

- N. B. :** (1) Question No. 1 is compulsory.
 (2) Answer any **three** questions from Q. 2 to Q. 6.
 (3) **Each** question carry **equal** marks.
 (4) **Non-programmable** calculator is **allowed**.

1. (a) Find $L^{-1} \left\{ \frac{e^{-4-3s}}{(s+4)^{3/2}} \right\}$ 5

(b) Find the constant a,b,c,d and e If. 5

$$f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - cxy^3 + 4xy)$$
 is analytic.

(c) Obtain half range Fourier cosine series for $f(x) = \sin x$, $x \in (0, \pi)$. 5

(d) If r and \bar{r} have their usual meaning and a is constant vector, prove that 5

$$\nabla \times \left[\frac{a \times \bar{r}}{r^n} \right] = \frac{(2-n)}{r^n} a + \frac{n(a \cdot \bar{r})\bar{r}}{r^n + 2}$$

2. (a) Find the analytic function $f(z) = u + iv$ If $3u + 2v = y^2 - x^2 + 16xy$. 6

(b) Find the z - transform of $\left\{ a^{|k|} \right\}$ and hence find the z - transform of $\left\{ \left(\frac{1}{2} \right)^{|k|} \right\}$ 6

(c) Obtain Fourier series expansion for $f(x) = \sqrt{1 - \cos x}$, $x \in (0, 2\pi)$ and hence 8

deduce that $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$.

3. (a) Find :-

(i) $L^{-1} \left\{ \frac{s}{(2s+1)^2} \right\}$ 3

(ii) $L^{-1} \left\{ \log \frac{s^2 + a^2}{\sqrt{s+b}} \right\}$ 3

(b) Find the orthogonal trajectories of the family of curves $e^{-x} \cos y + xy = \alpha$ 6
 where α is the real constant in xy - plane.

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- (c) Show that $\vec{F} = (ye^{xy} \cos z) \mathbf{i} + (xe^{xy} \cos z) \mathbf{j} - (e^{xy} \sin z) \mathbf{k}$ is irrotational and 8

find the scalar potential for \vec{F} and evaluate $\int_c \vec{F} \cdot d\mathbf{r}$ along the curve joining the points $(0, 0, 0)$ and $(-1, 2, \pi)$.

4. (a) Evaluate by Green's theorem. $\int e^{-x} \sin y dx + e^{-x} \cos y dy$ where c is the rectangle 6

whose vertices are $(0, 0)$, $(\pi, 0)$, $(\pi, \frac{\pi}{2})$ and $(0, \frac{\pi}{2})$.

- (b) Find the half range sine series for the function. 6

$$f(x) = \frac{2kx}{\ell}, \quad 0 \leq x \leq \frac{\ell}{2}$$

$$= \frac{2k}{\ell}(\ell - x), \quad \frac{\ell}{2} \leq x \leq \ell$$

- (c) Find the inverse z-transform of $\frac{1}{(z-3)(z-2)}$ 8

- (i) $|z| < 2$
 (ii) $2 < |z| < 3$
 (iii) $|z| > 3$.

5. (a) Solve using Laplace transform. $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 3y = e^{-x}$, $y(0) = 1$, $y'(0) = 1$. 6

- (b) Express $f(x) = \frac{\pi}{2} e^{-x} \cos x$ for $x > 0$ as Fourier sine integral and show that 6

$$\int_0^{\infty} \frac{w^3 \sin wx}{w^4 + 4} dw = \frac{\pi}{2} e^{-x} \cos x.$$

- (c) Evaluate $\iiint_s \vec{F} \cdot \mathbf{n} ds$, where $\vec{F} = x\mathbf{i} - y\mathbf{j} + (z^2 - 1)\mathbf{k}$ and s is the cylinder formed 8
 by the surface $z = 0$, $z = 1$, $x^2 + y^2 = 4$, using the Gauss - Divergence theorem.

6. (a) Find the inverse Laplace transform by using convolution theorem. 6

$$\mathcal{L}^{-1} \left\{ \frac{s^2 + 2s + 3}{(s^2 + 2s + 5)(s^2 + 2s + 2)} \right\}.$$

- (b) Find the directional derivative of $\phi = 4e^{2x - y + z}$ at the point $(1, 1, -1)$ in the direction towards the point $(-3, 5, 6)$. 6

- (c) Find the image of the circle $x^2 + y^2 = 1$, under the transformation $w = \frac{5 - 4z}{4z - 2}$. 8

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