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ROLL No. 

TEST BOOKLET No. **504**

TEST FOR POST GRADUATE PROGRAMMES

## MATHEMATICS

Time: 2 Hours

Maximum Marks: 450

### INSTRUCTIONS TO CANDIDATES

1. You are provided with a Test Booklet and an Optical Mark Reader (OMR) Answer Sheet to mark your responses. Do not soil the Answer Sheet. Read carefully all the instructions given on the Answer Sheet.

2. Write your Roll Number in the space provided on the top of **this page**.

3. Also write your Roll Number, Test Code, and Test Subject in the columns provided for the same on the **Answer Sheet**. Darken the appropriate bubbles with a **Ball Point Pen.**

4. The paper consists of 150 objective type questions. All questions carry equal marks.

5. Each question has four alternative responses marked **A, B, C** and **D** and you have to **darken** the bubble corresponding to the correct response fully by a **Ball Point Pen** as indicated in the example shown on the Answer Sheet.

6. Each correct answer carries **3** marks and each wrong answer carries **1** minus mark.

7. Space for rough work is provided at the end of this Test Booklet.

8. You should return the Answer Sheet to the Invigilator before you leave the examination hall. However, you can retain the Test Booklet.

9. Every precaution has been taken to avoid errors in the Test Booklet. In the event of any such unforeseen happening, the same may be brought to the notice of the Observer/Chief Superintendent in writing. Suitable remedial measures will be taken at the time of evaluation, if necessary.

## MATHEMATICS

1. The largest value of the function $f(x) = 2x^3 - 9x^2 + 12x + 3$ in the range $0 \leq x \leq 2$ is

   (A) 8
   (B) 7
   (C) 3
   (D) 0

2. The equation $x^3 - 30x^2 + 108x - 104 = 0$ has

   (A) no real roots
   (B) exactly one real root
   (C) three distinct real roots
   (D) a repeated root

3. The inequalities $x^2 + 3x + 2 > 0$ and $x^2 + x < 2$ are met by all $x$ in the region

   (A) $x < -2$
   (B) $-1 < x < 1$
   (C) $x > -1$
   (D) $x > -2$

4. The power of $x$ which has the greatest coefficient in the expansion of $\left(1 + \frac{1}{2}x\right)^{10}$ is

   (A) $x^2$
   (B) $x^3$
   (C) $x^5$
   (D) $x^{10}$

5. The four digit number 2652 is such that any two consecutive digits from it make a multiple of 13. Another number $M$ has the same property, is 100 digit long, and begins in a 9. What is the last digit of $M$?

   (A) 2
   (B) 3
   (C) 6
   (D) 9

6. Let $m$ and $n$ be integers. Then $\frac{6^{m+n} \times 12^{m-n}}{8^m \times 9^{m+2n}}$ is an integer if

(A) $m + n \leq 0$ (B) $n \leq 0$
(C) $m \leq 0$ (D) $m \geq n$

7. Given function $f(x)$ satisfies
$\int_0^1 3f(x)dx + \int_1^2 2f(x)dx = 7;\ \int_0^2 f(x)dx + \int_1^2 f(x)dx = 1.$
Then $\int_0^2 f(x)dx$ is equal to

(A) 1 (B) 0
(C) 1/2 (D) 2

8. The function $f(x) = 2x^3 - 6x^2 + 5x - 7$ has

(A) no stationary point (B) one stationary point
(C) two stationary points (D) three stationary points

9. If $1 + 3x + 5x^2 + 7x^3 + \cdots + 99x^{49}$ is divisible by $x - 1$, then the remainder is

(A) 2000 (B) 2500
(C) 3000 (D) 3500

10. Let $S(n)$ is the sum of digits in the positive integer $n$. Then
$S(1) + S(2) + S(3) + \cdots + S(99)$ is

(A) 746 (B) 862
(C) 900 (D) 924

11. The smallest value of $a$ in $\int_0^1 (x^2 - a)^2\, dx$ as $a$ varies is

(A) 3/20 (B) 4/45
(C) 7/13 (D) 1

12. The point on the circle $x^2 + y^2 + 6x + 8y - 75 = 0$ which is closest to the origin, is at what distance from the origin?

(A) 3
(B) 4
(C) 5
(D) 10

13. The smallest positive integer $n$ such that $1 - 2 + 3 - 4 + 5 - 6 + \cdots + (-1)^{n+1}n \geq 100$ is

(A) 99
(B) 101
(C) 199
(D) 300

14. The sum of the first $2n$ terms of 1, 1, 2, 1/2, 4, 1/4, 8, 1/8, 16, 1/16, .... is

(A) $2^n + 1 - 2^{1-n}$
(B) $2^n + 2^{-n}$
(C) $2^{2n} - 2^{3-2n}$
(D) $\frac{2^n - 2^{-n}}{3}$

15. A rectangle has perimeter $a$ cm and area $b$ sq.cm. Then the value of $a$ and $b$ satisfy

(A) $a^3 > b$
(B) $b^2 > 2a + 1$
(C) $a^2 \geq 16b$
(D) $ab \geq a + b$

16. The sequence $\langle x_n \rangle$ is defined by $x_n = n^3 - 9n^2 + 631$. Then the largest value of $n$ for which $x_n > x_{n+1}$ is

(A) 5
(B) 7
(C) 11
(D) 17

17. If $\alpha$ and $\beta$ are different complex numbers with $|\beta| = 1$, then $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$ is equal to

(A) 0
(B) 1/2
(C) 1
(D) 2

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18. Integral solutions of the equation $(1-i)^x = 2^x$ are

(A) 0 (B) $4n, n \in N$
(C) 0, 1 (D) $2n, n \in N$

19. The number of tangents to the parabola $y^2 = 8x$ through $(2,1)$ is

(A) 0 (B) 1
(C) 2 (D) 3

20. If $e_1$ and $e_2$ are the eccentricities of a hyperbola and its conjugate hyperbola, then $\frac{1}{e_1^2} =$

(A) $1+\frac{1}{e_2^2}$ (B) $1-\frac{1}{e_2^2}$
(C) $\frac{1}{e_2^2}-1$ (D) $\frac{1}{e_2^2}$

21. $\lim_{x\to 1}\frac{\sqrt{\{1-\cos 2(x-1)\}}}{x-1}$

(A) exists and it equals $\sqrt{2}$
(B) exists and it equals $-\sqrt{2}$
(C) does not exist because $(x-1) \to 0$
(D) does not exist because left hand limit is not equal to right hand limit

22. $\lim_{x\to\infty}\frac{\sqrt{(x^2-1)}}{2x+1} =$

(A) 1 (B) 0
(C) $-1$ (D) 1/2

23. If $f(x) = \begin{cases} ax^2 + b, & x \leq 0 \\ x^2, & x > 0 \end{cases}$ possesses the derivative at $x = 0$, then

(A) $a = 0, b = 0$
(B) $a > 0, b = 0$
(C) $a \in R, b = 0$
(D) $a \neq 0, b \neq 0$

24. If $f(x) = (x - x_0)g(x)$, where $g(x)$ is continuous at $x_0$, then $f'(x_0)$ is equal to

(A) $0$
(B) $x_0$
(C) $g(x_0)$
(D) $g'(x_0)$

25. If $f(x) = e^x$ and $g(x) = \ln x$, then $(gof)'(x)$ is equal to

(A) $0$
(B) $1$
(C) $e$
(D) $1 + e$

26. If $y = x^{x^{x\ldots}}$, then $x(1 - y\log x)\frac{dy}{dx} =$

(A) $x^2$
(B) $y^2$
(C) $xy^2$
(D) $x^2y$.

27. If $y = x^{\log x}$, then $\frac{dy}{dx}$ equals

(A) $\log x \ldots x^{\log x - 1}$
(B) $x^{\log x - 1}.2\log x$
(C) $x\log(\log x)$
(D) $\frac{1}{x\log x}.x^{\log x - 1}$

28. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where $p$ is constant. Then $f'''(0) =$

(A) $p$
(B) $p + p^2$
(C) $p + p^3$
(D) 0

29. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(A) $u$
(B) $\sin u$
(C) $\tan u$
(D) $\cot u$

30. The distance from origin to the normal of the curve $y = e^{2x} + x^2$ at the point $x = 0$ is

(A) $\frac{2\sqrt{5}}{5}$
(B) $2\sqrt{5}$
(C) $\sqrt{5}$
(D) 2

31. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then

(A) $a > 0, b > 0$
(B) $a < 0, b > 0$
(C) $a > 0, b = 0$
(D) $a < 0, b = 0$

32. For all $x \in (0,1)$, which one is true?

(A) $e^x < 1 + x$
(B) $\log_e(1 + x) < x$
(C) $\sin x > x$
(D) $\log_e x > x$

33. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As $b$ varies, the range of $m(b)$ is

(A) $[0, 1]$
(B) $(0, 1/2]$
(C) $[1/2, 1]$
(D) $(0, 1]$

34. $N$ characters of information are held on magnetic tape, in batches of $x$ characters each; the batch processing time is $\alpha + \beta x^2$ seconds; $\alpha$ and $\beta$ are positive constants. The optimal value of $x$ for fast processing is

(A) $\alpha/\beta$ (B) $\beta/\alpha$

(C) $\sqrt{\left(\frac{\alpha}{\beta}\right)}$ (D) $\sqrt{\left(\frac{\beta}{\alpha}\right)}$

35. A square piece of tin of side 18cm is to be made into a box without top, by cutting a square from the each corner and folding up the flaps to form the box. The side of the square, so that the volume of the box is the maximum possible is given by

(A) 9 (B) 6

(C) 3 (D) 1

36. $\int \frac{(\sin x+\cos x)dx}{\sqrt{(1+\sin 2x)}}$ equals

(A) $\log(\sin x + \cos x)$ (B) $x$

(C) $\log x$ (D) $\log \sin(\cos x)$

37. The value of $\int \frac{dx}{e^x+1}$ is

(A) $\log(e^x - 1) + c$ (B) $\log(e^x + 1) + c$

(C) $x - \log(e^x + 1) + c$ (D) $\log e^x + c$

38. $\int e^x(\sin hx + \cos hx)dx =$

(A) $e^x \sec hx$ (B) $e^x \cos hx$

(C) $\sin h\, 2x$ (D) $\cos h\, 2x$

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39. $f: R \to R, g: R \to R$ are one to one real valued functions. Then the value of $\int_{-\pi}^{\pi} |f(x) + f(-x)||g(x) - g(-x)|\, dx$ is

(A) 0
(B) $\pi$
(C) 1
(D) $-\pi$

40. Area bounded by the loop of the curve $ay^2 = x^2(a - x)$ is equal to

(A) $4a^2/15$
(B) $8a^2/15$
(C) $16a^2/15$
(D) $12a^2/15$

41. The differential equation representing the family of curves $y^2 = 2c\left(x + \sqrt{c}\right)$, where $c$ is positive parameter, is of

(A) order 1
(B) order 2
(C) degree 2
(D) degree 4

42. If $(A + B) \perp$ to $B$ and $(A + 2B) \perp$ to $A$, then

(A) $A = \sqrt{2}\, B$
(B) $A = 2B$
(C) $2A = B$
(D) $A = B$

43. If $a$ and $b$ are two unit vectors inclined at an angle of $60^\circ$ to each other, then

(A) $|a + b| > 1$
(B) $|a + b| < 1$
(C) $|a - b| > 1$
(D) $|a - b| < 1$

44. If $u = a - b, v = a + b$ and $|a| = |b| = 2$, then $|u \times v|$ is equal to

(A) $2\sqrt{[16 - (a.b)^2]}$
(B) $\sqrt{[16 - (a.b)^2]}$
(C) $2\sqrt{[4 - (a.b)^2]}$
(D) $[4 - (a.b)^2]$

45. The co-efficient of $x^r$ in
$(1 + x)^m + (1 + x)^{m+1} + \cdots + (1 + x)^n, m \leq r \leq n$, is

(A) $(n+1)C_{r+1}$
(B) $(n-1)C_{r-1}$
(C) $nC_r$
(D) $nC_{r+1}$

46. The remainder when $5^{99}$ is divided by 13 is

(A) 6
(B) 8
(C) 9
(D) 10

47. In the binomial expansion of $(a - b)^n, n \geq 5$, the sum of the $5^{th}$ and $6^{th}$ terms is zero. Then $a/b$ equals

(A) $(n - 5)/6$
(B) $(n - 4)/5$
(C) $5/(n - 4)$
(D) $6/(n - 5)$

48. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is

(A) $-4 < x \leq 0$
(B) $0 < x < 1$
(C) $-100 < x < 100$
(D) $0 < x < \infty$

49. All the letters of the word 'EAMCET' are arranged in possible ways. The number of such arrangement in which no two vowels are adjacent to each other is

(A) 360
(B) 144
(C) 72
(D) 54

50. A polygon has 44 diagonals. Then the number of its sides are

(A) 11
(B) 7
(C) 8
(D) 22

51. The number of divisors of 9600 including 1 and 9600 are

(A) 60
(B) 58
(C) 48
(D) 46

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52. The number of non-negative integral solution of $x + y + z = n$ ($n$ is a positive integer) is

(A) $(n+2)C_n$
(B) $(n+4)C_n$
(C) $nC_2$
(D) Sum of first $n$ natural numbers

53. If the probabilities that $A$ and $B$ will die within a year are $p$ and $q$ respectively, then the probability that only one of them will be alive at the end of the year is

(A) $p+q$
(B) $p+q-2pq$
(C) $p+q-pq$
(D) $p+q+pq$

54. A six faced dice is so biased that it is twice as likely to show an even number as an odd number when thrown. If it is thrown twice, the probability that the sum of two numbers thrown is even is

(A) 5/9
(B) 4/9
(C) 2/3
(D) 1/3

55. $A$ speaks truth 60% times, $B$ speaks truth 70% times. The probability that they say same thing while describing a single event is

(A) 0.42
(B) 0.46
(C) 0.54
(D) 0.12

56. If $A$ is symmetric as well as skew-symmetric matrix, then $A$ is

(A) Diagonal
(B) Null
(C) Triangular
(D) Unitary

57. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^t$, then

(A) $x = 0, y = 5$
(B) $x + y = 5$
(C) $x = y$
(D) $x \neq y$

58. If the system of equations
$x + 2y - 3z = 2,\ (k+3)z = 3,\ (2k+1)y + z = 2$
is inconsistent, then $k$ is

(A) $-3$ (B) $1/2$
(C) 1 (D) 2

59. If $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0, a \neq 0$, then

(A) $a$ is one of the cube roots of unity
(B) $b$ is one of the cube roots of unity
(C) $(a/b)$ is one of the cube roots of unity
(D) $(a/b)$ is one of the cube roots of $-1$

60. Two finite sets have $m$ and $n$ elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of $m$ and $n$ are

(A) 7, 6 (B) 6, 3
(C) 5, 1 (D) 8, 7

61. Let $f : R \to R : f(x) = \tan x$. Then $f^{-1}(1) =$

(A) does not exist (B) $\{n\pi + \frac{\pi}{4} : n \in Z\}$
(C) $\pi/4$ (D) $\pi/2$

62. Let $E = \{1,2,3\}$ and $F = \{1,2\}$. Then the number of onto functions from $E$ to $F$ is

(A) 14 (B) 16
(C) 12 (D) 18

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63. Let $R$ be the relation on the set $R'$ of all real numbers defined by $aRb$ if and only if $|a-b| \leq 1$. Then $R$ is

(A) reflexive
(B) equivalence
(C) transitive
(D) anti-symmetric

64. For real numbers $x$ and $y$, we write $xRy$ if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation $R$ is

(A) reflexive
(B) symmetric
(C) transitive
(D) equivalence

65. Let function $f: R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$. Then $f$ is

(A) One-to-one and onto
(B) one-to-one but not onto
(C) onto but not one-to-one
(D) neither one-to-one nor onto

66. The escape velocity for a body projected vertically upwards is 11.2km/sec. If the body is projected in a direction making an angle of $60^{\circ}$ with the vertical, then the escape velocity will be

(A) 11.2 km/sec
(B) $5.6\sqrt{2}$ km/sec
(C) 5.6 km/sec
(D) $\frac{5.6}{\sqrt{2}}$ km / sec

67. A body starts from rest with a uniform acceleration of 8 $m/sec^2$. Then the time it will take in traversing the second metre of its journey is

(A) 1/2 sec
(B) $(\sqrt{2}-1)/2$ sec
(C) $\sqrt{2}$ sec
(D) $(\sqrt{2}+1)/2$ sec

68. The train of length 200 km travelling at 30 m/sec overtakes another of length 300 m travelling at 20 m/sec. The time taken by the first train to pass the second train is

(A) 30 sec
(B) 50 sec
(C) 10 sec
(D) 40 sec

69. In order to keep a body in air above the earth for 12 seconds, the body should be thrown vertically up with a velocity of

(A) $\sqrt{6}$ g m/sec (B) $\sqrt{12}$ g m /sec
(C) $6g$ m/sec (D) $12g$ m/sec

70. Two balls are projected respectively from the same point in direction inclined at 60° and 30° to the horizontal. If they attain the same height, the ratio of their velocities of projection is

(A) $\sqrt{3}$:1 (B) 1:$\sqrt{3}$
(C) 1:1 (D) 1:2

71. Forces 7, 5 and 3 acting on a particle are in equilibrium. The angle between the last pair of force is

(A) 120° (B) 90°
(C) 60° (D) 30°

72. Two weights of 10 gms and 2 gms hang from the ends of a uniform lever one meter long and weighing 4 gms. The point in the lever about which it will balance is from the weight of 10 gms., at a distance of

(A) 5 cm (B) 25 cm
(C) 45 cm (D) 65 cm

73. The number of integers between 100 and 1000 that are divisible by 7 is

(A) 128 (B) 127
(C) 126 (D) 125

74. Product of three consecutive integers is divisible by

(A) 5 (B) 6
(C) 7 (D) 8

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75. The greatest common divisor of $2n + 1$ and $2n - 1$, for all integer $n$ is

(A) 4 (B) 3
(C) 2 (D) 1

76. Let $G$ a group and $a, b \in G$, and let the order of $a$ that is $O(a) = mn$. If $b = a^m$, then $O(b)$ is

(A) $m$ (B) $mn$
(C) $n$ (D) 1

77. If $G$ is a finite group with only one conjugate classes, then $O(G)$ is

(A) 2 (B) 4
(C) 6 (D) 3

78. The decomposition of $x^4 - 4$ over $Z_3$ is

(A) $(x - \sqrt{2})(x + \sqrt{2})(x^2 + 2)$
(B) $(x^2 - 2)(x^2 + 2)$
(C) $(x - 1)(x + 1)(x^2 + 1)$
(D) $(x + 1)(x - 2)(x^2 + 2)$

79. Let $A = \{x : x \in R, |x| < 1\}; B = \{x : x \in R, |x - 1| \geq 1\}$ and $A \cup B = R - D$. Then the set D is

(A) $\{x : 1 < x \leq 2\}$ (B) $\{x : 1 \leq x < 2\}$
(C) $\{x : 1 \leq x \leq 2\}$ (D) $\{x : x \geq 2\}$

80. Let $A = \{(x, y) : y = e^{x}, x \in R\}, B = \{(x, y) : y = e^{-x}, x \in R\}$. Then

(A) $A \cap B = \{(0,1)\}$ (B) $A \cap B \neq \phi$
(C) $A \cap B = R^2$ (D) $A \cup B = R^2$

81. Let $f: R \to R$ be a function defined as $f(x) = x|x|$; for each $x \in R, R$ being the set of real numbers. Which one of the following is correct?

(A) $f$ is one-one but not onto
(B) $f$ is onto but not one-one
(C) $f$ is both one-one and onto
(D) $f$ is neither one-one nor onto

82. If $A$ and $B$ are two non-empty sets of $R$ and if $C = \{x + y : x \in A, y \in B\}$, then

(A) $\text{Inf } C = \text{Inf} A + \text{Inf} B$
(B) $\text{Inf } C \neq \text{Inf} A + \text{Inf} B$
(C) $\text{Inf } C < \text{Inf} A + \text{Inf} B$
(D) $\text{Inf } C > \text{Inf} A + \text{Inf} B$

83. An integer $m$ is said to be related to another integer $n$, if $m$ is a multiple of $n$. Then the relation is

(A) reflexive and symmetric
(B) reflexive and transitive
(C) symmetric and transitive
(D) equivalence relation

84. The domain of convergence for $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ is

(A) $(-1, 1)$
(B) $(-1, 1]$
(C) $(-1, -2)$
(D) $[-1, 1)$

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85. Let $a_n = \begin{cases} 1+\frac{1}{n}, & n \text{ is even} \\ -1-\frac{1}{n}, & n \text{ is odd} \end{cases}$.

Then

(A) $\lim_{n\to\infty} \sup a_n = \lim_{n\to\infty} \inf a_n = 1$

(B) $\lim_{n\to\infty} \sup a_n = \lim_{n\to\infty} \inf a_n = -1$

(C) $\lim_{n\to\infty} \sup a_n = 1, \lim_{n\to\infty} \inf a_n = -1$

(D) $\lim_{n\to\infty} \sup a_n = -1, \lim_{n\to\infty} a_n = 1$

86. The sequence $\{x_n\}$, where $x_n = n^{\frac{1}{n}}$, $n = 1, 2, ....$ converges to

(A) 0
(B) 1
(C) 1/2
(D) $\sqrt{2}$

87. Let $f:[a,b] \to R$ be a continuous function and let $f(a) < f(b)$. Then, by intermediate value theorem

(A) $f([a,b]) = [f(a), f(b)]$
(B) $f([a,b]) \supseteq [f(a), f(b)]$
(C) $f([a,b]) \subseteq [f(a), f(b)]$
(D) $f([a,b]) \neq [f(a), f(b)]$

88. Let $f:[0,1] \to [0,1]$ be a continuous function. Then

(A) $f$ has atleast one fixed point
(B) $f$ has finitely many fixed points
(C) $f$ has infinitely many fixed points
(D) $f$ need not have any fixed point

89. Let $f:[a,b]\to R$ is continuous on $[a,b]$ and $f$ is differentiable on $(a,b)$. If $f(a)=f(b)=0$ there exist $c$ such that $f'(c)=0$, then

(A) $c\subset[a,b]$
(B) $c\in(a,b)$
(C) $c\subset[a,b)$
(D) $c\in(a,b]$

90. If $f$ is a non-negative continuous function on $[a,b]$ and $\int_a^b f(x)dx=0,\ \forall\ x\in[a,b]$, then

(A) $f(x)=c\neq 0,\forall\ x\in[a,b]$
(B) $f(x)=x,\forall x\in[a,b]$
(C) $f(x)=0,\forall x\in[a,b]$
(D) $f(x)=1,\forall x\subset[a,b]$

91. The $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2}$ exists and is the Riemann integral of the function

(A) $f(x)=x^2,\ 0\le x\le 1$

(B) $f(x)=k$ for all $x$ , $0\le x\le 1$

(C) $f(x)=3x,\ 0\le x\le \frac{1}{3}$

(D) $f(x)=x,\ 0\le x\le 1$

92. Consider the improper integrals

(I) $\int_0^1\frac{dx}{\sqrt{1-x}}$ (II) $\int_0^1\frac{dx}{x^2}$

(A) (I) is convergent but (II) is divergent
(B) (I) is divergent but (II) is convergent
(C) Both (I) and (II) are convergent
(D) Neither (I) nor (II) is convergent

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93. Let if possible, $\alpha = \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$, $\beta = \lim_{(x,y)\to(0,0)} \frac{\sin(x^2-y^2)}{x^2+y^2}$.

Then

(A) $\alpha$ exists but $\beta$ does not exist
(B) $\alpha$ does not exist but $\beta$ exist
(C) $\alpha, \beta$ do not exist
(D) both $\alpha, \beta$ exist

94. If $f(x,y)$ is differentiable at $(a,b)$, then the partial derivatives $f_x$ and $f_y$ at $(a, b)$

(A) $f_x$ exists but $f_y$ does not exist
(B) $f_x$ does not exist but $f_y$ exist
(C) $f_x$ and $f_y$ both exist
(D) $f_x$ and $f_y$ both do not exist

95. If A is an open set and B is a closed set in $R^n$, then

(A) B-A is closed set
(B) B-A is open set
(C) B-A is null set
(D) B-A is the whole of $R^n$

96. If $f:[a,b]\to R$ is monotonic, then

(A) $f$ is of bounded variation
(B) $f$ is unbounded
(C) the set of discontinuities of $f$ are uncountable
(D) $f$ is continuous

97. Let $T: R^3 \to R^3$ defined by $T(x, y, z) = (x, y, 0)$ and $S: R^2 \to R^2$ by $S(x, y) = (2x, 3y)$ be linear transformations on the real vector spaces $R^3$ and $R^2$ respectively. Then, which one of the following is correct?

(A) $T$ and $S$ are both singular
(B) $T$ and $S$ are both non-singular
(C) $T$ is singular and $S$ is non-singular
(D) $S$ is singular and $T$ is non-singular

98. Let $T$ be a linear transformation from 3-dimensional vector space $V$ into a 2-dimensional vector space $W$. Then $T$

(A) can be both injective and surjective
(B) can be neither injective nor surjective
(C) can be surjective but cannot be injective
(D) can be injective but cannot be surjective

99. The transformation $(x, y, z) \to (x+y, y+z): R^3 \to R^2$ is

(A) linear and has zero kernel
(B) linear and has a proper subspace as kernel
(C) neither linear nor one-one
(D) neither linear nor onto

100. Let $V_1, V_2, V_3$ be three non-zero vectors in $R^n$ which are linearly dependent. Then

(A) $V_3$ must be a linear combination of $V_1$ and $V_2$
(B) $V_2$ must be a linear combination of $V_1$ and $V_3$
(C) $V_1$ must be a linear combination of $V_2$ and $V_3$
(D) None of these can be linear combination of the other two

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101. The dimension of the vector space of all real numbers $R$ over the field of rational numbers is

(A) 1
(B) 2
(C) 3
(D) 0 by convention

102. If $f$ is integrable on $[a,b]$ and $\int_a^x f(t)dt = 0$ for all $x \in [a,b]$, then

(A) $f(t) = 0$ almost everywhere in $[a,b]$
(B) $f(t) = 0$ nowhere in $[a,b]$
(C) $f(t)$ is not equal to zero almost everywhere in $[a,b]$
(D) $f(t)$ is a non-zero constant everywhere

103. Let $C$ be the standard Cantor's middle thirds set. Then

(A) $C$ is uncountable and of measure zero
(B) $C$ is countable and of measure zero
(C) $C$ is uncountable and of positive measure
(D) $C$ is not measurable

104. If $|z| = |z-1|$, then

(A) $\text{Re}(z) = 1$
(B) $\text{Re}(z) = 1/2$
(C) $\text{Im}(z) = 1$
(D) $\text{Im}(z) = 1/2$

105. For complex number z, $|z+5|^2 + |z-5|^2 = 75$, represents

(A) a circle
(B) an ellipse
(C) a triangle
(D) a straight line

106. $u, v$ are called conjugate harmonic functions, if

(A) $u, v$ are harmonic functions and $u + iv$ is analytical function
(B) $u, v$ are harmonic functions and $u + iv$ may not be analytical function
(C) $u, v$ are harmonic functions
(D) $u + iv$ is analytical function

107. If $f : G \to C$ is a differentiable with $f'(z) = 0$ for all $z \in G$ and $G$ is open, then

(A) $f$ is constant function in each component of $G$
(B) $f$ is increasing function
(C) $f$ is decreasing function
(D) $f(z) \equiv 0$ for all $z$

108. If $z = a$ is an isolated singularity of $f$, then $a$ is the pole of $f$, if

(A) $\lim_{z \to a} |f(z)| = 0$
(B) $\lim_{z \to a} |f(z)| = a$
(C) $\lim_{z \to a} |f(z)| = \infty$
(D) $\lim_{n \to \infty} |f(z)| = 1$

109. A bounded entire function is constant. This statement is known as

(A) Cauchy's theorem
(B) Liouville's theorem
(C) Morera theorem
(D) Schwatz lemma

110. If C is the circle $|z - a| = r$, then $\int_C \frac{dz}{z - a}$ is

(A) $2\pi i$
(B) $-2\pi i$
(C) $\pi i$
(D) 0

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111. If $f(z)$ is an analytic function and $f'(z)$ is continuous at each point within or on a closed curve $C$, then $\int_C f(z)dz =$

(A) $\infty$ (B) 1
(C) 0 (D) $\pi$

112. Laurent's expansion of the function $1/(z^2-3z+2)$ for $|z|>2$ is

(A) $\sum_{n=0}^{\infty}\frac{2^n-1}{z^{n+1}}$ (B) $\sum_{n=0}^{\infty}\frac{2^n}{z^{n+1}}$
(C) $\sum_{n=0}^{\infty}\frac{2^n+1}{z^{n+1}}$ (D) $\sum_{n=0}^{\infty}\frac{2^n}{z^n}$

113. The symmetric point of $1+i$ with respect to the circle $|z-1|=2$ is

(A) $1-i$ (B) $1+4i$
(C) $1+2i$ (D) $-1-i$

114. The value of $\int_C \frac{e^{-z}}{z^2}dz$, where $C$ is the circle $|z|=1$ is

(A) $-1$ (B) $2\pi i$
(C) $-2\pi i$ (D) $2\pi$

115. A transformation of the type $w=\alpha z+\beta$, where $\alpha$ and $\beta$ are complex constants, is known as a

(A) translation
(B) magnification
(C) linear fractional transformation
(D) bilinear transformation

116. If $z = re^{i\theta}$, then the image of $\theta =$ constant under the mapping $w(z) = \mathrm{Re}^{i\phi} = iz^3$ is

(A) $\phi = 3\theta$
(B) $\phi = 3\theta + \pi/2$
(C) $\phi = 3\theta - \pi/2$
(D) $\phi = \theta^3$

117. Under the mapping $w = z + 2 - i$, the image of line $\mathrm{Im}\ z = 0$ is

(A) $\mathrm{Im}(w) = 1$
(B) $\mathrm{Im}(w) = -1$
(C) $\mathrm{Re}(w) = 1$
(D) $\mathrm{Re}(w) = 1$

118. The cross ratio of the four points $(z_1, z_2, z_3, z_4)$ is real if and only if the four points lie on a

(A) circle
(B) straight line
(C) circle and on a straight line
(D) circle or on a straight line

119. If $A$ and $B$ are finite sets, then

(A) $|A \cup B| = |A| + |B| - |A \cap B|$
(B) $|A \cap B| = |A| + |B| - |A \cup B|$
(C) $|A \cap B| = |A \cup B| - |A| + |B|$
(D) $|A \cup B| = |A| - |B| + |A \cap B|$

120. The unit digit of $2^{100}$ is

(A) 2
(B) 4
(C) 6
(D) 8

121. If $G$ is a group such that $a^2 = e, \forall a \in G$, then $G$ is

(A) abelian group
(B) non- abelian group
(C) ring
(D) field

122. If $G$ is a group, then for all $a, b \in G$

(A) $(ab)^{-1} = a^{-1}b^{-1}$
(B) $(ab)^{-1} = b^{-1}a^{-1}$
(C) $(ab)^{-1} = ab$
(D) $(ab)^{-1} = ba$

123. For two subgroups $H$ and $K$ of group $G$, $HK$ is a sub-group of $G$ if and only if

(A) $HK = KH$
(B) $HK \subset KH$
(C) $HK \supset KH$
(D) $HK \neq KH$

124. If $G$ is a finite group of order $n, a \in G$ and order of $a$ is $m$, if $G$ is cyclic, then

(A) $m = n$
(B) $m > n$
(C) $m < n$
(D) $m \leq n$

125. If order of group $G$ is $p^2$, where $p$ is prime, then $G$ is

(A) abelian
(B) not abelian
(C) ring
(D) cyclic

126. If $G$ is a finite group and $H$ is a normal subgroup of $G$, then $o(G/H)$ is

(A) $\frac{o(H)}{o(G)}$
(B) $\frac{o(G)}{o(H)}$
(C) $o(G).o(H)$
(D) $o(H)$

127. If U is an ideal of ring R, then

(A) U/R is a ring
(B) R/U is a ring
(C) R/U is an integral domain
(D) R/U is a field

128. Let $X$ and $Y$ are topological spaces. A function $f : X \to Y$ is a continuous function,

(A) if for each open subset $V$ of $Y$, the set $f^{-1}(V)$ is a closed subset of $X$

(B) if for each closed subset $V$ of $Y$, the set $f^{-1}(V)$ is an open subset of $X$

(C) if for each open subset $V$ of $Y$, the set $f^{-1}(V)$ is an open subset of $X$

(D) if for each closed set $V$ of $Y$, $f^{-1}(V)$ is both open and closed

129. If $X$ is any set, $T$ is a collection of all subsets of $X$, then topology $(X,T)$ is

(A) a discrete topology
(B) a trivial topology
(C) an indiscrete topology
(D) a metric space

130. If $Y$ is a subspace of $X$, $A \subset Y$ and $\overline{A}$ is a closure of $A$ in $X$. Then, closure of $A$ in $X$ is equal to

(A) $\overline{A} \cap Y$
(B) $Y$
(C) $A \cup Y$
(D) $A$

131. Every non-empty set of real numbers that has a lower bound has

(A) a supremum
(B) an infimum
(C) neither supremum nor infimum
(D) both supremum and infimum

132. If $x$ and $y$ are two real numbers with $x > 0$, then there exists positive integer $n$ such that $nx$ is

(A) $> y$
(B) $< y$
(C) $= y$
(D) $\geq y$

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133. Let $f(x) = \begin{cases} 1 \,;\, x \text{ rational} \\ -1 \,;\, x \text{ irrational} \end{cases}$ in [0,1]. Then in [0,1]

(A) $f(x)$ is continuous everywhere
(B) $f(x)$ is Riemann integrable
(C) $f(x)$ is not Riemann integrable
(D) $f(x)$ is continuous only at the rationals

134. Which of the following statements is not true?

(A) Every cyclic group is abelian
(B) Every subgroup of a cyclic group is cyclic
(C) Every group of prime order is cyclic
(D) Every abelian group is cyclic

135. Which of the following statements is not correct?

(A) Isomorphism is 1-1 onto homomorphism
(B) Onto homomorphism is epimorphism
(C) Isomorphism is an equivalence relation among groups
(D) Every isomorphism is an automorphism

136. Which of the following statements is correct?

(A) Every group is a subfield of a field
(B) Every group is a field
(C) Every integral domain is a field
(D) Every finite integral domain is a field

137. The polar form of the complex number $-5+5\,i$ is

(A) $5\sqrt{2}e^{\frac{3\pi i}{4}}$
(B) $5\sqrt{3}e^{\frac{3\pi i}{4}}$
(C) $25\sqrt{2}e^{\frac{3\pi i}{4}}$
(D) $5\sqrt{2}e^{\frac{-3\pi i}{4}}$

138. Log (1 + $i$) is equal to

(A) $\log\left(\sqrt{2}\right)+i(8n+1)\frac{\pi}{4}$

(B) $\log\left(\sqrt{2}\right)+i(6n+1)\frac{\pi}{4}$

(C) $\log\left(\sqrt{2}\right)+i(4n-1)\frac{\pi}{4}$

(D) $\log\left(\sqrt{2}\right)-i(n-1)\frac{\pi}{4}$

139. The real part of $\cos h(x+iy)$ is

(A) $\cos h\, x \cos y$

(B) $\sin x \sin h\, y$

(C) $\cos x \cos h\, y$

(D) $\sin h\, x \cos y$

140. If $y = \cos(x-y)$ , then $\frac{dy}{dx}$ is equal to

(A) $-\sin(x-y)$

(B) $\sin(x-y)\frac{dy}{dx}$

(C) $\frac{\sin(x-y)}{\sin(x-y)-1}$

(D) $\frac{\sin(x-y)}{\sin(x-y)+1}$

141. If $u = x^y$ , then $\frac{\partial u}{\partial x}$ is equal to

(A) $uxy$

(B) $u\frac{y}{x}$

(C) $u\frac{x}{y}$

(D) $ux^2y^2$

142. Which of the following statements is not correct for polynomials with real coefficients?

(A) Every polynomial of degree $\geq 1$ has at most one zero

(B) If $h$ is a zero of the polynomial $f(x)$, then $x-h$ is a factor of $f(x)$

(C) Every polynomial of degree $n$ has exactly $n$ roots

(D) Complex roots occur in conjugate pairs

143. If the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ are in A.P., then the roots are

(A) 1, 2, 3
(B) −1, 2, 3
(C) 1, −2, 3
(D) −1, −2, −3

144. The equation whose roots are three times those of the equation $2x^3 - 5x^2 + 7 = 0$ is

(A) $2x^3 + 5x^2 + 189 = 0$
(B) $2x^3 + 15x^2 + 197 = 0$
(C) $2x^3 - 15x^2 + 187 = 0$
(D) $2x^3 - 15x^2 + 189 = 0$

145. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, then $x$ is equal to

(A) 3
(B) 5
(C) 2
(D) 4

146. The value of $m$ so that vector $(m, 3, 1)$ is linear combination of the vectors $(3, 2, 1)$ and $(2, 1, 0)$, is

(A) 1
(B) 3
(C) 5
(D) 7

147. If $S$ is a set of vectors containing the zero vector, then

(A) $S$ is linearly dependent
(B) $S$ is linearly independent
(C) $S$ may be linearly dependent or linearly independent
(D) $S$ contains only one element

148. The scalar $\lambda$ is a character root of the matrix $A$ if

(A) $(A-\lambda I)$ is non-singular
(B) $(A-\lambda I)$ is singular
(C) $A$ is non-singular
(D) $A$ is singular

149. If $M(x,y)dx + N(x,y)dy = 0$ and $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, then the equation is

(A) exact
(B) not exact
(C) linear
(D) not linear

150. The integrating factor for the linear differential equation $\frac{dy}{dx} + Py = Q$ is

(A) $\int Pdx$
(B) $\int Qdx$
(C) $\exp(\int Qdx)$
(D) $\exp(\int Pdx)$

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