

Q.P. Code : 5067

(3 Hours)

[Total Marks : 80]

Instructions:

- 1) Question No. 1 is compulsory.
- 2) Attempt any THREE of the remaining.
- 3) Figures to the right indicate full marks.

Q 1. A) Find Laplace of $\{t^5 \cosh t\}$ (5)B) Find Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$ (5)

C) Find a, b, c, d, e if,

 $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic (5)D) Prove that $\nabla \left(\frac{1}{r} \right) = -\frac{\vec{r}}{r^3}$ (5)Q.2) A) If $f(z) = u + iv$ is analytic and $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$, find $f(z)$ (6)B) Find inverse Z-transform of $f(z) = \frac{z+2}{z^2 - 2z+1}$ for $|z| > 1$ (6)C) Find Fourier series for $f(x) = \sqrt{1 - \cos x}$ in $(0, 2\pi)$ Hence, deduce that $\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$ (8)Q.3) A) Find $L^{-1} \left\{ \frac{1}{(s-2)^2(s+3)} \right\}$ using Convolution theorem (6)B) Prove that: $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = (3x^2 - 1)/2$ are orthogonal over $(-1, 1)$ (6)C) Verify Green's theorem for $\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2 - y^2)\vec{i} + (x+y)\vec{j}$ and c is the triangle with vertices $(0,0)$, $(1,1)$, $(2,1)$ (8)

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Q.4) A) Find Laplace Transform of $f(t) = |\sin pt|, t \geq 0$ (6)

B) Show that $\vec{F} = (y \sin z - \sin x) \mathbf{i} + (x \sin z + 2yz) \mathbf{j} + (xy \cos z + y^2) \mathbf{k}$ is irrotational.

Hence, find its scalar potential. (6)

C) Obtain Fourier expansion of $f(x) = x + \frac{\pi}{2}$ where $-\pi < x < 0$

$$= \frac{\pi}{2} - x \text{ where } 0 < x < \pi$$

Hence, deduce that (i) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

$$(ii) \frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots \quad (8)$$

Q.5) A) Using Gauss Divergence theorem to evaluate $\iint_S \vec{N} \cdot \vec{F} ds$ where $\vec{F} = 4x\mathbf{i} - 2y^2\mathbf{j} + z^2\mathbf{k}$

and S is the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$ (6)

B) Find $Z\{2^k \cos(3k + 2)\}, k \geq 0$ (6)

C) Solve $(D^2 + 2D + 5)y = e^{-t} \sin t$, with $y(0) = 0$ and $y'(0) = 1$ (8)

Q.6) A) Find $L^{-1}\left\{\tan^{-1}\left(\frac{2}{s^2}\right)\right\}$ (6)

B) Find the bilinear transformation which maps the points 2, i, -2 onto points 1, i, -1 by using cross-ratio property. (6)

C) Find Fourier Sine integral representation for $f(x) = \frac{e^{-ax}}{x}$ (8)