RS

W'05 : 5 FN : AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a, b, etc) should be answered at one place.

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Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) State the Leibnitz's theorem and find y_n, if

$$y = \frac{x^n}{1+x}$$ 5

(*b*) State the Lagrange's mean value theorem and show that

$$|\sin b - \sin a| \le |b - a| \quad \text{for all real } a \text{ and } b.$$ 5

(*c*) Show that the arc of the upper half of the cardiod $r = a(1 - \cos\theta)$ is bisected by the line $\theta = 2\pi/3$. 5

(*Turn Over*)

(d) If $x^x y^y z^z = c$, show that at $x = y = z$

$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}.$$ 5

2. (a) The temperature T at any point (x, y, z) in space is $T = 400\, xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. 5

(b) In what direction from $(3, 1, -2)$ is the directional derivative of $\phi = x^2 y^2 z^4$ maximum and what is its magnitude? 5

(c) Test the convergence of the power series

$$\sum_0^\infty \frac{2n!}{n^n} \frac{2^n}{n!} x^n, \quad (x > 0)$$ 5

(d) Expand $f(x) = \tan x$ about $x = \pi/4$ up to first three terms by Taylor's series. 5

3. (a) State the Green's theorem and evaluate

$$\int_C [(y - \sin x)\, dx + \cos x\, dy],$$ where C plane triangle enclosed by line $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2}{\pi} x$. 5

(b) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$ 7

(c) Solve the following system of equation by using Gauss elimination method:

$$x - y + 2z = -8$$
$$x + y + z = -2$$
$$2x - 2y + 3z = -20$$ 8

4. (a) State the Stokes's theorem and evaluate

$$\oint_C \vec{F} \cdot d\vec{r},$$ where $\vec{F} = y^2 \hat{i} + x^2 \hat{j} - (x + z)\hat{k}$

and C is the boundary of the triangle with vertices at $(0, 0, 0)$, $(1, 0, 0)$ and $(1, 1, 0)$. 6

(b) For what value of λ and r, the system of equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = r$$

have (i) no solution, (ii) unique solution, (iii) more than one solution? 8

(c) Use divergence theorem to show

$$\oint_S \nabla r^2\, d\vec{s} = 6V,$$ where S is any closed surface enclosed by volume V. 6

Group B

5. (a) Define Bernoulli's equation and solve

$$xy(1 + xy^2) \frac{dy}{dx} = 1.$$ 6

(b) Solve $(xy^2 - e^{1/x^3})\, dx - x^2 y\, dy = 0$. 7

(c) Solve $(D^2 + 4)\, y = x \sin x$. 7

6. (*a*) Solve $(x^3D^3 + 3x^2D^2 + xD)\,y = x^3 \log x$. 7

(*b*) Solve

$$(D^2 - 6D + 9)y = e^{3x}/x^2$$

by using variation of parameter method. 6

(*c*) If $L\{f(t)\} = \bar{f}(s)$, then prove that

$$L\{f(at)\} = \frac{1}{a}\bar{f}(s/a)$$

and evaluate $L(\sin 2t \cdot \cos 3t)$ where L represents the Laplace transform. 7

7. (*a*) Write Newton's backward interpolation formula and evaluate value of $f(21)$ from the following data:

x	0	5	10	15	20
y	1·0	1·6	3·8	8·2	15·4

7

(*b*) Define Simpson's one-third rule and estimate the area bounded by the curve $y = f(x)$, x-axis and ordinate $x = 0$ and $x = 5$ from the following data:

x_i	0	0·5	1	1·5	2	2·5	3	3·5	4	4·5	5
$f(x_i)$	1	2·1	3·2	4	5·1	6·2	7·2	8·3	9	10·2	12

6

(*c*) A certain screw-making machine produces an average 2 defective screws out of 100, and pack them in boxes of 500. Find the probability that a box contains 15 defective screws. 7

8. (*a*) Construct the difference table from the following value of y, where value of x are (*i*) 1·05, and (*ii*) 1·45

x :	1·0	1·1	1·2	1·3	1·4	1·5
y :	0·24197	0·21875	0·19419	0·17137	0·14973	0·12952

6

(*b*) Estimate the value of π, correct up to three decimal places, from $\int_0^1 \frac{1}{1+x^2}\,dx$ by using Simpson's one-third rule when number of sub-intervals is 4. 7

(*c*) The incidence of occupational disease in an industry is such that the workers have 20% chances of suffering from it. What is the probability that out of six workers chosen at random, four or more will suffer from the disease? 7

Group C

9. Select most appropriate and unique choice for the following: 1 × 20

(*i*) In usual notation, for Binomial distribution, npq, is

(*a*) $< np$

(*b*) np

(*c*) $> np$

(*d*) none of the above

(*ii*) The particular integral of $\frac{d^2y}{dx^2} + 4y = \cos 2x$ is

(*a*) $\frac{x \sin 2x}{4}$

(*b*) $\frac{x \cos 2x}{4}$

(*c*) $\frac{\sin 2x}{4}$

(*iii*) The C.F. of $y'' - 2y' + y = xe^x \sin x$ is

(*a*) $(c_1 + c_2 x)\, e^x$

(*b*) $(c_1 + c_2)\, e^{-x}$

(*c*) none of the above

(*iv*) If n is a positive integer, then $L(t^n)$ is

(*a*) $(n+1)!/s^n$

(*b*) $n!/s^{n+1}$

(*c*) $n!/s^n$

(*v*) The value of $L^{-1}\{\frac{1}{s-a}\}$ is

(*a*) e^{at}

(*b*) e^{-at}

(*c*) e^{a-t}

(*vi*) For the Poisson's distribution if $p(x=2) = \frac{2}{3}\, p(x=1)$, then mean of Poisson's distribution will be

(*a*) 0

(*b*) 4/13

(*c*) 3/4

(*vii*) The differential equation

$$y' + y\cos x = \sin 2x \text{ is}$$

(*a*) exact

(*b*) linear

(*c*) homogeneous

(*viii*) The rank of the matrix

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix} \text{ is}$$

(*a*) 2

(*b*) 3

(*c*) 4

(*ix*) For any closed surface S,

$$\iint_S \text{curl } \vec{F} \cdot \hat{n}\ ds \text{ is}$$

(*a*) V

(*b*) S

(*c*) 0

(*x*) The directional derivative of the function $f = 4xz^3$ at the point $(1, -1, 1)$ along z-axis is

(*a*) 12

(*b*) 144

(*c*) 0

(*xi*) The unit normal vector to the surface $f = xyz$ at $(1, 1, 1)$ is

(*a*) $\frac{1}{\sqrt{3}}(i - j - k)$

(*b*) $\frac{1}{\sqrt{3}}(i + j + k)$

(*c*) θ

(*xii*) The integrating factor of the differential equation $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$ is

(*a*) $1/y$

(*b*) y

(*c*) $4/x$

(*xiii*) The equation $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$ is

(*a*) exact.

(*b*) linear.

(*c*) homogeneous.

(*xiv*) If expected and observed frequencies are equal, then x^2 is equal to

(*a*) 1

(*b*) 0

(*c*) 1·5

(*xv*) For standard normal variate mean μ is

(*a*) 1

(*b*) 0

(*c*) 6

(*d*) none of the above

(*xvi*) By using Green's theorem, value of

$$\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy,$$ where C is the boundary of the region defined by $y = \sqrt{x}$, $y = x^2$ is

(*a*) 3/2

(*b*) 3/10

(*c*) 8/3

(*d*) none of the above

(*xvii*) One of the eigenvalues of the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ is

(*a*) 0

(*b*) 1

(*c*) 6

(*d*) none of the above

(*xviii*) Given that

x:	1	2	3	4	5
y:	2	5	10	17	26, then

the value of $\nabla^2 y_5$ is

(*a*) 0

(*b*) 2

(*c*) 3

(*d*) 12

(*xix*) Consider the difference table

x:	0	1	2	3	4
y:	3	6	11	18	27,

then y

(*a*) $3x^2+2x+1$

(*b*) x^2+2x+3

(*c*) $2x^2+3x+1$

(*xx*) The equations of two regression lines are

$8x-10y+66=0$ and

$40x-18y=214$, then

mean of x and y are

(*a*) 13, 17

(*b*) 3, 7

(*c*) 17, 3

S'06 : 5 FN : AN 209 (1409)

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Group A

1. (*a*) A twice differential function f is such that $f(a) = f(b) = 0$ and $f(c) > 0$ for $a < c < b$. Prove that there is at least one value ξ between a and b for which $f''(\xi) < 0$. 6

(*b*) If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$ then evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is the straight line joining $(0, 0, 0)$ to $(1, 1, 1)$. 6

(c) Evaluate by Green's theorem in plane

$$\int_C e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$$

where C is the rectangle with vertices $(0,0)$, $(\pi,0)$, $(\pi,\pi/2)$ and $(0,\pi/2)$. 8

2. (a) Find the greatest value of the directional derivative of the function

$2x^2 - y - z^4$ at the point $(2, -1, 1)$. 6

(b) If $P = \frac{d^n}{dx^n}\{(x^2-1)^n\}$ show that

$$\frac{d}{dx}\{(1-x^2)\frac{dP}{dx}\} + n(n+1)P = 0.$$ 8

(c) Prove that $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y = b e^{-x/a}$ at the point where the curve crosses the y-axis. 6

3. (a) By using the methods of Langrange's undetermined multipliers. Find the

maxima and minima of $x^2+y^2+z^2$

subject to the conditions

$ax + by + cz = 1$, and

$a'x + b'y + c'z = 1$ 8

(b) Evaluate

$$\iint_A xy\,dx\,dy$$

where A is the domain bounded by x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$. 6

(c) Find the shortest distance from the origin to the hyperbola

$x^2 + 8xy + 7y^2 = 225,\ z = 0$ 6

4. (a) Find the basis of eigenvectors and then diagonalize the matrix, A

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$ 8

(b) Find the interpolating polynomial to interpolate the following data :

x :	2	2·1	2·2	2·3	2·4
$f(x)$:	1·414214	1·449138	1·48324	1·516575	1·549193

Use it to compute the value of $f(2\cdot15)$. 6

(c) Examine the consistency of the system of equations

$2x - 3y + 5z = 1$

$3x + y - z = 2$

$x + 4y - 6z = 1.$ 2

(d) Find the inverse of the matrix

$$A = \begin{pmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{pmatrix}$$ 4

Group B

5. (a) Solve the differential equation

$y'' + 2y' + y = e^{-x}\cos x$

by using the method of variation of parameters. 8

(*b*) Use Laplace's transform to solve the differential equation

$$y'' + 2y' + 2y = 2$$
$$y(0) = 0, \quad y'(0) = 1.$$ 6

(*c*) Solve the differential equation

$$\frac{dy}{dx}(x^2y^3 + xy) = 1.$$ 6

6. (*a*) Find the surface passing through the lines

$z = x = 0$ and $z - 1 = x - y = 0$

and satisfying the differential equation

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0.$$ 8

(*b*) Solve the differential equation

$$(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy.$$ 6

(*c*) If δ is the central difference operator and μ be the average operator then show that

$$\mu = \sqrt{1 + \frac{1}{4}\delta^2}$$ 6

7. (*a*) A solid of revolution is formed by rotating about the x-axis the area between the x-axis, the lines $x = 0$ and $x = 1$, and a curve through the points with the following coordinates:

x	: 0·00	0·25	0·50	0·75	1·0
y	: 1·000	0·9896	0·9589	0·9089	0·8415

Estimate the volume of the solid formed, giving the answer to three decimal places. 6

(*b*) Use Lagrange's interpolation formula to find the value of $\log_{10} 301$ from the following data values

x	: 300	304	305	307
$\log_{10} x$	: 2·4771	2·4829	2·4843	2·4871

8

(*c*) Solve the differential equation

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right).$$ 6

8. (*a*) An incomplete frequency function values distribution is given as follows:

Variable	*Frequency*	*Variable*	*Frequency*
10-20	12	50-60	?
20-30	30	60-70	25
30-40	?	70-80	18
40-50	65	Total:	229

Given that the median value is 46, determine the missing frequencies using the median formula. 8

(*b*) A car hire firm has two cars which it hires out day by day. The number of cars demanded on each day is distributed as Poisson variate with mean 1·5. Calculate the proportion of days on which (*i*) neither car is used, (*ii*) some demand is refused. 6

(*c*) The theory predicts the proportion of beans in the four groups *A*, *B*, *C*, and *D* should be 9 : 3 : 3 : 1. In an experiment among 1600 beans, the number in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory? 6

Group C

). Answer the following: 2 × 10

(*i*) Test the convergence of the series

$$\frac{1}{(\log 2)^p}+\frac{1}{(\log 3)^p}+\dots+\frac{1}{(\log n)^p}+\dots$$

where $p>0$.

(*ii*) If $z=e^{xy^2}$, $x=t\cos t$ and $y=t\sin t$ then compute $\frac{dz}{dt}$ at $t=\frac{\pi}{2}$.

(*iii*) Find the length of the curve

$$x=a\cos^3 t,\ \ y=a\sin^3 t.$$

(*iv*) Find the directional derivative of

$$f(x,y,z)=2x^2+3y^2+z^2$$

at the point (2, 1, 3) in the direction of the vector $\vec{a}=\vec{i}-2\vec{k}$

(*v*) Let T be the function from R^3 into R^3 defined by

$$T(x_1,x_2,x_3)=(x_1-x_2+2x_3,\ 2x_1+x_2,\ -x_1-2x_2+2x_3)$$

Verify whether T is a linear transformation? If it is find the associated matrix.

(*vi*) Solve the differential equation

$$\frac{dy}{dx}+\frac{y}{x}=\frac{y^2}{x}\log x$$

(*vii*) Check for the linear independence or dependence of the following set of vectors

$$\{(1,1,1),(4,1,1),(1,-1,2)\}$$

(*viii*) If $\vec{v}=x\vec{i}+y\vec{j}+z\vec{k}$ then prove that

$$\text{div}\left(\text{grad}\frac{1}{\vec{v}}\right)=0$$

(*ix*) Find the mode for the following distribution:

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

(*x*) Form a partial differential equation by eliminating f from $z=e^{mx}f(x+y)$.

S'07 : 5 FN : AN 209 (1409)

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Group A

1. (*a*) If $y=\{x+\sqrt{(1+x^2)}\}^m$, prove that

$$(1+x^2)\,y_{n+2}+(2n+1)\,xy_{n+1}+(n^2-m^2)\,y_n=0.$$ 5

(*b*) If $r^2=x^2+y^2+z^2$ and $v=r^m$, prove that

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=m\,(m+1)\cdot r^{m-2}.$$ 5

(c) Are the following vectors linearly dependent?

$\vec{x}_1 = (1, 2, 4)$, $\vec{x}_2 = (2, -1, 3)$, $\vec{x}_3 = (0, 1, 2)$, $\vec{x}_4 = (-3, 7, 2)$

If so, find the relation between them. 4

(d) Examine the convergence of the series

$$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \ldots + \frac{2^{n+1}-2}{2^{n+1}+1}x^n + \ldots$$

where $x > 0$. 6

2. (a) Use Maclaurin's theorem to show that

$$e^{x\cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} + \ldots$$

5

(b) If $I_n = \int_0^{\pi/2} x \sin^n x \, dx$ and $n > 1$, prove that

$$I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}$$

Hence deduce that $I_5 = \frac{149}{225}$ 6

(c) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. 4

(d) Test for consistency and solve:

$$3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$$
$$2x_1 + 5x_2 + 5x_3 - 18x_4 = 9$$
$$4x_1 - x_2 - x_3 + 8x_4 = 7$$

5

3. (a) The smaller segment of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cut off by the chord $x = \frac{a}{2}$ revolves completely about this chord. Find the volume generated. 5

(b) If $f(x, y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2}$, prove that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = -2f(x, y) \text{ and}$$

$$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = 6f(x, y)$$

5

(c) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

and verify that it is satisfied by A and hence find the inverse of A. 5

(d) Divide 24 into three parts such that the continued product of the first, the square of the second and the cube of the third may be a maximum. 5

4. (a) Reduce the matrix

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

to the diagonal form and find the modal matrix. 5

(b) A vector field is given by

$$\vec{F} = (x^2 + xy^2)\,\hat{i} + (y^2 + x^2y)\,\hat{j}$$

Show that the field is irrotational, and find the scalar potential such that $\vec{F} = \text{grad}\,\phi$. 5

(c) Use Stokes' theorem to evaluate

$$\int_C (e^x\,dx + 2y\,dy - dz)$$

where C is the curve $x^2 + y^2 = 4$, $z = 2$. 4

(d) Change the order of integration in the double integral

$$\int_0^a \int_0^x \frac{\phi'(y)\,dy\,dx}{\sqrt{(a-x)(x-y)}}$$

and hence find its value (in terms of ϕ). 6

Group B

5. (a) Solve the differential equation

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$$ 4

(b) Solve the differential equation

$$(D^3 - 2D^2 + 3)\,y = \cos x + x^{-x}$$ 6

(c) Solve

$$x(y^2 - z^2)\,p + y(z^2 - x^2)\,q = z(x^2 - y^2)$$

where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$ 4

(d) Using Laplace transforms, find the solution of the equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = e^{-t}$$

with $x = \dfrac{dx}{dt} = 1$ at $t = 0$. 6

6. (a) Solve the differential equation using the method of variation of parameters

$$\frac{d^2y}{dx^2} + y = x\sin x$$ 6

(b) Solve the differential equation:

$$x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 13y = \log x$$ 6

(c) A bar with insulated sides is initially at temperature 0 °C throughout. The end $x = 0$ is kept at 0 °C and heat is suddenly applied at the end $x = l$ so that $\dfrac{\partial u}{\partial x} = A$ for $x = l$ where A is a constant. Find the temperature function $u(x, t)$: 8

7. (a) The following data are taken from the steam table:

Temp. °C	140	160	170	190
Pressure kgf/cm^2	3·685	6·302	8·076	12·575

Use Lagrange's interpolation formula to compute the pressure at temperature 165 °C. 5

(b) A company generally purchases large lots of a certain kind of electronic device. A method is used that rejects a lot if two or more defective units are found in a random sample of 100 units.

(i) What is the probability of rejecting a lot that is 1% defective?

(ii) What is the probability of accepting a lot that is 5% defective? 8

(c) In an explosive factory cordite sticks are manufactured by two processes A and B. After inspection the results found were:

	Defective	*Nondefective*
Process A	100	900
Process B	60	440

Do these data indicate that process A and process B are of same quality? (5% values of χ^2 for d.f. 1, 2 and 3 are 3·84, 5·99 and 7·82 respectively). 7

(a) Find the first and second derivatives of the functions tabulated below at $x = 0 \cdot 40$:

x :	0·40	0·50	0·60	0·70	0·80
y :	1·5836	1·7974	2·0442	2·3275	2·6511

5

(b) Find an approximate value of $\log_e 5$ up to 4 decimal places, by evaluating the integral

$$\int_0^5 \frac{dx}{4x+5},$$

by Simpson's one–third rule, dividing the range into ten equal parts. 5

(c) In an industrial process the diameter of a ball bearing is an important component part. The buyer sets specifications on the diameter to be $3 \cdot 0 \pm 0 \cdot 01$ cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean 3·0 cm and standard deviation 0·005 cm. On the average, what percentage of manufactured ball bearings will be scrapped? (Given that $\Phi(2) = 0 \cdot 9772$ where

$$\Phi(z) = \frac{1}{\quad} \int^{z} e^{-\frac{1}{2}x^2} dx).$$

(d) For determining the ash contents of six samples of coal, each sample was divided into two equal parts and a part given to each of two analysts A and B. The percentage of ash contents found by them are given below samplewise:

Sample No. :	1	2	3	4	5	6
Analyst A :	8	10	11	9	12	13
Analyst B :	9	13	12	10	11	12

Do the two analysts differ significantly? ($t_{0 \cdot 05}$ for d.f. $5 = 2 \cdot 57$). 5

Group C

9. Select most appropriate and unique choice for the following: 1×20

(i) If $f(x) = (x-a)^{5/2}$ and $f(x+h) = f(x) + hf'(x) + \frac{h^2}{\lfloor 2} f''(x + \theta h)$, then the value of θ as x tends to a is

(a) $\frac{8}{225}$

(b) $\frac{16}{225}$

(c) $\frac{32}{225}$

(d) $\frac{64}{225}$

(*ii*) If $u = \log(x^3 + y^3 - x^2y - xy^2)$, then the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ is

(*a*) $\frac{2}{x-y}$

(*b*) $\frac{2}{x+y}$

(*c*) $\frac{2}{x^2-y^2}$

(*d*) $\frac{2}{x^2+y^2}$

(*iii*) If $z = f(u, v)$ where $u = x^2 - y^2$ and $v = 2xy$, with $(x, y) \neq (0, 0)$ then the differential equation

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0 \text{ is equivalent to}$$

(*a*) $\frac{\partial z}{\partial u} = 0$

(*b*) $\frac{\partial z}{\partial v} = 0$

(*c*) $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = 0$

(*d*) $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = 0$

(*iv*) If A be a 2×2 matrix over a field, the $\det(I + A) = 1 + \text{Trace } A$ when A is

(*a*) singular

(*b*) non-singular

(*c*) symmetric

(*d*) orthogonal

(*v*) If two of the eigenvalues of a 3×3 matrix, whose determinant equals 4, are -1 and 2. Then the third eigenvalue of the matrix is

(*a*) 2

(*b*) -2

(*c*) 3

(*d*) -3

(*vi*) The angle between the tangent vectors to the curve

$$x = t^2 + 1,\ y = 4t - 3,\ z = 2t^2 - 6t$$

at the points $t = 1$ and $t = 2$ is

(*a*) $\cos^{-1}\left(\frac{5}{\sqrt{6}}\right)$

(*b*) $\cos^{-1}\left(\frac{5}{2\sqrt{6}}\right)$

(*c*) $\cos^{-1}\left(\frac{5}{3\sqrt{6}}\right)$

(*d*) $\cos^{-1}\left(\frac{5}{4\sqrt{6}}\right)$

(*vii*) The value of n for which the vector $r^n \vec{r}$ is solenoidal is

(*a*) -3

(*b*) 3

(*c*) -2

(*d*) 2

(*viii*) The value of the line integral

$$\int_C [\sin y dx + x(1+\cos y)\, dy]$$

taken over the circular path given by $x^2+y^2=a^2,\ z=0$ is

(*a*) πa^2

(*b*) $2\pi a^2$

(*c*) $3\pi a^2$

(*d*) $4\pi a^2$

(*ix*) The solution of the differential equation $\cos^2 x \dfrac{d^2y}{dx^2}=1$ such that $y=\dfrac{dy}{dx}=0$ when $x=0$ is

(*a*) $y=\tan x$

(*b*) $y=\sin x$

(*c*) $\log(\tan x+1)$

(*d*) $y=\log\sec x$

(*x*) The particular integral of the equation

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x$$

is

(*a*) $\frac{1}{2}e^x$

(*b*) $\frac{1}{2}xe^x$

(*c*) $\frac{1}{2}x^2e^x$

(*d*) $\frac{1}{2}x^3e^x$

(*xi*) If $f(t)$ is the staircase function such that $f(t)=b$ when $0<t<a$, $f(t)=2b$ when $a<t<2a$, and so forth, then Laplace transform of $f(t)$ is

(*a*) $\dfrac{b}{s(1-e^{as})}$

(*b*) $\dfrac{b}{s(1+e^{-as})}$

(*c*) $\dfrac{b}{s(1+e^{as})}$

(*d*) $\dfrac{b}{s(1-e^{-as})}$

(*xii*) The inverse Laplace transform of $\dfrac{e^{-as}}{s^n}$ is

(*a*) $\dfrac{(t-a)^{n-1}}{\lfloor n-1}u(t-a)$

(*b*) $\dfrac{(t-a)^{n-1}}{\lfloor n}u(t-a)$

(*c*) $\dfrac{(t-a)^{n}}{\lfloor n-1}u(t-a)$

(*d*) $\dfrac{(t-a)^{n}}{\lfloor n}u(t-a)$

(*viii*) The value of the line integral

$$\int_C [\sin y dx + x(1+\cos y)\, dy]$$

taken over the circular path given by $x^2 + y^2 = a^2$, $z = 0$ is

(*a*) πa^2

(*b*) $2\pi a^2$

(*c*) $3\pi a^2$

(*d*) $4\pi a^2$

(*ix*) The solution of the differential equation

$\cos^2 x \dfrac{d^2y}{dx^2} = 1$ such that $y = \dfrac{dy}{dx} = 0$ when $x = 0$ is

(*a*) $y = \tan x$

(*b*) $y = \sin x$

(*c*) $\log(\tan x + 1)$

(*d*) $y = \log \sec x$

(*x*) The particular integral of the equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$$

is

(*a*) $\frac{1}{2}e^x$

(*b*) $\frac{1}{2}xe^x$

(*c*) $\frac{1}{2}x^2 e^x$

(*d*) $\frac{1}{2}x^3 e^x$

(*xi*) If $f(t)$ is the staircase function such that $f(t) = b$ when $0 < t < a$, $f(t) = 2b$ when $a < t < 2a$, and so forth, then Laplace transform of $f(t)$ is

(*a*) $\dfrac{b}{s(1-e^{as})}$

(*b*) $\dfrac{b}{s(1+e^{-as})}$

(*c*) $\dfrac{b}{s(1+e^{as})}$

(*d*) $\dfrac{b}{s(1-e^{-as})}$

(*xii*) The inverse Laplace transform of $\dfrac{e^{-as}}{s^n}$ is

(*a*) $\dfrac{(t-a)^{n-1}}{\lfloor n-1}u(t-a)$

(*b*) $\dfrac{(t-a)^{n-1}}{\lfloor n}u(t-a)$

(*c*) $\dfrac{(t-a)^{n}}{\lfloor n-1}u(t-a)$

(*d*) $\dfrac{(t-a)^{n}}{\lfloor n}u(t-a)$

(*xiii*) The inverse Laplace transform of $\log \frac{s+1}{s-1}$ is

(*a*) $\frac{\sinh t}{t}$

(*b*) $\frac{2 \sinh t}{t}$

(*c*) $\frac{\sin t}{t}$

(*d*) $\frac{2 \sin t}{t}$

(*xiv*) The value of $\Delta^3 (1-x)(1-2x)(1-3x)$ with unit interval of differencing is

(*a*) -24

(*b*) 24

(*c*) -36

(*d*) 36

(*xv*) If $y_0 = 2$, $y_1 = 6$, $y_2 = 8$, $y_3 = 9$ and $y_4 = 17$, then $\Delta^4 y_0$ is

(*a*) 5

(*b*) 6

(*c*) 7

(*d*) 8

(*xvi*) In Simpson's one-third rule, $y(x)$ is a polynomial of degree

(*a*) one

(*b*) two

(*c*) three

(*d*) four

(*xvii*) If $x = 4y + 5$, $y = kx + 4$ are the regression lines of x and y and y on x respectively, then

(*a*) $0 < k \leq \frac{1}{4}$

(*b*) $-\frac{1}{4} \leq k < 0$

(*c*) $k \geq \frac{1}{4}$

(*d*) $k \leq -\frac{1}{4}$

(*xviii*) Two events A and B have probabilities 0·25 and 0·50 respectively. The probability that both A and B occur simultaneously is 0·14. Then the probability that neither A nor B occurs is

(*a*) 0·36

(*b*) 0·37

(*c*) 0·38

(*d*) 0·39

(*xix*) If the function

$$f(x) = ae^{-cx}, \quad \text{when } x \geq 0$$
$$= 0 \quad , \quad \text{when } x < 0$$

is the probability density function of a continuous distribution, then the value of C is

(*a*) a

(*b*) $-a$

(*c*) $2a$

(*d*) $-2a$

(*xx*) The life-lengths of two electronic devices D_1 and D_2 are normally distributed with means 44 hours and 48 hours, and standard deviation of 6 hours and 3 hours respectively. In a particular situation if one of these devices is to be used for a 52-hour period, then

(*a*) D_1 is preferable

(*b*) D_2 is preferable

(*c*) D_1 and D_2 are equally good

(*d*) None of these

W'07 : 5 FN : AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a, b, etc) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answer may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) Prove that

$$\vec{F} = (2xy + z^3)\,\vec{i} + x^2\,\vec{j} + 3xz^2\,\vec{k}$$

is a conservative force field. Also, find the scalar potential. 6

(*b*) Using Stoke's theorem, evaluate

$$\int_S \vec{n}\,.\,\text{curl}\,\vec{F}\,ds$$

where $\vec{F} = y\,\vec{i} + z\,\vec{j} + x\,\vec{k}$ and S is a portion of the surface $x^2 + y^2 - 2ax + a^2 = 0$ above the plane $Z = 0$. 8

(c) Evaluate the integral

$$\int_S \left(yz\vec{i} + zx\vec{j} + xy\vec{k}\right)\cdot ds$$

over the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. 6

2. (a) Show that

$$f(x,y,z) = (x+y+z)^3 - 3(x+y+z) - 24xyz + a^3$$

has a minima at (1,1,1) and a maxima at (-1,-1,-1). 6

(b) If the functions $f(x)$, $\phi(x)$ and $\psi(x)$ satisfy the conditions of mean value theorem in $[a, b]$, show that these exist at least one point ξ such that

$$\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f(b) & \phi(b) & \psi(b) \\ f'(\xi) & \phi'(\xi) & \psi'(\xi) \end{vmatrix} = 0.$$ 6

(c) Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$ 8

3. (a) For what values of λ and μ, the following system of equations has (i) no solution, (ii) a unique solution, and (iii) an infinite number of solutions: 6

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

(b) Prove that

$$\iint_R \sqrt{|y - x^2|}\, dx\, dy = (3\pi + 8)/6$$

where $R = [-1, 1, 0, 2]$. 8

(c) Find the equations of the tangent line and normal plane to the curve

$$x^2 + 2y^2 + 3z^2 = 3$$
$$2x + 3y + 4z = 5$$

at (1, 1, 0). 6

4. (a) Find the volume of the solid enclosed between the surfaces $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. 6

(b) Find the absolute maximum and minimum values of the function $f(x, y) = 3x^2 + y^2 - x$ over the region $2x^2 + y^2 \le 1$. 8

(c) The part of the laminscate $r^2 = a^2 \cos 2\theta, 0 \le \theta \le \pi/4$ is revolved about the x-axis. Find the surface area of the solid generated. 6

Group B

5. (a) By constructing a difference table and taking second order differences as constants, find the sixth term of the series

8, 12, 19, 29, 42, ... 8

(b) Find the value of

$$\int_1^5 \log_{10} x\, dx$$

taking 8 sub-intervals correct to 4 decimal places by Trapezoidal rule of integration. 6

(c) From the values given in the table, compute dy/dx for $x = 1$:

x	1	2	3	4	5	6
y	1	8	27	64	125	216

6

6. (a) Using Lagrange's interpolation formula, find a polynomial which passes through the points (0, - 12), (1, 0), (3, 6), (4, 12). 6

(b) Solve

$$xy\frac{dy}{dx} = \frac{1+y^2}{1+x^2}(1+x+x^2)$$ 6

(c) Let a random sample of 10 boys has the following IQ:

70, 120, 110, 101, 88, 83, 95, 98, 107, 100

Do these data support the assumption of a population mean IQ of 100? 8

7. (a) Let T be a linear transformation on R_4 defined by

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_2 + x_3 \\ x_1 + x_2 \\ x_1 \\ x_1 - x_2 \end{pmatrix}$$

Describe the image of T, denoted by $V(T)$, and determine its rank. 6

(b) If $Z = ax + by$ and r is the correlation coefficient between x and y, show that

$$\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\gamma\sigma_x\sigma_y.$$

Hence, prove that

$$\gamma = \left(\sigma_x^2 + \sigma_y^2 - \sigma_{xy}^2\right)/2\sigma_x\sigma_y.$$ 8

(c) If $x = 4y + 5$ and $y = kx + 4$ are the regression lines of x on y and y on x respectively, show that $0 \le k \le 1/4$ 6

8. (a) Find the general solution of the differential equation

$$(D^2 + 1)y = \text{cosec } x.$$ 6

(b) Solve

$$y'' - ty' + y = 1$$
$$y(0) = 1, \; y'(0) = 2.$$ 6

(c) Solve the system of equations

$$\frac{d^2x}{dt^2} + \frac{dy}{dt} = e^{2t}$$

$$\frac{dx}{dt} + \frac{dy}{dt} - x - y = 0.$$ 8

Group C

9. Answer the following: 2 × 10

(i) Suppose T_1 and T_2 are two rotation linear transformations which rotate vectors counter clockwise through angles θ_1 and θ_2 about the origin respectively. Find the matrices of $T_1 T_2$ and $T_2 T_1$.

(ii) Obtain the expansion of

$$f(x) = 1/(x-2)(x-3)$$

in powers of $(x - 1)$.

(iii) Determine the eigenvalues of the matrix A and then diagonalize it:

$$A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

(*iv*) Solve

$$x^2 y'' + 2xy' = 0.$$

(*v*) Test whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

(*vi*) If Δ be the forward difference operator; δ, the central difference operator; and E, the shift operator, then show that

$$E^{1/2}\delta = \Delta$$

(*vii*) Find the directional derivative of

$$u(x, y) = x^2 + y^2$$

at $(1, 1)$ in the direction making an angle $30°$ with the positive direction of x-axis.

(*viii*) If $\vec{r}$ is a position vector of any point in three-dimensional space, find ∇r^n.

(*ix*) Test the linear dependence or independence of the given vectors

$$\{(4, 0, 2), (2, 2, 0), (1, 1, 0)\}.$$

(*x*) Show that in the curve $y = be^{-(a/x)}$, the subtangent varies as the square of the abscissa.

S'08 : 5 FN : AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

All parts of a question (a, b, etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answers may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) If $y = \tan^{-1} x$, show that

$$(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$$ 5

(*b*) Is the mean value theorem valid for the function $f(x) = x^2 + 3x + 2$ in $1 \leq x \leq 2$. Find ξ, if the theorem is applicable. 4

(*c*) Obtain the Taylor's polynomial expansion of the function $f(x) = \sin x$ about the point $x = \pi/4$. Further, show that the error tends to zero as $n \to \infty$. 6

(*d*) Is the series

$$\sum_{1}^{\infty} \frac{1}{(2n+1)\sqrt{n}}$$

convergent ? Name the series with which you compare the given series to test its convergence. 5

2. (*a*) Suppose that u and v are two functions of x and y and satisfy the relations

$$\left.\begin{aligned} u^2 - v^2 &= x \\ 2uv &= y \end{aligned}\right\}$$

Then compute $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$, assuming that they exist. 6

(*b*) Let $r = \sqrt{x^2 + y^2}$ be the radius vector from the origin to the point (x, y) in the xy-plane and θ be the angle which r makes with x-axis. Show that

(*i*) $\frac{\partial^2 r}{\partial y^2} = \frac{\cos^2\theta}{r}$ 3

(*ii*) $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left\{\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right\}$ 3

(*c*) Find the directional derivative of the function $f = x^4 + x^3 y + z^3$ at the point $P(1, 2, 5)$ towards the point $(5, 0, 8)$. 8

3. (*a*) The cylinder $x^2 + z^2 = 1$ is cut by the planes $y = 0$, $z = 0$ and $x = y$. Find the volume of the region in the first octant. 6

(*b*) Find the minimum value of $(x^2 + y^2 + z^2)$ subject to the condition $xyz = a^3$. 6

(*c*) Verify the Green's theorem for $f(x, y) = e^{-x}\sin y$, $g(x, y) = e^{-x}\cos y$ and the contour C is the square with vertices at $(0,0)$, $(\pi/2, 0)$, $(\pi/2, \pi/2)$, $(0, \pi/2)$. 8

4. (*a*) Show that the vectors $\vec{v}_1 = (1, -1, 0)$, $\vec{v}_2 = (0, 1, -1)$, $\vec{v}_3 = (0, 2, 1)$ and $\vec{v}_4 = (1, 0, 3)$ are linearly dependent. 5

(*b*) If $u = xf(y/x) + g(y/x)$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0.$$ 5

(*c*) Reduce the matrix

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{bmatrix}$$

to row echelon form and determine its rank. 5

(*d*) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$ 5

Group B

5. (a) Solve the differential equation

$$(5x^3 + 12x^2 + 6y^2)\, dx + 6xy\, dy = 0.$$ 5

(b) Solve the differential equation

$$\cot 3x \frac{dy}{dx} - 3y = \cos 3x + \sin 3x\,;\ 0 < x < \pi/2.$$ 5

(c) Solve the first order partial differential equation

$$x(y^2 - z^2)\frac{\partial z}{\partial x} + y(z^2 - x^2)\frac{\partial z}{\partial y} = z(x^2 - y^2).$$ 5

(d) Find the Laplace transform of the function $(t \sin t)$ and use the result to evaluate $\int_0^\infty t\, e^{-3t} \sin t\, dt$. 5

6. (a) Using $F(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-iwx}\, dx$ as the definition of Fourier transform of $f(x)$, show that Fourier transform of $e^{-4ix} f(x)$ equals $F(w+4)$. 6

(b) Solve the differential equation

$$y''(x) + 3y'(x) + 2y(x) = e^x$$

for its general solution, using the method of variation of parameters. 6

(c) Solve the initial value problem

$$y''(t) + y(t) = 2\cos t\,;\ y(0) = 0,\ y'(0) = -1$$

using Laplace transform procedure. 8

7. (a) Write down the (i) probability distribution function (PDF), (ii) mean, and (iii) variance for the following selected distributions: (i) Binomial (discrete), (ii) Poisson (discrete) and the Gaussian (continuous). 8

(b) (i) If S^2 is the variance of a random sample of size n taken from a normal population having the variance σ^2, then write down the expression for the random variable having the chi-square distribution with a parameter $\upsilon = n - 1$. 3

(ii) State the central limit theorem. 3

(c) If a one gallon can of paint covers on the average 513·3 square feet with a standard deviation of 31·5 square feet, what is the probability that the sample mean area covered by a sample of 40 of these one gallon cans will be anywhere from 510·0 square feet to 520·0 square feet. 6

8. (a) Using Lagrange polynomial, find the value of $f(x)$ at $x = 4$, given the table of x versus $f(x)$ as

x :	1·5	3	6
$f(x)$:	−0·25	2	20

6

(*b*) Consider the function $f(x)$ given by

x :	0·1	0·2	0·3	0·4	0·5	0·6
$f(x)$:	0·425	0·475	0·400	0·450	0·575	0·675

Use composite Trapezoidal rule to evaluate the integral

$$\int_{0.1}^{0.6} [f(x)]\, dx.$$ 6

(*c*) Evaluate the integral

$$\int_{-1}^{1} x^2 e^{-x}\, dx$$

by composite Simpson's one-third rule with spacing $h = 0.25$. You may use any other known numerical method to evaluate the given integral. 8

Group C

9. Answer the following: 2×10

(*i*) If $y = \sin(2\sin^{-1} x)$, then find the value of $(1 - x^2)y_2 - xy_1$ in terms of y.

(*ii*) Determine the region, in which the function

$z = \sqrt{(1 - x^2 - y^2)}$ is defined.

(*iii*) If $f(x, y) = \dfrac{x + y - 1}{x + y + 1}$, then evaluate $\left(\dfrac{\partial f}{\partial x}\right)$ at the point (2, 1).

(*iv*) Find the gradient of the scalar field

$f(x, y) = y^2 - 4xy$ at the point (1, 2).

(*v*) Find the solution of the differential equation $y\,dx - x\,dy + e^{1/x}\,dx = 0$ by suitably choosing an integrating factor.

(*vi*) Write down the Laplace transform of

$f(t) = e^{-3t}\cos 3t$.

(*vii*) What is the total number of arithmetic operations required to interpolate by Lagrangian form ?

(*viii*) A bag contains 10 white balls and 15 black balls. Two balls are drawn in succession. What is the probability that one of them is black and the other white ?

(*ix*) Distinguish between 'round-off error' and 'truncation error'.

(*x*) $X_1, \ldots, X_n$ are independent normal random variables, each of which has mean μ and variance σ^2. What is the mean and variance of the random variable

$$Z = \sqrt{n}\,\frac{\bar{X} - \mu}{\sigma}$$

where the symbols have their usual meanings.

W'08 : 5 FN : AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

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Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answer may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) Show that if $0 < u < v$, then

$$(v-u)/(1+v^2) < \tan^{-1} v - \tan^{-1} u < (v-u)/(1+u^2)$$

and deduce that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}(4/3) < \frac{\pi}{4} + \frac{1}{6}.$$ 8

(*b*) Examine the function $(x-3)^5 (x+1)^4$ for extreme values. 8

(*c*) If $y = (\tan^{-1} x)^2$, prove that

$$(1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2.$$ 4

2. (*a*) If the normal to the curve

$$x^{2/3} + y^{2/3} = a^{2/3}$$

makes an angle θ with x-axis, show that its equation is

$$y\cos\theta - x\sin\theta = a\cos 2\theta.$$ 8

(*b*) Solve the following system of linear equations

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

by Gauss elimination method. 6

(*c*) Find the unit tangent vector, $\vec{t}$, for the curve

$$\vec{r} = t^2\,\vec{i} + t\,\vec{j} + (2t+3)\,\vec{k}$$ 6

3. (*a*) Prove that the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

is $8abc/3\sqrt{3}$. 8

(*b*) Evaluate $\iint_R f(x,y)\; dxdy$ over the rectangle $R = [0,1;0,1]$, where

$$f(x,y) = \begin{cases} x+y, & \text{if } x^2 < y < 2x^2 \\ 0, & \text{elsewhere} \end{cases}$$ 6

(*c*) Show that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

for $-1 < x \le 1$. 6

4. (*a*) Prove that the line integral

$$\int_C \frac{xdy - ydx}{x^2+y^2}$$

taken in the positive direction over any closed contour with the origin inside it is equal to 2π. 8

(*b*) Find the equation of the tangent plane and normal line to the surface $x^2y + xz^2 = z - 1$ at the point $(1,-3,2)$. 6

(*c*) Find the unit tangent vector $\vec{t}$ for the curve

$$r = t^2\,\vec{i} + t\,\vec{j} + (2t+3)\,\vec{k}.$$ 6

Group B

5. (*a*) Solve the differential equation

$$\frac{d^2x}{dt^2} = 3x + 4y$$

$$\frac{d^2y}{dt^2} = -x - y$$ 8

(*b*) Solve the differential equation

$$y(2xy + e^x)\,dx - e^x dy = 0.$$ 6

(*c*) The table below gives values of $\tan x$ for $0{\cdot}1 \le x \le 0{\cdot}3$:

0·1	0·15	0·2	0·25	0·3
0·1003	0·1511	0·2027	0·2553	0·3093

Use Newton's forward difference interpolation formula to obtain the values of (*i*) $\tan 0{\cdot}12$, and (*ii*) $\tan 0{\cdot}5$. 6

6. (*a*) Evaluate the value of the integral

$$I = \int_0^1 \frac{1}{1+x}\,dx$$

correct up to three decimal places by using Simpson's one-third rule with step size $h = 0{\cdot}25$. 8

(*b*) Solve the differential equation

$$(D^4 - 2D^3 + D^2)\,y = x^3$$

where $D = d/dx$. 6

(*c*) It has been claimed that in 60% of all solar heat installations the utility bill is reduced by at least one-third. Accordingly, what are the probabilities that the utility bill will be reduced by at least one-third in (*i*) four of five installations; and (*ii*) at least four of five installations? 6

7. (*a*) Find the mean and standard deviations of the following data: 6

19·7	21·5	22·5	22·2	22·6
21·9	20·5	19·3	19·9	21·7
22·8	23·2	21·4	20·8	19·4
22·0	23·0	21·1	20·9	21·3

(*b*) Use Lagrange's interpolation formula to find the unique polynomial of degree 2 or less such that

$$f(0) = 1,\ f(1) = 3 \text{ and } f(3) = 55.$$ 6

(*c*) Solve the equation by using Laplace transform

$$x' + 2x - 3y = t$$

$$y' - 3x - 2y = e^{2t}$$

which satisfies the conditions $x(0) = y(0) = 0$. 8

8. (*a*) If $D = \partial/\partial x$ and $D' = \partial/\partial y$, then solve the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 0$$ 6

(*b*) Solve the differential equation

$$(D^4 + 2D^2 + 1)\,y = x^2 \cos x.$$ 6

(c) The following are measurements of the air velocity and evaporation coefficients of burning fuel droplets in an impulse engine:

Air Velocity (cm/s)		Evaporation Coefficients (mm²/s)	
x	x	y	y
20	220	0·18	0·75
60	260	0·37	1·18
100	300	0·35	1·36
140	340	0·78	1·17
180	380	0·56	1·65

Construct a 95% confidence interval for the mean evaporation coefficient, when the air velocity is 190 cm/s. 8

Group C

9. Answer the following: 2×10

(i) Obtain the inverse transform of

$$f(p) = (p+2)/(p-1)^2 p^3.$$

(ii) Solve $(dy/dx) = (x+y+4)/(x-y-6)$.

(iii) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, find the value of grad $(\log|\vec{r}|)$.

(iv) Find the constants a, b and c so that

$$\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy-2z)\vec{k}$$

is irrotational.

(v) Find the particular integral of

$$(D^2+4)y = x\sin x.$$

(vi) Find the directional derivative of

$$\varphi = x^2yz + 4xz^2$$

at $(1,-2,-1)$ in the direction of $2\vec{i} - \vec{j} - 2\vec{k}$.

(vii) Find the mean and median of 15, 14, 2, 27, 13.

(viii) If a random variable has the probability density

$$f(x) = \begin{cases} 2e^{-2x}, & \text{for } x > 0 \\ 0, & \text{for } x \leq 0 \end{cases}$$

find the probability that it will take on a value between 1 and 3.

(ix) Show that

$$\mu = \sqrt{1 + \delta^2/4}$$

where μ and δ are usual notation.

(x) Solve the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} + 6\frac{\partial^3 z}{\partial y^3} = 0.$$

S'09 : 5 FN : AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

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Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) If $x = \tan(\log y)$, prove that

$$(1 + x^2)\, y_n = \{1 - 2(n-1)\, x\}\, y_{n-1} - (n-1)(n-2)\, y_{n-2}$$ 6

(*b*) Obtain, by Maclaurin's theorem, the first four terms of the expansion of $e^{x\cos x}$. Hence or otherwise find the value of

$$\lim_{x \to 0} \left(\frac{e^x - e^{x\cos x}}{x - \sin x}\right).$$ 8

(*c*) If $f(x, y) = (1 - 2xy + y^2)^{-1/2}$, show that

$$\frac{\partial}{\partial x}\{(1 - x^2)\frac{\partial f}{\partial x}\} + \frac{\partial}{\partial y}(y^2 \frac{\partial f}{\partial y}) = 0$$ 6

2. (a) Find the maximum or minimum value of the function

$$f(x,y) = (x^2 + y^2)\, e^{-(x^2+y^2)}$$

for points not on the circle $x^2 + y^2 = 1$. 6

(b) If $I_{m,n} = \int \cos^m x \sin nx\, dx$, prove that

$$I_{m,n} = -\frac{\cos^m x \cos nx}{m+n} + \frac{m}{m+n} I_{m-1,\, n-1}$$

Hence or otherwise, evaluate

$$\int_0^{\pi/2} \cos^5 x \sin 3x\, dx$$

8

(c) Find the inner products $<\vec{x}, \vec{y}>$, $<\vec{y}, \vec{x}>$, the norms $\|\vec{x}\|$, $\|\vec{y}\|$, the normalized $\vec{x}$, $\vec{y}$ and the distance between $\vec{x}$ and $\vec{y}$, where

$$\vec{x} = [2, i, 3]^T, \quad \vec{y} = [1+2i, 2i, 1]^T$$

6

3. (a) Find the volume generated by revolving about the axis of x. The area bounded by the curves

$$x^2 + y^2 = 25,\; 3x - 4y = 0,\; y = 0$$

lying in the first quadrant. 5

(b) Find a linear transformation $T: R^2 \to R^2$ such that $T(1,0) = (1,1)$ and $T(0,1) = (-1,2)$. Also, prove that T maps the square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$ into a parallelogram. 5

(c) Using matrix method, show that the equations

$$3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$$
$$2x_1 + 5x_2 + 5x_3 - 18x_4 = 9$$
$$4x_1 - x_2 - x_3 + 8x_4 = 7$$

are consistent and hence solve them. 5

(d) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that

(i) div $(r^n \vec{r}) = (n+3)\, r^n$, and

(ii) $\vec{r} / r^3$ is solenoidal.

5

4. (a) Solve the following system of equations by Gauss elimination method: 6

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

(b) The eigenvectors of a 3×3 matrix A corresponding to the eigen values 1, 2, 3 are $[-1, -1, 1]^T$, $[0, 1, 0]^T$ and $[0, -1, 1]^T$, respectively. Find the matrix A. 6

(c) Show that the line integral

$$\int_C (3x^2\, dx + 2yz\, dy + y^2\, dz)$$

is independent of the path in any domain in space and find its value, if C has the initial point $A\,(0, 1, 2)$ and terminal point $B\,(1, -1, 7)$. 8

Group B

5. (a) Solve the differential equations:

$$\frac{dy}{dx} = (4x + y + 1)^2, \text{ if } y(0) = 1.$$

6

(b) Using Laplace transform method, solve the initial value problem

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 4e^{2t},\; x = -3 \text{ and } \frac{dx}{dt} = 5 \text{ at } t = 0.$$

6

(c) The following data are taken from the steam table:

Temperature, °C:	140	150	160	170	180
Pressure, kg/cm^2:	3·685	4·854	6·302	8·076	10·225

Find the pressure at temperature $t = 142$ °C. 8

6. (a) Find general solution of the differential equation

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \sin(2x - 3)$$ 6

(b) A curve passes through the points (1, 2), (1·5, 2·4), (2, 2·7), (2·5, 2·8), (3, 3), (3·5, 2·6), (4, 2·1) and (4·5, 1·9)
Use Simpson's one-third rule to obtain the volume of the solid of revolution got by revolving this area about x-axis. 6

(c) Find the Fourier sine transform of $e^{-|x|}$. Hence, show that

$$\int_0^\infty \frac{x \sin x}{1 + x^2} dx = \frac{\pi}{2} e^{-m}, \; m > 0.$$ 8

7. (a) Using Lagrange's interpolation formula, find the missing term in the table:

x:	2	5	8	14
y:	94·8	87·9		68·7

6

(b) If the proportions of a brand of television set requiring service during the first year of operation is a random variable having a beta distribution with $\alpha = 3$ and $\beta = 2$, what is the probability that at least 80% of the new models sold this year will service during the first year of operation? 6

(c) The finished inside diameter of a piston ring is normally distributed with a mean of 10 cm and a standard deviation of 0·03 cm. (i) What proportion of ring will have inside diameters exceeding 10·075 cm? (ii) What is the probability that a piston ring will have an inside diameter between 9·97 cm and 10·03 cm?
(Given: $\phi(2\cdot5) = 0\cdot9938$, $\phi(1) = 0\cdot8413$, where

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2} dx.$$ 8

8. (a) Find the general solution of the partial differential equation

$$(x^2 - yz)\frac{\partial z}{\partial x} + (y^2 - zx)\frac{\partial z}{\partial y} = z^2 - xy$$ 6

(b) Find the temperature distribution $u(x, t)$ in a laterally insulated bar 80 cm long, if the initial temperature is 100 sin $(\pi x / l)$ °C and the ends are kept at 0°C. How long will it take for the maximum temperature in the bar to drop to 50°C? Given that

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

where $c^2 = 1\cdot158$ cm^2/sec. 8

(c) For determining the ash contents of six samples of coal, each sample was divided into two parts and a part was given to each of two analysts A and B. The percentage ash contents found by them are given below sample-wise:

Sample no.:	1	2	3	4	5	6
Analyst A:	8	10	11	9	12	13
Analyst B:	9	13	12	10	11	12

Do the two analysts differ significantly? Given 5% calculated value of t for d.f. 5 is 2·23. 6

Group C

9. Answer the following: 10 × 2

(*i*) If $f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(\theta x)$, find the value of θ as x tends to 1, $f(x)$ being $(1-x)^{5/2}$.

(*ii*) Find the magnitude of the vector drawn perpendicular to the surface $x^2 + 2y^2 + z^2 = 7$ at the point $(1, -1, 2)$.

(*iii*) Two eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

are equal to 1 each. Find the eigenvalues of A^{-1}.

(*iv*) If $u = x^2 \tan^{-1}(y/x) - y^2 \sin^{-1}(x/y)$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$

(*v*) Find the inverse Laplace transform of $\log[(1+s)/s]$.

(*vi*) Evaluate

$$\iint xy\, dxdy$$

over the area between $y = x^2$ and $y = x$.

(*vii*) Prove that

$$\left(\frac{\Delta^2}{E}\right) u_x \neq \frac{\Delta^2 u_x}{E u_x}$$

(*viii*) If the regression equations of two variables x and y are $x = 3 - y/4$ and $y = 5 - 2x$, find the correlation coefficient between them.

(*ix*) If the probabilities of happening of some events are $p_1, p_2, \ldots, p_n$, then find the probability of happening of at least one of them.

(*x*) If a random variable X has a Poisson distribution such that $P(X=1) = P(X=2)$, then find $P(X=4)$.

W'09 : 5 FN : AN 209 (1409)

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Group A

1. (*a*) If $y(x) = \sin(\alpha \sin^{-1} x)$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-\alpha^2)y_n = 0$$

where y_r represents r^{th} order derivative of $y(x)$. 8

(*b*) Is the mean-value theorem valid for the function $f(x) = x^2 + 3x + 2$ in $1 \leqslant x \leqslant 2$? Find a point $x = c$ in $[1, 2]$, if the theorem is applicable. 6

(*c*) Find the maximum and minimum values of the function $f(x) = xy(a - x - y)$ at the point $(0, a)$. 6

2. (*a*) Compute the area of the figure bounded by the parabolas $x = -2y^2$ and $x = 1 - 3y^2$. 8

(*b*) Find the volume of the solid generated by revolving the figure bounded by the parabola $y^2 = 8x$ and the straight line $x = 2$, about the line $y = -4$. 6

(*c*) Find the centre of gravity of a uniform lamina in the form of a quadrant of the ellipse $x^2/a^2 + y^2/b^2 = 1$. 6

3. (*a*) Prove that the series $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ is divergent. 6

(*b*) Investigate the value of x for which the ratio-test fails while examining the convergence of the series

$$\frac{1}{1^p} + \frac{x}{3^p} + \frac{x^2}{5^p} + \ldots + \frac{x^{n-1}}{(2n-1)^p} + \ldots$$ 6

(*c*) (*i*) Find the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ when $f(x, y) = \tan^{-1}(y/x)$. 4

(*ii*) If $u = e^{xyz}$, examine that the result

$$\partial^3 u/\partial x\,\partial y\,\partial z = (1 + 3xyz + x^2y^2z^2)\, e^{xyz}$$

is correct. 4

4. (*a*) Obtain the reduction formulae for

(*i*) $\int_0^1 x^m (1-x)^n\, dx$, and

(*ii*) $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$, where m and n are positive integers. 8

(*b*) If $u = xf(y/x) + g(y/x)$, then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x\,\partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0.$$ 8

(*c*) If $u = f(x-y, y-z, z-x)$, then prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$ 4

Group B

5. (*a*) Divide a given positive number N into three parts (positive) such that their sum is N and product is maximum. 6

(*b*) Find a point within a triangle such that the sum of squares of its distances from three vertices is minimum. 6

(*c*) At what point on the curve $xy = 20$ is the tangent parallel to the line $5x + y = 1$? 8

6. (*a*) Verify whether the two integrals

$$I_1 \equiv \int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3}\, dy$$

and

$$I_2 \equiv \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3}\, dx$$

are equal. 8

(*b*) Evaluate $\iint_R \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) dx\, dy$

where R consists of points in the positive quadrant of the ellipse $x^2/a^2 + y^2/b^2 = 1$. 8

(*c*) If $u = x^2 - y^2 - z^2 - 2$, find the gradient ∇u at the point $(1, -1, 2)$. 4

7. (*a*) Show that $f = (2xy + z^3)i + x^2 j + 3xz^2 k$ is a conservative force field. Also, find the (*i*) scalar potential ϕ, and (*ii*) work done by the force f in moving an object from $(1, -2, 1)$ to $(3, 1, 4)$. 10

(*b*) Use Green's theorem to evaluate

$$\int_C (x^2 y\,dx + y^3\,dy),$$

where C is the closed path formed by $y = x$ and $y = x^3$ from $(0, 0)$ to $(1, 1)$. 10

8. (*a*) Use Stokes' theorem to evaluate

$$\int_C (e^x\,dx + 2y\,dy - dz)$$

where C is the curve $x^2 + y^2 = 4$, $z = 2$. 6

(*b*) Find the eigenvalues and eigenvectors of the matrix 6

$$A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

(*c*) Determine for what values of k the system $x + 2y = 1$; $5x + ky = 5$ has (*i*) unique solution, and (*ii*) infinite number of solutions. 8

Group C

9. Answer the following: 10×2

(*i*) Evaluate $\int_0^{\pi/2}\int_0^{\pi} \cos(x + y)\,dx\,dy$.

(*ii*) Find the sum of the eigenvalues of

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 2 & 4 \\ 1 & 2 & 7 \end{bmatrix}$$

(*iii*) If $f = \nabla(x^3 + y^3 + z^3 - 3xyz)$, find Curl f.

(*iv*) If $\nabla\phi$ is solenoidal, find the value of $\nabla^2\phi$.

(*v*) Find the volume generated by revolving about OX, the area bounded by $y = x^3$ between $x = 0$ and $x = 2$.

(*vi*) Let $y(x) = (2 - 3x)^{10}$, then evaluate d^9y/dx^9.

(*vii*) Write a reduction formula for $\int x^n e^{2x}\,dx$.

(*viii*) The infinite series $\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\right)$ is divergent. Is this statement true or false.

(*ix*) If Curl $F = 0$, then F is called a solenoidal vector. Is this statement true or false?

(*x*) When a system of linear algebraic equations is said to be consistent for solution?

S'10 : 5 FN : AN 209 (1409)

ENGINEERING MATHEMATICS

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Group A

1. (*a*) If $x = \cosh(\log y/m)$, prove that

$$(x^2 - 1)y_2 + xy_1 = m^2 y. \quad 8$$

(*b*) If $y = l^{a \sin^{-1} x}$, prove that

$$(1 - x^2)y_{n+2} - (2n+1)\, xy_{n+1} - (n^2 + a^2)y_n = 0. \quad 6$$

(*c*) Find the area of the region lying above x-axis, and included between the circle $x^2 + y^2 = 2ax$ and the parabola $y^2 = ax$. 6

2. (*a*) Test for the convergence of series

$$\frac{1}{2\sqrt{2}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^5}{5\sqrt{4}}+\ldots+\infty$$ 6

(*b*) Find the volume of solid obtained by revolving the cardiod $r=(1+\cos\theta)$ about the initial line. 8

(*c*) Show that maximum and minimum values of $r^2=x^2+y^2$, where $ax^2+2hxy+by^2=1$, are given by the roots of the equation

$$[a-(1/r^2)][b-(1/r^2)]=h^2$$ 6

3. (*a*) Show that all the points on the curve

$$y^2=4a[x+a\sin(x/a)]$$

at which the tangent is parallel to x-axis be on a certain parabola. 6

(*b*) Apply Green's theorem in the plane to evaluate

$$\int_C[(2x^2-y^2)\,dx+(x^2+y^2)\,dy]$$

where C is boundary of the curve enclosed by x-axis and the semi-circle $y=(1-x^2)^{1/2}$. 6

(*c*) Evaluate the integral $\int\vec{F}\cdot d\vec{r}$, along the curve $x^2+y^2=1$, $z=1$ in the positive direction from $(0,1,1)$ to $(1,0,1)$ for $\vec{F}$ given by

$$(2x+yz)\vec{i}+xz\vec{j}+(xy+2z)\vec{k}.$$ 8

4. (*a*) Solve the following system of linear equations by Gauss elimination method:

$$x-2y-3z=-1$$
$$2x-y-2z=2$$
$$3x-y-3z=3$$ 6

(*b*) Find the eigenvalues and eigenvectors of the matrix

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$ 8

(*c*) A vector function $\vec{F}$ is a product of a scalar function and the gradient of a scalar function. Show that

$$\vec{F}\cdot\text{curl}(\vec{F})=0.$$ 6

Group B

5. (*a*) In a population, the probability of selecting (*i*) a male or a smoker is 7/10; (*ii*) a male smoker is 2/5; and (*iii*) a male if a smoker is already selected is 2/3. Find the probability of selecting a male. 6

(*b*) Solve the following differential equations: 3+3

(*i*) $y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$

(*ii*) $\frac{dy}{dx}=\frac{2y-x-4}{y-3x+3}$.

(*c*) A survey of 800 families, with four children each, revealed the following distribution:

No. of boys:	0	1	2	3	4
No. of girls:	4	3	2	1	0
No. of families:	32	178	290	236	64

Is this result consistent with the hypothesis that male and female births equal probable? Assume that the tabulated $\chi^2_{0.5}$ for 4 degrees of freedom (d.f.) is 9.488. 8

6. (a) In a Poisson frequency distribution, the frequency corresponding to three successes is 2/3 times the frequency corresponding to four successes. Find the mean and standard deviation (s.d.) of the distribution. 6

(b) The following is a table of values of a polynomial of degree 5. It is given that $f(3)$ is in error. Correct the error. 8

x	0	1	2	3	4	5	6
$y = f(x)$	1	2	33	254	1025	3126	7777

(c) Solve the following differential equation:

$$x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log_e x.$$ 6

7. (a) Using a Lagrange's interpolation formula, find a polynomial which passes through the points (0, – 12), (1, 0), (3, 6) and (4, 12). 6

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4y = x \sin x$$ 6

(c) Use Simpson's one-third rule to find the value of the integral

$$\int_0^{\pi/3} e^{\sin x}\, dx$$

correct up to four decimal places. Compare the results with that obtained by Trapezoidal rule of integration. 8

8. (a) From the following table of values of x and y, obtain dy/dx for $x = 1.2$: 8

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.025

(b) Solve the following equation:

$$\frac{dy}{dx} + \frac{xy}{1-x^2} = xy^{1/2}$$ 6

(c) Solve the differential equation

$$x\,dx + y\,dy + (x\,dy - y\,dx)/(x^2 + y^2) = 0$$ 6

Group C

9. Answer the following: 10 × 2

(i) Prove that between any two real roots of $e^x \sin x = 1$, there exists at least one root of $e^x \cos x + 1 = 0$.

(ii) If x, y, z are connected to equations $\phi(x, y, z) = 0$ and $\psi(x, y, z) = 0$, find dy/dx.

(iii) Solve $(x + y)\,dx + dy = 0$.

(*iv*) Solve $(x^2 + y^2)\,dx - 2xy\,dy = 0$.

(*v*) Find the inverse Laplace transform of

$$(s-1)/(s^2 - 6s + 25).$$

(*vi*) Find one vector in R^3 which generates the intersection of V and W, where

$$V = \{(a, b, 0) \mid a, b \in R\}$$

and W be the space generated by the vectors

$$(1, 2, 3) \text{ and } (1, -1, 1).$$

(*vii*) Let $A = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}$ be the matrix over the real field R. Find eigenvalues and linearly independent eigenvectors.

(*viii*) If $\vec{f} = f(x, y, z, t)$, show that

$$d\vec{f} = (d\vec{r}\,.\nabla)\,\vec{f} + (\partial \vec{F}/\partial t)\,dt$$

(*ix*) Use Green's theorem to evaluate

$$\int_C (x^2 + xy)\,dx + (x^2 + y^2)\,dy$$

where C is the square formed by the lines $x = \pm 1$ and $y = \pm 1$.

(*x*) Find the shortest distance from the point $(3, 0)$ to the parabola $y = x^2$.

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Group A

1. (*a*) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$ 8

(*b*) Evaluate

$$\int_C [(x+y)\,dx - x^2\,dy + (y+z)\,dz]$$

where C is $x^2 = 4y,\ z = x,\ 0 \leq x \leq 2$. 6

(*c*) Find the surface area of the solid generated by revolving the curve $x^2 + (y-b)^2 = a^2$ about x-axis. 6

2. (*a*) Find the normal vector and the equation of tangent plane to the surface $z=\sqrt{x^2+y^2}$ at the point (3,4,5). Also, find the divergence of the vector field $(x^2y^2-z^3)i+(2xyz)j+(e^{xyz})k$. 8

(*b*) Verify whether the vectors $x_1=(1\ 3\ 4\ 2)$, $x_2=(3\ -5\ 2\ 2)$ and $x_3=(2\ -1\ 3\ 2)$ are linearly-dependent. If so, express one of these vectors as a linear combination of others. 6

(*c*) Find the total length of the curve $r=a\sin^3(\theta/3)$. 6

3. (*a*) Discuss the convergence of the following series: 2 × 4

(*i*) $1+\frac{2!}{2^2}+\frac{3!}{3^3}+\frac{4!}{4^4}+....$

(*ii*) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt{n+1}}$

(*b*) Apply Green's theorem to evaluate

$$\int_C[(2x^2-y^2)\,dx+(x^2+y^2)\,dy]$$

where C is the boundary of the area enclosed by the x-axis and upper-half of the circle

$$x^2+y^2=a^2.$$ 6

(*c*) Find the value(s) of λ for which the equations

$$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$$
$$(\lambda-1)x+2(2\lambda-1)y+(\lambda+3)z=0$$
$$2x+(3\lambda+1)y+3(\lambda-1)z=0$$

have non-trivial solution(s). Find the ratios $x:y:z$ when λ has the smallest of these values. What happens when λ has the greatest of these values? 6

4. (*a*) Find the shortest distance between the line $2x+y=10$ and the ellipse $9x^2+4y^2=36$. 10

(*b*) Evaluate the integral

$$\iint_R e^{x^2}\,dx\,dy$$

where the region R is given by $R: 2y\le x\le 2$ and $0\le y\le 1$. 10

Group B

5. (*a*) Solve the following differential equations: 2 × 7

(*i*) $(1+y^2)\,dx=(\tan^{-1}y-x)\,dy$

(*ii*) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$

(*b*) Use difference operator technique to get missing values in the following table: 6

x	45	50	55	60	65
y	3.0	—	2.0	—	-2.4

6. (*a*) Solve

$$\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0,\ y=0 \text{ when } x=0.$$ 6

(*b*) Solve the equation

$$y'' + 3y' + 2y = 2e^x$$

by using the method of variation of parameters. 8

(*c*) Find the Fourier series expansion of the periodic function

$$f(x) = x,\ -\pi \leq x \leq \pi,\ f(x + 2\pi) = f(x).$$ 6

7. (*a*) Find the cubic polynomial which takes the following values:

x	0	1	2	3
$f(x)$	1	2	1	10

Hence or otherwise, evaluate $f(4)$. 8

(*b*) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.
[Given: $P(z \geq 0.5) = 0.31$, $P(z \leq 1.4) = 0.92$, where z is standard normal variate.] 6

(*c*) Given that

x	1.1	1.2	1.3	1.4	1.5	1.6
y	8.403	8.781	9.129	9.451	9.750	10.031

Find dy/dx at $x = 1.1$. 6

8. (*a*) The data given below show the test scores made by 10 salesmen on an intelligence test and their weekly sales:

Salesman	1	2	3	4	5	6	7	8	9	10
Test Score	40	70	50	60	80	50	90	40	60	60
Sales (in '000)	2.5	6.0	4.5	5.0	4.5	2.0	5.5	3.0	4.5	3.0

Calculate the regression line of sales on test scores and estimate the most probable weekly sales if a salesman makes a score of 70. 8

(*b*) Given the values:

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

Evaluate $f(9)$ using Lagrange's formula. 6

(*c*) Evaluate

$$\int_0^6 dx/(1 + x^2)$$

by using Trapezoidal rule. Take $h = 1$. 6

Group C

9. Answer the following: 10×2

(*i*) A binomial distribution has mean 20 and standard deviation 4. Find the parameters of the distribution.

(*ii*) Verify whether the vector field

$$v = xyz\,(yzi + xzj + xyk)$$

is conservative.

(*iii*) Find the value(s) of k for which the vectors $(1, k, 5)$, $(1, -3, 2)$, $(2, -1, 1)$ will form a basis in $\mathbb{R}^3$.

(*iv*) Verify whether the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ is diagonalizable.

(*v*) Consider the function $f(x, y) = \tan^{-1}(y/x)$. Find the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$.

(*vi*) Evaluate $\lim_{x \to 0} [\log_e (1 + x)]/\sin x$.

(*vii*) Find the coefficient of x^3 in the expansion of $e^{\sin x}$.

(*viii*) Find the Laplace transform of the function $e^{-2t} . t^3$.

(*ix*) Obtain the function whose first difference is $9x^2 + 11x + 5$.

(*x*) Find complementary function of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \sec 2x.$$

S'12: 7FN: AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

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All parts of a question (a,b,etc.) should be answered at one place.

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Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) Discuss the continuity of the following function at $x = a$: **6**

$$f(x) = \begin{cases} (x-a)\sin(1/x-a), & \text{when } x \neq a \\ 0, & \text{when } x = a \end{cases}$$

(*b*) Discuss the convergence of the following series : **6**

$$\frac{1^2 \cdot 2^2}{\lfloor 1} + \frac{2^2 \cdot 3^2}{\lfloor 2} + \frac{3^2 \cdot 4^2}{\lfloor 3} + \cdots$$

(*c*) By using Taylor's theorem, show that

$$1 + x + \frac{x^2}{2} < e^x < 1 + x + \frac{x^2}{2}e^x, x > 0.$$ **8**

(*Turn Over*)

2. (*a*) Find the relative maximum and minimum of the function

$$f(x, y) = 2(x^2 - y^2) - x^4 + y^4.$$ 6

(*b*) Using Cauchy mean value theorem, find the value of C for the following pair of functions:

$$f(x) = e^x \text{ and } g(x) = e^{-x}, x \in [a, b].$$ 6

(*c*) Find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$ 8

3. (*a*) Show that the matrix

$$A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}.$$

is diagonalizable. Find P such that $P^{-1}AP$ is a diagonal matrix. 8

(*b*) Solve the following system of equations by Gauss elimination method: 6

$$2x + y - z = 4$$
$$x - y + 2z = -2$$
$$-x + 2y - z = 2$$

(*c*) Verify Stoke's theorem for the vector

$$\vec{v} = (3x - y)\vec{i} - 2yz^2\vec{j} - 2y^2z\vec{k}$$

where S is the surface of the sphere

$$x^2 + y^2 + z^2 = 16, z > 0.$$ 6

4. (*a*) Find the extreme values of

$$f(x, y, z) = 2x + 3y + z$$

such that $x^2 + y^2 = 5$ and $x + z = 1$. 8

(*b*) Evaluate the line integral of

$$\vec{v} = x^2\vec{i} - 2y\vec{j} + z^2\vec{k}$$

over the straight line from (–1, 2, 3) to (2, 3, 5). 6

(*c*) Find the eigenvalues and corresponding eigenvectors of the following matrix: 6

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

Group B

5. (*a*) Solve

$$\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}.$$ 6

(*b*) Solve

$$y'' + 3y' + 2y = x + \cos x$$

by the method of variation of parameters. 8

(*c*) Find the Lagrange's interpolating polynomial that fits the following data values: 6

x	1	2	3
$f(x)$	1	7	61

6. (*a*) Determine the step size in an equidistant table for

$$f(x) = (1 + x)^6;\ x \in [0, 2]$$

if error in magnitude in linear interpolation is 5×10^{-5}. 6

(*b*) Evaluate the following integral by Simpson's one-third rule with $n = 4$:

$$\int_0^1 \frac{1}{(3+2x)} dx$$

Compare the result with the exact solution. 8

(*c*) For two events A and B, if $P(A) = 0{\cdot}5$, $P(B) = 0{\cdot}6$ and $P(A \cap B) = 0{\cdot}8$, then find the conditional probability $P(A/B)$ and $P(B/A)$. 6

7. (*a*) Find the mean variance and standard deviation of the distribution 6

X_i	2	3	8
$f(X_i)$	1/4	1/2	1/4

(*b*) Form a partial differential equation by eliminating the arbitrary function f and g from $z = y\,f(x) + x\,g(y)$. 6

(*c*) Solve

$$\frac{dx}{dt} + 2x - 3y = t$$

and

$$\frac{dy}{dt} - 3x + 2y = e^{2t}$$ 8

8. (*a*) Solve

$$\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 x$$ 6

$$(1 + y^2)\, dx = (\tan^{-1} y - x)dy$$ 6

(*c*) Form a partial differential equation by eliminating the arbitrary function ϕ from

$$\phi(x^2 + y^2 + z^2,\ z^2 - 2xy) = 0.$$ 8

Group C

9. Solve the following : 10 × 2

(*i*) If $f(x) = (x-1)(x-2)(x-3)$, find c for $a = 0$, $b = 4$.

(*ii*) Find the first order partial derivative of the function

$$f(x, y) = x^4 - x^2y^2 + y^4$$

at (– 1, 1).

(*iii*) Find dy/dx from $f(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$.

(*iv*) Find the gradiant of $f(x, y) = y^2 - 4xy$ at (1, 2).

(*v*) Let $v_1 = (1, -1, 0)$, $v_2 = (0, 1, -1)$ and $v_3 = (0, 0, 1)$. Show that $\{v_1, v_2, v_3\}$ is linearly independent.

(*vi*) Solve

$$3e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$$

(*vii*) Solve

$$(ax + hy + g)\, dx + (hx + by + f)\, dy = 0$$

S'12:7 FN:AN 209 (1409) (5) (*Turn Over*) (*viii*) Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

(*ix*) Find the divergence of the vector

$$\vec{v} = \left(x^2y^2 - z^3\right)\vec{i} + 2xyz\,\vec{j} + e^{xyz}\,\vec{k}$$

(*x*) If A, B, C are mutually exclusive and exhaustive events associated with a random experiment and $P(B) = 0{\cdot}6\ P(A)$ and $P(C) = 0{\cdot}2\ P(A)$, find $P(A)$.

W'12: 7 FN: AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,** ANY TWO *from* **Group B** and ALL *from* **Group C.**

All parts of a question (a, b, etc.) should be answered at one place.

Answer should be brief and to-the-point and be supplemented with neat sketches. Unnecessary long answer may result in loss of marks.

Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) Let $y(x) = e^{4\sin^{-1}x}$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+4)y_n = 0.$$

Also, show that

$$(y_n)_{x=0} = \{(n-2)^2 + 4\}(y_{n-2})_{x=0}$$ 8

(*b*) Is the mean value theorem valid for $f(x) = x^2 + 3x + 2$ in the interval $1 \le x \le 2$? If the theorem is applicable, find a point C in $[1, 2]$. 6

(*c*) Use the appropriate test to examine whether the series

$$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+....+\frac{n}{2^n}+....$$

is convergent. 6

2. (*a*) Find the point at which the function $f(x)=(1/x)^x$ attains its extremum. Examine further that the maximum value of $f(x)$ is $(e)^{1/e}$. 5

(*b*) Let $u=\phi(y-z;\ z-x;\ x-y)$, then evaluate

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}.$$ 5

(*c*) Expand the function $f(x)=\tan^{-1}x$ in a finite series in powers of x, with remainder in the Lagrange's form. 5

(*d*) Determine the value of a such that divergence of

$$f=(x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$$

vanishes. 5

3. (*a*) Evaluate

$$\oint_C[(2x-y+4)\ dx+(5y+3x-6)\ dy]$$

around C: a triangle in the xy plane with vertices at (0, 0), (3, 0) and (3, 2) traversed in the counter-clockwise direction. 8

(*b*) Given $\vec{F}=(2x+3z)\hat{i}-(xz+y)\hat{j}+(y^2+2z)\hat{k}$, use Gauss's theorem to evaluate $\iint_S \vec{F}.\vec{n}\ dS$, where S is the surface of a sphere having centre at $(3,-1,-2)$ and radius 3. 6

(*c*) Evaluate $\iint_R\sqrt{x^2+y^2}\ dxdy$, where R is the region in the xy-plane bounded by $x^2+y^2=4$ and $x^2+y^2=9$. 6

4. (*a*) Let A be a matrix such that

$$[1,0]A=[1,2,0]$$

and $$[0,1]A=[0,-1,1].$$

Then find the product matrix

$$A\begin{bmatrix}-1\\1\\1\end{bmatrix}$$ 8

(*b*) Find the matrix A representing the linear transformation that maps (x_1,x_2) on to $(2x_1-5x_2,\ 3x_1+4x_2)$ and verify the result. Does the inverse of this matrix A exists ? 6

(*c*) Of the three eigenvalues of the matrix

$$A=\begin{bmatrix}-2&2&-3\\2&1&-6\\-1&-2&0\end{bmatrix},$$

two are repeated. What are these eigenvalues ? Further, find the eigenvector corresponding to the non-repeated eigenvalue. 6

Group B

5. (*a*) Solve:

$$dy/dx=1-xy-y+x,\ \text{given}\ y(0)=1.$$ 5

(*b*) Use the method of variation of parameters to solve

$$y''(x) + y(x) = \sec x.$$ 5

(*c*) Using the method of undetermined coefficient, solve the equation

$$y''(x) + 2y'(x) + y(x) = e^{-x}$$

subject to the conditions $y(0) = -1$ and $y'(0) = 1$. 10

6. (*a*) Eliminate the arbitrary functions *f* and *g* from the relation

$$z = f(x+10y) + g(x-10y)$$

to develop a partial differential equation of an appropriate order. Name the differential equation and its type. 6

(*b*) If the Laplace transform of a given function $f(t)$ is $F(s)$, then find the Laplace transform of the function

$$\{f(t-4)\ u(t-4)\},$$

where $u(t-a)$ represents the unit function with jump at $t = a$. 6

(*c*) Use the Laplace transform procedure to solve the initial value problem

$$\left.\begin{array}{l} y''(x) + 4y'(x) - 32y(x) = 0; \\ y(0) = 6, y'(0) = 0 \end{array}\right\}$$ 8

7. (*a*) State the Simpson's one-third rule and use it to evaluate

$$\int_0^6 \frac{dx}{1+x^2}.$$ 6

(*b*) Express $f(x) = 2x^3 - 3x^2 + 3x - 10$ in factorial form and hence evaluate $\Delta^3 f(x)$. 6

(*c*) Derive the following :

(*i*) $\delta = E^{1/2} - E^{-1/2}$

(*ii*) $E = e^{hD}$

(*iii*) $D = (1/h)\ \log\ (1+\Delta)$.

where symbols have their usual meanings. 3 + 3 + 2

8. (*a*) A pointer moves along a fixed straight rod. Its distance *x* cm along the rod is given below for various values of the time *t* sec. Developing an appropriate difference table, find the velocity and the acceleration of the pointer : 10

t =	0	0.1	0.2	0.3	0.4	0.5	0.6
x =	30.13	31.62	32.87	33.64	33.95	33.81	33.24

(*b*) Compute the probability of obtaining at least two 'six' in rolling a fair die four times. 5

(*c*) Define the Normal distribution or the Gauss distribution in its standard form. Is the curve $f(x)$ in the defn. is symmetric with respect to $x = \mu$. If so, why ? What happens to the curve when $\mu = 0$? 5

Group C

9. Answer the following : 10×2

(*i*) Is the series $\sum_{r=1}^{\infty} (-1)^{r+1}$ convergent or divergent or oscillates finitely ?

(*ii*) Find the point at which the function

$$f(x) = a\sin^2 x + b\cos^2 x (a > b)$$

attains an extremum value.

(*iii*) Determine the region where the function $z = \log(x^2 + y^2 - 1)$ is defined.

(*iv*) . Find the rank of the matrix

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$

using the concept of 'linear independency'.

(*v*) The line integral

$$\int_{AB} [(y+z)dx + (z+x)dy + (x+y)dz]$$

is independent of the path from A to B. Is this statement *true* or *false* ?

(*vi*) Find the eigenvector corresponding to the eigenvalue 6 of the matrix

$$\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$$

(*vii*) Does there exists a function ϕ such that $\vec{F} = \nabla\phi$, where $\vec{F} = (2x-3)\hat{i} - z\hat{j} + \cos z\hat{k}$?

(*viii*) Find the inverse Laplace transform of

$$F(s) = 1/(s^2 + 100).$$

(*ix*) Name *any two* iterative methods by which one may solve a system of linear equations.

(*x*) Let the random variable X = number of heads in a single toss of a fair coin, has the possible values $X = 0$ and $X = 1$ with probabilities $P(X = 0) = 1/2$ and $P(X = 1) = 1/2$. Then find the mean and variance.

S'13: 7FN: AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

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Group A

1. (*a*) If $y(x) = \log\left\{x + \sqrt{1 + x^2}\right\}$, then prove that

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0.$$

Also, find out $(y_6)_{x=0}$ and $(y_7)_{x=0}$. 8

(*b*) If $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \cdots + \frac{a_{n-1}}{2} + a_n = 0$

then prove that the equation

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0$$

has least one real root between 0 and 1. 6

(*c*) The curve $y^2(a + x) = x^2(3a - x)$ revolves about the axis of x. Find the volume generated by the loop. 6

2. (*a*) Discuss the convergence of the infinite series

$$\frac{1}{1\cdot2\cdot3}+\frac{3}{2\cdot3\cdot4}+\frac{5}{3\cdot4\cdot5}+\cdots ad \text{ inf.} \qquad 5$$

(*b*) If $V = (x^2 + y^2 + z^2)^{-1/2}$, show that

$$x\frac{\partial V}{\partial x}+y\frac{\partial V}{\partial y}+z\frac{\partial V}{\partial z}=-V \qquad 5$$

(*c*) Show that $\sin^p\theta \cos^q\theta$ attains a maximum value when $\theta = \tan^{-1}\sqrt{p/q}$. 5

(*d*) Find the unit normal to the surface $\phi(x, y, z) \equiv x^4 - 3xyz + z^2 = 0$ at the point $p(1, 1, 1)$. 5

3. (*a*) Evaluate

$$\iint_S \vec{F}\cdot\hat{n}\, ds,$$

where $\vec{F} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and s is the surface of cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. 8

(*b*) Verify Green's theorem in plane for

$$\oint_C \{(xy+y^2)dx + x^2dy\}$$

where C is bounded by the curves $y = x$ and $y = x^2$. 6

(*c*) Expand $\log \sin x$ in powers of $(x - 2)$ up to 4 terms containing cube of $(x - 2)$. 6

4. (*a*) Show that the matrix

$$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

satisfies its characteristic equation. Does A^{-1} exist ? If yes, find A^{-1} using Cayley-Hamilton theorem. 8

(*b*) Prove that two similar matrices of same order have same eigenvalues. 6

(*c*) If P_n denotes the space of all polynomials of degrees less than or equal to n, with real coefficients, then find the matrix of differential operator $T: P_4 \to P_4$ defined by

$$T(f(x)) = \frac{d}{dx}f(x)$$

under usual basis of P_4. 6

Group B

5. (*a*) Show that the equation

$$(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$$

represents a family of hyperbolas having as asymptotes the lines $x + y = 0$ and $2x + y + 1 = 0$. 8

(*b*) By method of variation of parameters, solve the differential equation

$$D^2y(x) - 2Dy(x) + y(x) = \frac{e^x}{2x}$$

where $D \equiv d/dx$. 6

(*c*) Obtain the Laplace transform of the function, $f(t)$, given by

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$$ 6

6. (*a*) Solve

$$(x^2y^2 + xy + 1)\, y dx + (x^2y^2 - xy + 1)\, x dy = 0$$ 5

(*b*) Obtain a partial differential equation by eliminating the arbitrary function, f, from equation

$$x + y + z = f(x^2 + y^2 + z^2)$$ 5

(*c*) Solve the simultaneous differential equation

$$\frac{dx}{dt} + ax = y$$

$$\frac{dy}{dt} + ay = x$$

with initial conditions $x(0) = 0$, $y(0) = 1$ and using the laplace transforms. 5

(*d*) Solve the partial differential equation $p + q = pq$ where

$$p = \partial z/\partial x \text{ and } q = \partial z/\partial y.$$ 5

7. (*a*) The population of a town in the decennial census was as given below. Estimate the population for the year 1895. 6

Year : x	1891	1901	1911	1921	1931
Population : y (in thousand)	46	66	81	93	101

(*b*) State and derive trapezoidal rule for numerical integration and also derive the expression for error formula. 6

(*c*) From the following table of values of x and y, obtain dy/dx and d^2y/dx^2 for $x = 1{\cdot}2$: 8

x	1·0	1·2	1·4	1·6	1·8	2·0	2·2
y	2·7183	3·3201	4·0552	4·9530	6·0496	7·3891	9·0250

8. (*a*) Prove that if E and F are two independent events, then the events E and F' are also independent. 6

(*b*) Show that, in Poisson distribution with unit mean, mean deviation about mean is $(2/e)$ times the standard deviation. 6

(*c*) If the random variables X_1 and X_2 are independent and follow chi-square distribution with n d.f., show that $\sqrt{n}\ (X_1 - X_2)/2\sqrt{X_1X_2}$ is distributed as student's t with n d.f. independently of $X_1 + X_2$. 8

Group C

9. Answer the following : 10 × 2

(*i*) Evaluate

$$\int e^x (x \cos x + \sin x) dx.$$

(*ii*) Is the series

$$\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$$

convergent ? Give reason for it.

(*iii*) Find div $\left(\frac{\vec{r}}{|\vec{r}|}\right)$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(*iv*) Show that

$$E \equiv 1 + \Delta \text{ and } \Delta \equiv \nabla(1 - \nabla)^{-1}.$$

(*v*) If

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

then find the sum and product of the eigenvalues of matrix A.

(*vi*) Change the order of integration in

$$\int_0^a \int_{mx}^{lx} f(xy)\, dy\, dx.$$

(*vii*) Find

$$\mathcal{L}^{-1}\left[\frac{s-4}{s^2 - 4s + 13}\right].$$

(*viii*) Under what condition the differential equation

$$M(x, y)\, dx + N(x, y)\, dy = 0$$

will be exact and why ?

(*ix*) Write two basic characteristics of poisson distribution.

(*x*) Find length of the arc of semi-cubical parabola $ay^2 = x^3$ from the vertex to the point (a, a).

W'13:7FN:AN 209 (1409)

ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

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Any missing or wrong data may be assumed suitably giving proper justification.

Figures on the right-hand side margin indicate full marks.

Group A

1. (*a*) Prove that

$$\frac{d^3y}{dx^3} = -\frac{\frac{d^3y}{dx^3}\cdot\frac{dy}{dx} - 3\left(\frac{d^2y}{dx^2}\right)^2}{\left(\frac{dy}{dx}\right)^5}$$ 8

(*b*) Find the minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$. 8

(*c*) If $\vec{f} = 3xy\vec{i} - y^2\vec{j}$, evaluate

$$\int_C \vec{F}\cdot d\vec{r}$$

where C is the curve in the (x, y) plane $y = 2x^2$ from $(0, 0)$ to $(1, 2)$. 4

2. (*a*) Apply Maclaurin's theorem to obtain the expansion of sec x. 6

(*b*) Verify Green's theorem for $f(x, y) = e^{-x}\sin y$, $g(x, y) = e^{-x}\cos y$ and C is the square with vertices at $(0, 0)$, $(\pi/2, 0)$, $(\pi/2, \pi/2)$ and $(0, \pi/2)$. 8

(*c*) Determine the rank of the matrix

$$A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$ 6

3. (*a*) Show that the matrix A given by

$$A = \begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$$

is diagonalizable. Find a matrix P such that $P^{-1}AP$ is a diagonal matrix. 10

(*b*) Discuss the convergence of the following series :

$$\sum x^n/3^n n^2, \text{ for } x > 0$$ 5

(*c*) Evaluate

$$\nabla e^{r^2}, \text{ where } r^2 = x^2 + y^2 + z^2.$$ 5

4. (*a*) Evaluate

$$\iint_A xy\,dy\,dx$$

where A is the domain bounded by x-axis, ordinate $x = 2a$, and the curve $x^2 = 4ay$. 8

(*b*) Prove that

$$\frac{x}{a} + \frac{y}{b} = 1$$

touches the curve $y = be^{-x/a}$ at the point where the curve crosses the x-axis. 6

(*c*) Show that for $0 < u < v$, $(v - u)/(1 + v^2) < \tan^{-1} v - \tan^{-1} u < (v - u)/(1 + u^2)$. 6

Group B

5. (*a*) Solve

$$(1 + y^2) + (x - e^{-\tan^{-1}y})\, dy/dx = 0$$ 5

(*b*) For the following given values of x and $\log_{10} x$

(300, 2·4771), (304, 2·4829), (305, 2·4843) and (307, 2·4871)

find the value of $\log_{10}301$ using Lagrange's interpolation formula. 7

(*c*) An integer is chosen at random from 200 digits. What is the probability that the integer is divisible by 6 or 8 ? 8

6. (*a*) Find the general solution of the differential equation

$$(D^3 + 3D^2 + 2D)\, y = x^2$$ 6

(*b*) Find the Fourier transform of the function

$$f(t) = e^{-a|t|}, -a < t < \infty, a > 0$$

What is the inverse transform for it ? 7

(*c*) Let X be a normal variate with mean 30 and standard deviation 5. Find the probability that $26 \le X \le 40$. 7

7. (*a*) Evaluate the integral

$$\int_0^1 \frac{dx}{3+2x}$$

using Simpson's one-third rule with $n = 2, 4$. Compare the result with the exact solution. 7

(*b*) Find the partial differential equation of the set of all right circular cones whose axes coincide with z-axis. 6

(*c*) Apply the method of variation of parameters to solve

$$y_2 + a^2y = \text{cosec } ax.$$ 7

8. (*a*) The integrating factor of the following equation is of the form y^n. Find n and solve the equation

$$y^{n+1}\sec^2 x\, dx + \{3y^n \tan x - \sec^2 y\, y^{n-2}\}\, dy = 0$$ 8

(*b*) Solve the simultaneous differential equations

$$\frac{dx}{dt} - y = t$$

$$\frac{dy}{dt} + x = 1$$ 6

(*c*) Find the Laplace transform of the function

$$f(t) = \begin{cases} 0 & , \quad 0 \le t < 2, \\ K & , \quad t \ge 2 \end{cases}$$

and K is a constant. 6

Group C

9. Answer the following : 10 × 2

(*i*) Solve the differential equation

$$\frac{dy}{dx} = \frac{y}{\sqrt{x}}; \quad y(2) = 4$$

(*ii*) Let A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. If $P(B) = (3/2)\,P(A)$ and $P(C) = (1/2)\,P(B)$, the value of $P(A)$ is ____ .

(*iii*) A continuous random variable X has a p.d.f. $f(x) = 3x^2$, $0 \le x \le 1$. What will be the value of $P(X \le a)$?

(*iv*) Find the interval in which the equation

$$f(x) = x^6 - x - 1 = 0$$

has exactly one positive real root.

(*v*) If Δ and ∇ are the forward and backward difference operators respectively, then the value of $\Delta - \nabla$ is ____ .

(*vi*) Solve the differential equation

$$\frac{dy}{dx} + y\frac{d\phi}{dx} = \phi(x)\left(\frac{d\phi}{dx}\right)$$

(*vii*) State Rolle's theorem.

(*viii*) Discuss the applicability of Lagrange's mean value theorem for

$$f(x) = (1/x) \ln [-1, 1]$$

(*ix*) Find the directional derivative of $f(x, y, z) = xy^2 + 4xyz + z^3$ at $(1, 2, 3)$ in the direction of $3\vec{i} + 4\vec{j} - 5\vec{k}$.

(*x*) Find the gradient of the following scalar field :

$$f(x, y) = y^2 - 4xy \text{ at } (1, 2).$$