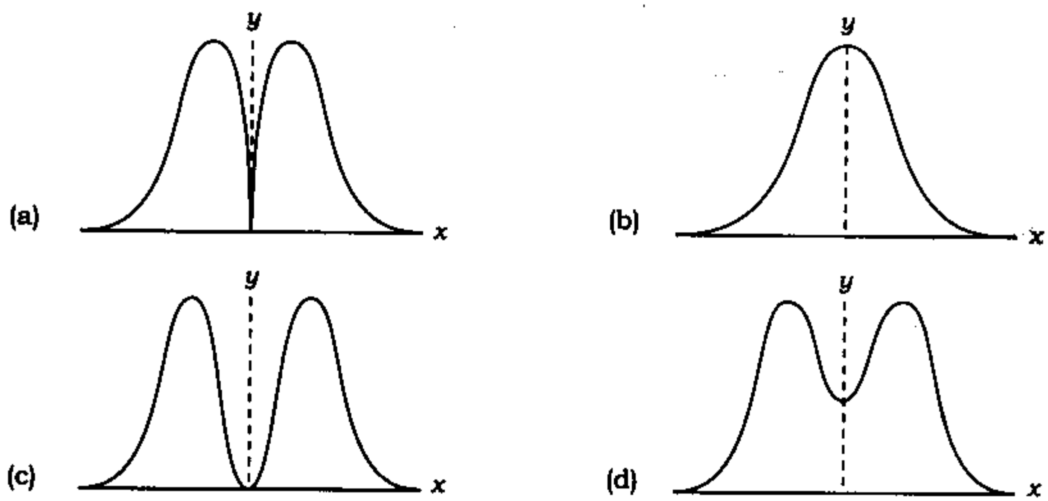
(a) $1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \cdots$

(b) $1 - \frac{1}{2}x^2 + \frac{5}{24}x^4 - \frac{61}{720}x^6 + \cdots$

(c) $1 - \frac{1}{2}x^2 + \frac{11}{24}x^4 - \frac{331}{720}x^6 + \cdots$

(d) $1 - \frac{1}{2x^2} + \frac{1}{24x^4} - \frac{1}{720x^6} + \cdots$

5. Which of the following graphs gives the best representation of the real-valued function $y = x^2 e^{-x^2}$?



6. An observer O uses the coordinate system (x, t) to describe non-relativistic motion in one dimension. Another observer O', moving with respect to O with a uniform velocity v (much smaller than the speed of light c) along the positive x-direction, uses (x', t'), such that at $t = 0$, $t' = 0$ and that instant x and x' coincide. Then

(a) $x' = x - vt,\ t' = t,\ \frac{\partial}{\partial x'} = \frac{\partial}{\partial x} - \frac{1}{v}\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t}$

(b) $x' = x - vt,\ t' = t,\ \frac{\partial}{\partial x'} = \frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial x}$

(c) $x' = x + vt,\ t' = t,\ \frac{\partial}{\partial x'} = \frac{\partial}{\partial x} + \frac{1}{v}\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t}$

(d) $x' = x + vt,\ t' = t,\ \frac{\partial}{\partial x'} = \frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} - v\frac{\partial}{\partial x}$

7. A ball dropped from a height h can only attain the height $4h/5$ after bouncing off the floor. If the ball is dropped from a height of 1 m, the time it will take to come to rest is, approximately

[Ignore air resistance and the finite radius of the ball.]

(a) 1·9 s

(b) 3·8 s

(c) 8·0 s

(d) 4·1 s

8. A small raindrop of mass m experiences a viscous drag force $F_d = bv$, proportional to its instantaneous speed v. If it starts from rest at a height h, its speed after a time t is

(a) $v(t) = \frac{mg}{b} \tanh\left(\frac{bt}{m}\right)$

(b) $v(t) = \frac{mg}{b} e^{-bt/m}$

(c) $v(t) = \frac{mg}{2b} (1 - e^{-2bt/m})$

(d) $v(t) = \frac{mg}{b} (1 - e^{-bt/m})$

9. The nature of flow in a viscous liquid is characterised by the dimensionless Reynolds' number Re proportional to v (the flow velocity) : $Re \propto v$. Given that Re also depends on *(i)* the density ρ of the fluid, *(ii)* the dynamical viscosity η and *(iii)* a characteristic length l, of the flow. By dimensional analysis, we find that

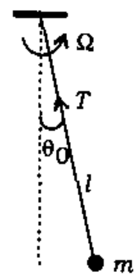
(a) $Re = \frac{\eta l v}{\rho}$

(b) $Re = \frac{\rho l v}{\eta}$

(c) $Re = \frac{\rho \eta v}{l}$

(d) $Re = \frac{\rho v}{\eta l}$

10. A ball of mass m is hung from a support by a massless wire of length l. The support is rotated with an angular speed $\Omega > \sqrt{g/l}$ around a vertical axis through the point of suspension as shown in the figure. The ball rests in equilibrium at an angle θ_0. Which of the following statements concerning θ_0 and the tension T, is true?



(a) $\theta_0 = 0$ and $T = mg$

(b) $\theta_0 = \tan^{-1}\left(\frac{g}{\Omega^2 l}\right)$ and $T < mg\cos\theta_0$

(c) $\theta_0 = \sin^{-1}\left(\frac{g}{\Omega^2 l}\right)$ and $T > mg\cos\theta_0$

(d) $\theta_0 = \cos^{-1}\left(\frac{g}{\Omega^2 l}\right)$ and $T > mg\cos\theta_0$

11. In a wire loop of resistance R and inductance L, an e.m.f. $\mathscr{E}$ is switched on at $t = 0$. The magnetic flux through the loop is given by

(a) $\frac{L\mathscr{E}}{R}(1 - e^{-tR/L})$

(b) $\frac{L\mathscr{E}}{R}e^{-tR/L}$

(c) $\frac{L\mathscr{E}}{R}\left(1 - \frac{L}{tR}\right)$

(d) $\frac{L\mathscr{E}}{R}$

12. The electric and magnetic fields of an electromagnetic wave in vacuum are given by $\boldsymbol{E} = \hat{i}\,E_0 \sin(kz - \omega t)$ and $\boldsymbol{B} = \hat{j}\,B_0 \sin(kz - \omega t)$ respectively. Which of the following relations is correct?

(a) $k^2 E_0 = \omega^2 B_0$

(b) $\omega E_0 = kB_0$

(c) $kE_0 = \omega B_0$

(d) $E_0 B_0 = \omega k$

13. The radius of the nucleus of the Ra atom, which carries an electric charge +88e, is $7 \cdot 0 \times 10^{-15}$ m. What should roughly be the speed of a proton, if it has to reach as close as $1 \cdot 0 \times 10^{-14}$ m from the centre of the nucleus? [The radius of the cloud of orbital electrons of the Ra atom is approximately $5 \cdot 0 \times 10^{-11}$ m.]

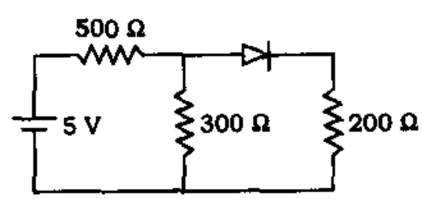
(a) $6 \cdot 7 \times 10^{9}$ m/s

(b) $3 \cdot 1 \times 10^{8}$ m/s

(c) $1 \cdot 4 \times 10^{5}$ m/s

(d) $4 \cdot 9 \times 10^{7}$ m/s

14. In the circuit shown below, the diode is non-ideal and has a voltage drop of 0·7 V. What is the value of the diode current?



(a) 4·84 mA

(b) 8·06 mA

(c) 3·03 mA

(d) 6·25 mA

15. The Doppler width $\Delta\lambda$ of the orange line (for which λ = 6058 Å) of Kr is 0·0055 Å. What is the spread in frequency of this spectral line?

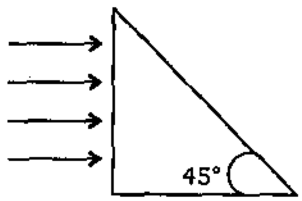
(a) $2 \cdot 7 \times 10^{7}$ Hz

(b) $2 \cdot 7 \times 10^{9}$ Hz

(c) $4 \cdot 5 \times 10^{6}$ Hz

(d) $4 \cdot 5 \times 10^{8}$ Hz

16. A beam of light, consisting of red (R), green (G) and blue (B) colours, is incident normal to a face on a right-angled prism (see figure). The refractive indices of the material of the prism for R, G and B wavelengths are 1·39, 1·44 and 1·47 respectively. Then



(a) R, G and B get transmitted (without undergoing total internal reflection)

(b) R and G undergo total internal reflection and B gets transmitted

(c) R gets transmitted, while G and B undergo total internal reflection

(d) All of R, G and B undergo total internal reflection

17. The two slits in a Young's double-slit experiment are of unequal width, one being four times wider than the other. If I_{max} and I_{min} denote the intensities at a neighbouring maximum and a minimum, then the ratio I_{min}/I_{max} is

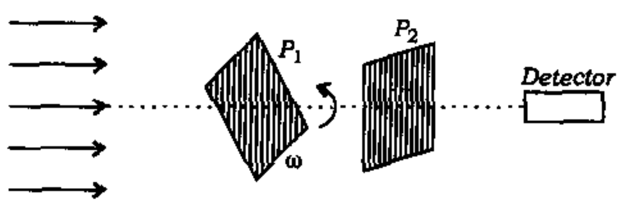
(a) $\frac{1}{9}$

(b) $\frac{1}{4}$

(c) $\frac{3}{5}$

(d) 0

18. A linear beam of unpolarised light passes through two plane polarisers, the planes of which are perpendicular to the direction of propagation of the beam. The first polariser rotates around this direction with an angular velocity of 20π radians per second. If the initial intensity of the light beam is I_0, then the intensity when it leaves the second polariser



(a) is periodic with frequency of 20 Hz and maximum of $I_0/4$

(b) is periodic with frequency of 20 Hz and maximum of $I_0/2$

(c) is periodic with frequency of 10 Hz and maximum of $I_0/4$

(d) is periodic with frequency of 10 Hz and maximum of $I_0/2$

19. The Boolean expression $B \cdot (A+B) + A \cdot (A+\overline{B})$ can be realised using a minimum number of

(a) 1 OR gate

(b) 1 AND gate

(c) 2 OR gates

(d) 2 AND gates

20. An ideal diatomic gas (of $\gamma = 5/3$) is expanded adiabatically so that its volume is doubled. By what ratio is its temperature reduced in this process?

(a) $1/2$

(b) $1/2^{1/3}$

(c) $1/2^{2/3}$

(d) $1/2^{5/3}$

21. Two buckets B_1 and B_2, each containing 25 litres of water, are initially at temperatures T_1 and T_2, respectively. Now take 1 litre of water from B_1, put it in B_2 and allow thermal equilibrium to be established. Then take 1 litre of water from B_2, put it back in B_1 and again allow it to come to thermal equilibrium. At the end of this cycle the amount of water in each bucket does not change, but their temperatures will change. When this process is repeated, the difference in temperature reduces by the same factor after each cycle. If $|T_1 - T_2|$ was 40 °C to begin with, what would be its value after 5 cycles?

(a) 27 °C

(b) 10 °C

(c) 19 °C

(d) 35 °C

22. A flat plate is constantly being bombarded from one side by particles of mass m. If the number density of the particles is ρ and they strike the plate with speed v along the normal to the plate, the pressure exerted on the plate is

(a) $m\rho v^2$

(b) $2m\rho v^2$

(c) $m\rho v$

(d) $2m\rho v$

23. Helium atoms at low temperatures make a perfect closed pack structure of hexagonal lattice with parameters $a = 0 \cdot 36$ nm and $c = 0 \cdot 59$ nm. The density of the crystal is approximately

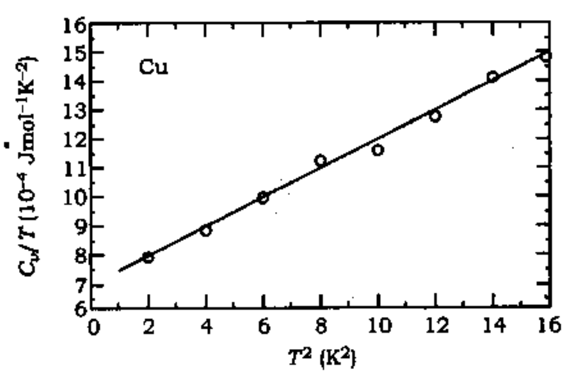
(a) 2000 kg/m^3

(b) 100 kg/m^3

(c) 123 kg/m^3

(d) 200 kg/m^3

24. The ratio of the specific heat capacity and temperature, C_v/T, of Cu is plotted as a function of T^2, the square of the absolute temperature, in the graph below :



The values of γ and β (the coefficients corresponding to the electronic and the vibrational components of the specific heat) are, approximately

(a) $\gamma = 7 \cdot 0 \times 10^{-4}$ J mol^{-1} K^{-2} and $\beta = 5 \cdot 0 \times 10^{-5}$ J mol^{-1} K^{-4}

(b) $\gamma = 5 \cdot 0 \times 10^{-5}$ J mol^{-1} K^{-2} and $\beta = 7 \cdot 0 \times 10^{-4}$ J mol^{-1} K^{-4}

(c) $\gamma = 1 \cdot 4 \times 10^{-3}$ J mol^{-1} K^{-2} and $\beta = 7 \cdot 0 \times 10^{-4}$ J mol^{-1} K^{-4}

(d) $\gamma = 5 \cdot 0 \times 10^{-4}$ J mol^{-1} K^{-2} and $\beta = 7 \cdot 0 \times 10^{-5}$ J mol^{-1} K^{-4}

25. A paramagnetic gas at room temperature is placed in an external magnetic field of 1·5 T (tesla). Each atom of the gas has a magnetic moment $\mu = 1 \cdot 0 \mu_B$, where $\mu_B = 9 \cdot 3 \times 10^{-24}$ J/T is the Bohr magneton. The difference in energy when an atom is aligned along the magnetic field and opposite to it, is

(a) $2 \cdot 8 \times 10^{-23}$ J

(b) $1 \cdot 4 \times 10^{-23}$ J

(c) $18 \cdot 6 \times 10^{-24}$ J

(d) $9 \cdot 3 \times 10^{-24}$ J

26. The Fermi energy ε_F in metals depends on the number density n_e of mobile electrons, which may be thought of as a free Fermi gas. If n_e of one metal is larger by a factor of 1000 compared to another, then in comparison, its Fermi energy is

(a) 1000 times larger

(b) smaller by a factor of 1/100

(c) 100 times larger

(d) 10 times larger

27. The kinetic energy of a proton and an α-particle (not under the influence of any force) are given to be equal. If we denote the de Broglie wavelengths of the proton by λ_p and that of the α-particle by λ_α, then

(a) $\lambda_p \simeq \lambda_\alpha$

(b) $\lambda_p \simeq 4\lambda_\alpha$

(c) $\lambda_p \simeq \frac{1}{2}\lambda_\alpha$

(d) $\lambda_p \simeq 2\lambda_\alpha$

28. When a monochromatic point source of light is placed at a distance of 0·2 m from a photoelectric cell, the stopping potential V_s and the saturation current I_s are found to be 0·6 V and 18·0 mA, respectively. If the same source is now placed 0·6 m away from the photoelectric cell, one finds

(a) $V_s = 0 \cdot 2$ V and $I_s = 6 \cdot 0$ mA

(b) $V_s = 0 \cdot 6$ V and $I_s = 6 \cdot 0$ mA

(c) $V_s = 0 \cdot 6$ V and $I_s = 2 \cdot 0$ mA

(d) $V_s = 0 \cdot 2$ V and $I_s = 18 \cdot 0$ mA

29. The graph in the figure below shows the intensity I as a function of frequency ν of a perfect blackbody at a fixed temperature T :

The corresponding graph at temperature $2T$ can be obtained by which of the following operations?

For every point of the graph

(a) multiply the ν-coordinate by 1/2 and the I-coordinate by 8

(b) multiply the ν-coordinate by 2 and the I-coordinate by 8

(c) multiply the ν-coordinate by 1/2 and the I-coordinate by 16

(d) multiply the ν-coordinate by 2 and the I-coordinate by 16

30. What is the maximum theoretical accuracy ΔE to which an ideal experiment may determine the energy levels of the hydrogen atom?

[Hint : Use the fact that the age of the universe is estimated to be approximately $1 \cdot 4 \times 10^{10}$ years.]

(a) $4 \cdot 7 \times 10^{-26}$ eV

(b) $9 \cdot 4 \times 10^{-33}$ eV

(c) $1 \cdot 2 \times 10^{-63}$ eV

(d) [illegible] $\times 10^{-70}$ eV

31. A particle in one dimension is in the ground state (lowest energy quantum state) of the potential well given by

$$V(x) = \begin{cases} 0 & \text{for } |x| < \frac{L}{2} \\ \infty & \text{otherwise} \end{cases}$$

Let P_+ be the probability that the particle is found to move along the positive x-direction and p be the magnitude of the momentum for that state of motion. Then

(a) $P_+ = 0$ and $p = 0$

(b) $P_+ = \frac{1}{2}$ and $p = \frac{\pi}{2L}$

(c) $P_+ = \frac{1}{2}$ and $p = \frac{\pi}{L}$

(d) $P_+ = 1$ and $p = \frac{\pi}{L}$

32. A particle of mass m is moving in a three-dimensional potential

$$V(x, y, z) = \frac{1}{2} m\omega^2 (x^2 + 2y^2 + 4z^2)$$

The energy of the particle in the ground state (lowest energy quantum state) is

(a) $\frac{\sqrt{7}}{2}\hbar\omega$

(b) $\frac{3}{2}\hbar\omega$

(c) $\frac{7}{2}\hbar\omega$

(d) $\frac{(3+\sqrt{2})}{2}\hbar\omega$

33. A nucleus may be modelled as a drop of liquid consisting of the nucleons (protons and neutrons). In this model, the dominant contribution to the nuclear binding energy is from the volume, which is proportional to A, the total number of nucleons. Then the two important subdominant contributions from the surface tension and the Coulomb repulsion of the protons are, proportional to

(a) $A^{2/3}$ and $Z / A^{1/3}$ respectively

(b) $A^{2/3}$ and $Z^2 / A^{1/3}$ respectively

(c) $A^{1/3}$ and $Z^2 / A^{2/3}$ respectively

(d) $A^{1/2}$ and $Z^2 / A^{1/3}$ respectively

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ENTRANCE EXAMINATION, 2011

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M.Sc. PHYSICS

[Field of Study Code : SPSM (227)]

Time Allowed : 3 hours

Maximum Marks : 90

Weightage : 100

INSTRUCTIONS FOR CANDIDATES

(i) All questions are compulsory.

(ii) For each question, one and only one of the four choices given is the correct answer.

(iii) For each question, **answer must be given in the space provided in the SHEET–I (Answer Table)**. Answer given in any other place will not be evaluated.

(iv) Each correct answer will be given +3 marks.

(v) Each wrong answer will be given −1 mark.

(vi) Use of calculators is permitted.

(vii) Extra pages are attached at the end of the Question Paper for Rough Work.

1. You are given the following transformation of the coordinates (x, y) of a point in the two-dimensional plane :

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Which of the following statements describes the transformation?

(a) Rotation by an angle of 45° about the z-axis followed by translation by one unit along both x- and y-axes

(b) Rotation by an angle of 45° about the z-axis followed by an expansion of both the x- and y-coordinates by a factor of $\sqrt{2}$

(c) Translation by one unit along both x- and y-axes followed by an expansion of both the x- and y-coordinates by a factor of $\sqrt{2}$

(d) Only a rotation by an angle of 45° about the z-axis

2. The elements of the infinite sequence 0, 1, 1, $\frac{3}{2}$, $\cdots$ satisfy the recursion relation $F_{n+1} = F_n + \frac{1}{2} F_{n-1}$, where F_n denotes the nth element. What is the value of $\lim_{n \to \infty} F_n / F_{n-1}$?

(a) 1·366

(b) 1·575

(c) 1·618

(d) The limit does not exist

3. For which of the following matrices both the eigenvalues are positive?

(a) $\begin{pmatrix} -7 & 2 \\ 2 & -7 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

B:

(c) $\begin{pmatrix} 4 & -3 \\ -3 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} -1 & 4 \\ 4 & -1 \end{pmatrix}$

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