Board question paper (Maths) : March 2013

Note:
i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Solution of L.P.P. should be written on graph paper only.
iv. Answers to both the sections should be written in the same answer book.
v. Answer to every new question must be written on a new page.

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following: **(6) [12]**

i. The principal solution of the equation $\cot x = -\sqrt{3}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $-\frac{5\pi}{6}$

ii. If the vectors $-3\hat{i}+4\hat{j}-2\hat{k}, \hat{i}+2\hat{k}, \hat{i}-p\hat{j}$ are coplanar, then the value of p is

(A) – 2 (B) 1

(C) –1 (D) 2

iii. If the line $y = x + k$ touches the hyperbola $9x^2 - 16y^2 = 144$, then k = ……

(A) 7 (B) – 7

(C) $\pm\sqrt{7}$ (D) $\pm\sqrt{19}$

(B) Attempt any THREE of the following: **(6)**

i. Write down the following statements in symbolic form:
(a) A triangle is equilateral if and only if it is equiangular.
(b) Price increases and demand falls.

ii. If $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$, then find A^{-1} by adjoint method.

iii. Find the separate equations of the lines represented by the equation $3x^2 - 10xy - 8y^2 = 0$.

iv. Find the equation of the director circle of a circle $x^2 + y^2 = 100$.

v. Find the general solution of the equation $4\cos^2 x = 1$.

Q.2. (A) Attempt any TWO of the following: (6)[14]

i. Without using truth table show that
$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

ii. If θ is the measure of acute angle between the pair of lines given by $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$, $a + b \neq 0$

iii. Show that the line $x + 2y + 8 = 0$ is tangent to the parabola $y^2 = 8x$, Hence find the point of contact.

(B) Attempt any TWO of the following: (8)

i. The sum of three numbers is 9. If we multiply third number by 3 and add to the second number, we get 16. By adding the first and the third number and then subtracting twice the second number from this sum, we get 6. Use this information and find the system of linear equations. Hence, find the three numbers using matrices.

ii. Find the general solution of $\cos x + \sin x = 1$.

iii. If $\bar{a}$ and $\bar{b}$ are any two non-zero and non-collinear vectors, then prove that any vector $\bar{r}$ coplanar with $\bar{a}$ and $\bar{b}$ can be uniquely expressed as $\bar{r} = t_1\bar{a} + t_2\bar{b}$, where t_1 and t_2 are scalars.

Q.3. (A) Attempt any TWO of the following: (6)[14]

i. Using truth table examine whether the following statement pattern is tautology, contradiction or contingency.
$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$

ii. Find k if the length of the tangent segment from $(8, -3)$ to the circle $x^2 + y^2 - 2x + ky - 23 = 0$ is $\sqrt{10}$ units.

iii. Show that the lines given by
$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1}$ and $\frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4}$ intersect.
Also find the co-ordinates of the point of intersection.

(B) Attempt any TWO of the following: (8)

i. Find the equation of the locus of the point of intersection of two tangents drawn to the hyperbola $\frac{x^2}{7} - \frac{y^2}{5} = 1$ such that the sum of the cubes of their slopes is 8.

ii. Solve the following L.P.P. graphically:
Maximize : $Z = 10x + 25y$
Subject to: $x \leq 3$, $y \leq 3$, $x + y \leq 5$, $x \geq 0$, $y \geq 0$

iii. Find the equations of the planes parallel to the plane $x + 2y + 2z + 8 = 0$ which are at the distance of 2 units from the point (1,1,2).