Reg. No. : | | | | | | | | | | | | |

**Question Paper Code : 97107**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Third Semester

Civil Engineering

MA 6351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches except Environmental Engineering, Textile Chemistry, Textile Technology, Fashion Technology and Pharmaceutical Technology)

(Regulation 2013)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating the arbitrary function $f$ from $z = f\left(\frac{y}{x}\right)$.

2. Find the complete solution of $p + q = 1$.

3. State the sufficient conditions for existence of Fourier series.

4. If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$ in $0 < x < 2\pi$, then deduce that value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$.

5. Classify the partial differential equation $(1 - x^2)z_{xx} - 2xy\, z_{xy} + (1 - y^2)z_{yy} + xz_x + 3x^2 y\, z_y - 2z = 0$.

6. Write down the various possible solutions of one dimensional heat flow equation.

7. State and prove modulation theorem on Fourier transforms.

8. If $F\{f(x)\} = F(s)$, then find $F\{e^{iax} f(x)\}$.

9. Find the $Z$ transform of $n$.

10. State initial value theorem on $Z$ transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the singular solution of $z = px + qy + p^2 - q^2$. (8)

(ii) Solve $(D^2 - 2DD')z = x^3 y + e^{2x-y}$. (8)

Or

(b) (i) Solve $x(y - z)p + y(z - x)q = z(x - y)$. (8)

(ii) Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y)$. (8)

12. (a) (i) Find the Fourier series of $f(x)=x^2$ in $-\pi<x<\pi$. Hence deduce the value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$. (8)

(ii) Find the half range cosine series expansion of $(x-1)^2$ in $0<x<1$. (8)

Or

(b) (i) Compute the first two harmonics of the Fourier series of $f(x)$ from the table given. (8)

| $x$ | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1.0 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

(ii) Obtain the Fourier cosine series expansion of $f(x)=x$ in $0<x<4$. Hence deduce the value of $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}$..... to $\infty$. (8)

13. (a) If a tightly stretched string of length $l$ is initially at rest in equilibrium position and each point of it is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0}=v_0\sin^3\frac{\pi x}{l}$, $0<x<l$, determine the transverse displacement $y(x,t)$. (16)

Or

(b) A square plate is bounded by the lines $x=0$, $x=a$, $y=0$ and $y=b$. Its surfaces are insulated and the temperature along $y=b$ is kept at 100°C. while the temperature along other three edges are at 0°C. Find the steady - state temperature at any point in the plate. (16)

14. (a) Find the Fourier transform of $f(x)=\begin{cases}1-|x|, \text{ if } |x|<1\\ 0, \text{otherwise}\end{cases}$. Hence, deduce the values (i) $\int_0^{\infty}\frac{\sin^2 t}{t^2}dt$ (ii) $\int_0^{\infty}\frac{\sin^4 t}{t^4}dt$. (16)

Or

(b) (i) Find the Fourier transform of $e^{-a^2x^2}, a>0$. Hence show that $e^{-\frac{x^2}{2}}$ is self reciprocal under the Fourier transform. (8)

(ii) Evaluate $\int_0^{\infty}\frac{dx}{(x^2+a^2)(x^2+b^2)}$, using Fourier transforms. (8)

15. (a) (i) Find $Z(\cos n\theta)$ and $Z(\sin n\theta)$. (8)

(ii) Using Z-transforms, solve $u_{n+2}-3u_{n+1}+2u_n=0$ given that $u_0=0, u_1=1$. (8)

Or

(b) (i) Find the Z-transform of $\frac{1}{n(n+1)}$, for $n\geq 1$. (8)

(ii) Find the inverse Z-transform of $\frac{z^2+z}{(z-1)(z^2+1)}$, using partial fraction. (8)

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