

**Second Year B.Sc. Degree Examination**  
**Aug/Sept 2009**  
**Directorate of Correspondence Course**

**PAPER - II : MATHEMATICS**

Time : 3 Hours

Max. Marks : 90

**Note :** Answer any SIX full questions of the following choosing atleast ONE from each part.

**PART - A**

1. a) i) Find the order and degree of the differential equation
- $$\left[ 1 - \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}} = 6 \frac{d^3y}{dx^3} \quad 2$$
- ii) Solve :  $\sec^2x \tan y \, dx + \sec^2y \tan x \, dy = 0$  2
- b) Solve :  $\frac{dy}{dx} (\tan y) = \sin(x+y) + \sin(x-y)$  5
- c) Solve :  $(1+y^2) + (x-e^{\tan^{-1}y}) \frac{dy}{dx} = 0$  6
2. a) i) Solve :  $x = y + a \log P.$  2
- ii) Solve :  $\frac{dy}{dx} + y \cot x = 5e^{\cos x}$  2
- b) Find the general and singular solution of  $(px-y)(py+x) = a^2p$  by using  $x^2=u$  and  $y^2=v.$  5
- c) Find the orthogonal trajectories of the family of circles through the origin with centres on the x-axis. 6

**PART - B**

3. a) i) Solve :  $(D^3+3D^2+3D+1)y=0$  where  $D = \frac{d}{dx}$  2
- ii) Solve :  $\frac{d^2y}{dx^2} + y = 3 + 5e^x$  2
- b) Solve :  $(D^2 - 2D + 1) y = x^3 e^{3x}$  5
- c) Solve :  $(x^2D^2 - 3xD + 5) y = x^2 \sin(\log x)$  6
4. a) i) Evaluate :  $\lim_{x \rightarrow 0} \frac{\sinh x + x}{\sin x - x \cos x}$  2
- ii) Evaluate :  $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\log(\theta - \frac{\pi}{2})}{\tan \theta}$  2
- b) Verify Rolle's theorem for the function  $f(x) = \log \left[ \frac{x^2 + ab}{x(a+b)} \right]$  in  $[a,b]$ ,  $0 \notin [a,b]$ . 5
- c) Show that  $\sec x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \dots$  using Maclaunn's Expansion. 6

**PART - C**

5. a) i) Prove that a group of order two is Abelian. 2  
 ii) Find the order of every element of the multiplicative group  $G = \{1, -1, i, -i\}$  2  
 b) Prove that the order of an element of a group is same as that of its inverse. 5  
 c) State and prove Fermat's theorem. 6
6. a) i) If  $a > b$  and  $b > c$  then prove that  $a > c$  a, b, c being numbers. 2  
 ii) Solve the inequality  $x-6 < 2x-5 \leq x-3$ . 2  
 b) Find the order of the Permutation  

$$\phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 6 & 7 & 8 & 1 & 2 \end{pmatrix}$$
  
 Also find whether  $\phi$  is even or odd. 5  
 c) Find the envelope of the family of Ellipses  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , where a and b are connected by the relation  $ab=c^2$ , where is a known constant. 6

**PART - D**

7. a) i) If  $\lim_{n \rightarrow \infty} \{x_n\} = l$  then prove that  

$$\lim_{n \rightarrow \infty} \{|x_n|\} = |l|$$
 2  
 ii) Discuss the convergence of the sequence whose  $n^{\text{th}}$  term is  $n[\log(n+1) - \log n]$ . 2  
 b) Prove that a monotonic increasing sequence bounded above is convergent. 5  
 c) Show that the sequence  $\{x_n\}$  defined by  $x_1 = \sqrt{7}$ ,  $x_{n+1} = \sqrt{7+x_n}$  converges to  $\frac{1 + \sqrt{29}}{2}$  6
8. a) i) Prove that if a series  $\sum u_n$  is convergent then  $\lim_{n \rightarrow \infty} u_n = 0$ . 2  
 ii) Test the convergence of the series  

$$\frac{1}{\sqrt{1^2+1}} + \frac{1}{\sqrt{2^2+2}} + \frac{1}{\sqrt{3^2+3}} + \dots \infty$$
 2  
 b) Examine the convergence of the series  

$$\sum \frac{4 \cdot 7 \cdot 10 \dots (3n+1)}{5 \cdot 8 \cdot 11 \dots (3n+2)}$$
 5  
 c) Find the sum to infinity of the series  

$$1 + \frac{1^2 + 2^2}{2!} + \frac{1^2 + 2^2 + 3^2}{3!} + \dots$$
 6  
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