JNU-ENTRANCE EXAMINATION- 2013

M.Sc. PHYSICS

Maximum Marks: 100

INSTRUCTIONS FOR CANDIDATES

1. All questions are compulsory.
2. For each question, one and only one of the four choices given is the correct answer.
3. Each correct answer will be given +4 marks.
4. Each wrong answer will be given -4/3 mark.
5. Use of calculator is permitted.

Q1. A train of mass M is traveling with uniform velocity on a level line. The last carriage, whose mass is m, becomes uncoupled at an instant. The driver discovers it after traveling a distance l and stops the engine. The resistance to the motion is uniform and proportional to the weight. The pull of the engine is uniform. When both parts of the train come to rest, the distance between them is

(a) $\left[\frac{M}{M+m}\right]l$ (b) $\left[\frac{M}{M-m}\right]l$ (c) $\left[\frac{m}{M+m}\right]l$ (d) $\left[\frac{m}{M-m}\right]l$

Q2. Let μ, P, T be the set of standard thermodynamic variables respectively denoting the chemical potential, pressure and temperature of a fluid. If s and v be respectively the entropy and volume per particle and $\mathrm{n} = 1/v$, then identify the correct thermodynamic relation.

(a) $d\mu = -sdT + vdP$ (b) $d\mu = -Tds + vdP$

(c) $d\mu = sdT + Pdv$ (d) $d\mu = Tds - Pdv$

Q3. The equation of state of an ideal gas is $p = nk_B T$, where p is the thermodynamic pressure and $\mathrm{n} = N/V$ is the thermodynamic variable for the number of particles per unit volume. The n dependence of the free energy f per unit volume of the ideal gas is obtained by the following expression, where c_T is temperature-dependent constant

(a) $nk_B T\lfloor \mathrm{In}\,\mathrm{n} + C_T \rfloor$ (b) $2nk_B T\lfloor \mathrm{nIn}\,\mathrm{n} + C_T \rfloor$

(c) $\frac{3}{2}nk_B T$ (d) $3nk_B T$

Q4. Identify the correct statement.

(a) The entropy of a system always increases when it undergoes an irreversible process

(b) The entropy of a system always decreases when it undergoes an irreversible process

(c) The second law of thermodynamics follows directly from principle of conservation of energy

(d) The internal energy of an ideal gas depends on its temperature

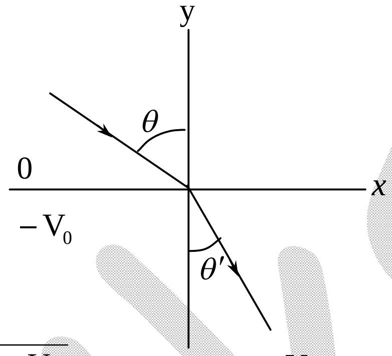
Q5. An ideal gas undergoes an isothermal expansion (at a constant temperature T) to double its volume. The change in the entropy per mole is

(a) $2R$ (b) $-R\mathrm{In}2$ (c) $R\mathrm{In}2$ (d) $\frac{R}{2}$

Q6. A simple pendulum consisting of a point mass m tied to a massless string of length l executes small oscillations of frequency ω and amplitude $A \approx l\theta_0$. The average (over a complete time period $T = 2\pi/\omega$ of the pendulum) tension on the string is

(a) $mg + \frac{mA^2\omega^2}{4l}$ (b) $-\frac{mA^2\omega^2}{4l}$ (c) $mg + \frac{mA^2\omega^2}{2l}$ (d) $-\frac{mA^2\omega^2}{2l}$

Q7. Consider the potential field $V(x, y)$ which is 0 and $-V_0(V_0 > 0)$ respectively in the regions of y greater and less than zero. Let θ and θ' be the angles of incidence and refraction of the particle with the y-axis at the point of incidence as it crosses the x-axis. The ratio $\sin(\theta)/\sin(\theta')$ is given (in terms of $\Delta = V_0/E$) by



(a) $\sqrt{1+\frac{2V_0}{E}}$ (b) $\sqrt{1+\frac{V_0}{E}}$ (c) $1+\frac{V_0}{E}$ (d) $1+\frac{2V_0}{E}$

Q8. Consider the simple Bohr model of the H atom. Its ground state $(n=1)$ has energy of $-13.6\,\text{eV}$. The wavelength of the electromagnetic radiation emitted when the atom makes a transition from the first excited $(n=2)$ state to the ground state is

(a) 1.22×10^{-5} cm (b) 2.44×10^{-5} cm

(c) 6.22×10^{-5} cm (d) 4.32×10^{-5} cm

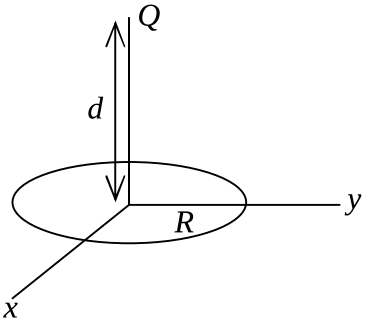
Q9. Radon has a half-life of 3.8 days. If we start with 10.24 gm of radon, the amount of it which will be left after 38 days is

(a) 10^{-4} gm (b) 10^{-2} gm (c) 10^{-6} gm (d) 10^{-3} gm

Q10. A capacitor with capacitance C is charged to a voltage V. If it is fully discharged by shorting through a resistor R, the total heat generated in the resistor is

(a) $\frac{1}{2}CV^2$ (b) $\frac{1}{4}CV^2$ (c) CV^2 (d) dependent on R

Q11. The flux of electric field through a circle of radius R placed in the x-y plane with its centre at the origin due to a point charge Q placed at $(0,\ 0,\ d)$ is



(a) $\frac{Q}{2\varepsilon_0}\left[1-\frac{d}{\left(d^2+R^2\right)^{1/2}}\right]$ (b) $\frac{Q}{2\varepsilon_0}\frac{d^3}{\left(d^2+R^2\right)^{3/2}}$

(c) $\frac{Q}{4\pi\varepsilon_0}\frac{d}{R}$ (d) $\frac{Q}{4\pi\varepsilon_0}\frac{R^2}{d^2}$

Q12. The electric field of an electromagnetic wave is given by

$$\vec{E}=\hat{i}E_0\sin(ky-\omega t)$$

where k and ω respectively denote the wave vector and angular frequency of the wave. $\hat{i},\hat{j}$ and $\hat{k}$ respectively denote the unit vectors in the x, y and z directions. The magnetic field $\vec{B}$ of the wave is given by

(a) $\hat{k}E_0 k\sin(ky-\omega t)$ (b) $\hat{j}E_0\cos(ky-\omega t)$

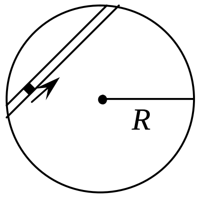
(c) $-\hat{k}\frac{E_0\omega}{k}\cos(ky-\omega t)$ (d) $-\hat{k}\frac{E_0 k}{\omega}\sin(ky-\omega t)$

Q13. The contour integral

$$\oint_C \frac{z\,dz}{(z-1)^2}$$

(a) $2\pi i$ (b) 0 (c) πi (d) $4\pi i$

Q14. Consider the earth as a uniform (density) sphere of total mass M and radius R. A small object slides along a tunnel connecting two points on the surface of the earth and is acted upon only by the gravitational force due to the earth. Using the value of the acceleration due to gravity as $9.81\ \text{meter/sec}^2$ and the radius of the earth $R = 6.37\times10^6\ \text{m}$, the time to slide between the two points is



(a) 21.1 min (b) 84.4 min (c) 42.2 min (d) 63.3 sec

Q15. What is the value of the definite integral $\int_0^1 (x\,\text{In}x)^4\,dx$?

(a) $\frac{4!}{5^5}$ (b) $\frac{6!}{5^4}$ (c) $\frac{2!}{5^2}$ (d) $\frac{1}{5^3}$

Q16. Let $\lambda_i\ (i=1,2,3)$ be the eigenvalues of the matrix $\begin{bmatrix} 2 & -1 & -3 \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{bmatrix}$

The sum $\sum_{i=1}^{3}\lambda_i^2$ is equal to

(a) 14 (b) 42 (c) 6 (d) 0

Q17. Consider the magnetic field $\vec{B} = \alpha_0\left(\hat{i}y + \hat{j}x\right)$ and two vector potentials $\vec{A}_1 = \alpha_0\left(\hat{i}xz - \hat{j}yz\right)$ and $\vec{A}_2 = -\hat{k}\frac{\alpha_0}{2}\left(x^2 - y^2\right)$, where α_0 is a constant and $\hat{i}, \hat{j}, \hat{k}$ represent the unit vectors along the cartesian axes. Identify the correct statement.

(a) Only $\vec{A}_1$ produces the magnetic field $\vec{B}$ but not $\vec{A}_2$

(b) Only $\vec{A}_2$ produces the magnetic field $\vec{B}$ but not $\vec{A}_1$

(c) Neither $\vec{A}_1$ nor $\vec{A}_2$ produces the magnetic field $\vec{B}$

(d) Both $\vec{A}_1$ and $\vec{A}_2$ produces the magnetic field $\vec{B}$

Q18. The kinetic energy of a free relativistic particle is defined as $E - m_0c^2$, where E and m_0 are respectively its total energy and rest mass. Let v be the speed of the particle when its kinetic energy is half of its rest mass energy. Then the ratio v/c is

(a) $\frac{\sqrt{5}}{3}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

Q19. If the age of the universe is 10^{10} years, the humans have existed for 10^6 years. If we take the age of the universe to be a day, how many seconds have the humans existed?

(a) 2.40 sec (b) 86.40 sec (c) 8.64 sec (d) 24 sec

Q20. The mass density and momentum density in a fluid at position $\vec{r}$ at time t, respectively denoted by $\rho\left(\vec{r},t\right)$ and $\vec{g}\left(\vec{r},t\right)$, are related by the continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{g} = 0$$

The above equation is a consequence of the law of conservation of

(a) linear momentum (b) mass

(c) energy (d) angular momentum

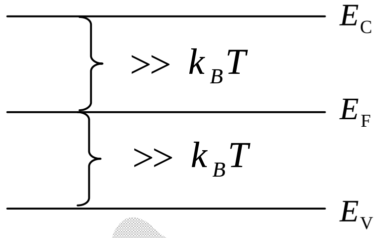
Q21. If the number of seconds in a year of 365 days is taken as $\pi \times 10^7$ seconds, the percentage error in this approximation would be close to

(a) 4.8 (b) 0.048 (c) 0.38 (d) 2.3

Q22. Consider a regular arrangement of identical spheres in a face-centred cubic (fcc) structure in which the centres of the respective spheres are located at each of the eight corners and the centres of six surfaces of a unit cube. The fraction of each cubic unit cell occupied by the spheres in the close-pack configuration is

(a) 0.50 (b) 0.62 (c) 0.74 (d) 0.88

Q23. In a simple model of an intrinsic semiconductor without doping, we assume that there are N_v states (all having same energy E_v) in valence band and N_c states (all having same energy E_c) in the conduction band. The probability of



occupying an electronic state of energy E at temperature T is given by the Fermi-Dirac distribution $1/\left(\exp\left(-\frac{E-E_F}{k_B T}\right)+1\right)$, where E_F is the Fermi energy. At equilibrium, the average number of electrons and holes in the conduction band and the valence band respectively are equal. Assuming that the Fermi level E_F lies in the gap between the valence and conduction band and is far from both compared to $k_B T$, it follows that the difference $E_F - (E_v + E_c)/2$ is given by

(a) $\frac{k_B T}{2} \ln \frac{N_v}{N_c}$ (b) $\frac{k_B T}{4} \ln \frac{N_v}{N_c}$ (c) $k_B T \frac{N_v}{N_c}$ (d) $2k_B T \frac{N_v}{N_c}$

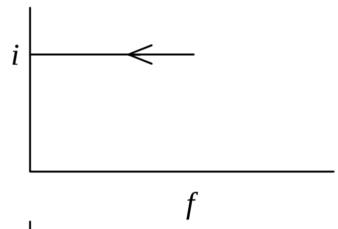
Q24. For an ideal *p-n* junction diode with sharp boundary between two semiconducting materials, the current i is related to the potential difference V across the diode by the relation

$$i = i_0\left(e^{eV/k_B T} - 1\right)$$

where e is the electronic charge and T is the temperature; i_0 is a material-dependent constant. The junction is termed respectively as forward and reverse biased depending on whether V is positive or negative. At temperature T, the magnitude of the current changes by a factor f for changing from forward to backward bias of the voltage which is equal to

(a) $\frac{2k_B T}{e} e^{2f}$ (b) $\frac{2k_B T}{e} \ln f$ (c) $\frac{k_B T}{e} e^{f}$ (d) $\frac{k_B T}{e} \ln f$

Q25. In a photoelectric effect experiment, a stream of photons of frequency f and intensity (energy per unit time) is incident on the photocathode to produce a photocurrent i. If the frequency f is steadily reduced without any change of intensity, the plot of current vs. frequency looks like

a)


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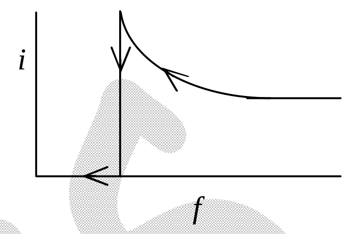

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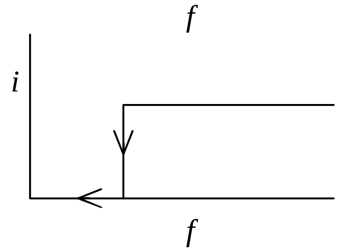

(d)