

QP Code : 14544

[3 Hours]

[Total Marks: 80

N.B. (1) Question no. 1 is compulsory.

(2) Attempt any three from the remaining.

(3) Figures to the right indicate full marks.

1. (a) Find the Laplace Transform of $\sin t \cos 2t \cos ht$. 5
(b) Find the Fourier series expansion of $f(x) = x^2$ $(-\pi, \pi)$ 5

(c) Find the z-transform of $\left(\frac{1}{3}\right)^{|k|}$ 5

(d) Find the directional derivative of $4xz^2 + x^2yz$ at $(1, -2, -1)$ in the direction of $2\bar{i} - \bar{j} - 2\bar{k}$ 5

2. (a) Find an analytic function $f(z)$ whose real part is $e^x(x \cos y - y \sin y)$ 6

(b) Find inverse Laplace Transform by using convolution theorem $\frac{1}{(s-3)(s+4)^2}$ 6

(c) Prove that $\bar{F} = (6xy^2 - 2z^3)\bar{i} + (6x^2y + 2yz)\bar{j} + (y^2 - 6z^2x)\bar{k}$ is a conservative field. Find the scalar potential ϕ such that $\nabla \phi = \bar{F}$. Hence find the workdone by \bar{F} in displacing a particle from $A(1,0,2)$ to $B(0,1,1)$ along AB. 8

3. (a) Find the inverse z-transform of $F(z) = \frac{z^3}{(z-3)(z-2)^2}$ 6

(i) $2 < |z| < 3$ (ii) $|z| > 3$

(b) Find the image of the real axis under the transformation $w = \frac{2}{z+i}$ 6

(c) Obtain the Fourier series expansion of $f(x) = \pi x; 0 \leq x \leq 1$
 $= \pi(2-x); 1 \leq x \leq 2$ 8

Here deduce That $\frac{1}{1^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{8}$

4. (a) Find the Laplace Transform of $f(t) = E; 0 \leq t \leq \frac{p}{2}$ 6

$= -E; \frac{p}{2} \leq t \leq p,$ $f(t+p) = f(t)$

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(b) Using Green's theorem evaluate $\int_c \frac{1}{y} dx + \frac{1}{x} dy$ where c is the boundary of the region bounded by $x=1$, $x=4$, $y=1$, $y=\sqrt{x}$ 6

(c) Find the Fourier integral for $f(x) = 1-x^2$, $0 \leq x \leq 1$
 $= 0$ $x > 1$ 8

Hence evaluate $\int_0^{\infty} \frac{\lambda \cos \lambda - \sin \lambda}{\lambda^3} \cos\left(\frac{\lambda}{2}\right) d\lambda$

5. (a) If $\vec{F} = x^2\vec{i} + (x-y)\vec{j} + (y+z)\vec{k}$ moves a particle from $A(1, 0, 1)$ to $B(2, 1, 2)$ along line AB . Find the workdone. 6

(b) Find the complex form of Fourier series $f(x) = \sin^2 x$ $(-\ell, \ell)$ 6

(c) Solve the differential equation using Laplace Transform.
 $(D^2+2D+5)y = e^{-t} \sin t$ $y(0) = 0$ $y'(0) = 1$ 8

6. (a) If $\int_0^{\infty} e^{-2t} \sin(t+\alpha) \cos(t-\alpha) dt = \frac{3}{8}$ find the value of α . 6

(b) Evaluate $\iint_s (y^2z^2\vec{i} + z^2x^2\vec{j} + z^2y^2\vec{k}) \cdot \vec{n} ds$ where s is the hemisphere $x^2+y^2+z^2=1$ above xy - plane and bounded by this plane. 6

(c) Find Half range sine series for $f(x) = \ell x - x^2$ $(0, \ell)$ 8

Hence prove that $\frac{1}{1^6} + \frac{1}{3^6} + \dots = \frac{\pi^6}{960}$