MATHEMATICS

(Maximum Marks: 100)

(Time allowed: Three hours)

*(Candidates are allowed additional 15 minutes for **only** reading the paper.*

They must NOT start writing during this time.)

Section A - *Answer* **Question 1** *(compulsory) and* **five** *other questions.*

Section B and Section C - *Answer* **two** *questions from* **either** *Section B* **or** *Section C.*

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

Slide rule may be used.

SECTION A (80 Marks)

[10×3]

Question 1

(i) Find the matrix X for which:

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

(ii) Solve for x, if:

$$\tan(\cos^{-1}x) = \frac{2}{\sqrt{5}}$$

(iii) Prove that the line $2x - 3y = 9$ touches the conics $y^2 = -8x$. Also, find the point of contact.

(iv) Using L'Hospital's Rule, evaluate:

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{\cot x}{x}\right)$$

(v) Evaluate: $\int tan^3 x\, dx$

(vi) Using properties of definite integrals, evaluate:

$$\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}\, dx$$

(vii) The two lines of regressions are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ and the variance of x is 12. Find the variance of y and the coefficient of correlation.

(viii) Express $\dfrac{2 + i}{(1 + i)(1 - 2i)}$ in the form of $a + ib$. Find its modulus and argument.

This Paper consists of 5 printed pages and 1 blank page.

(ix) A pair of dice is thrown. What is the probability of getting an even number on the first die or a total of 8?

(x) Solve the differential equation:

$$x\frac{dy}{dx} + y = 3\,x^2 - 2$$

Question 2

(a) Using properties of determinants, prove that: **[5]**

$$\begin{vmatrix} b+c & a & a \\ b & a+c & b \\ c & c & a+b \end{vmatrix} = 4abc$$

(b) Solve the following system of linear equations using matrix method: **[5]**

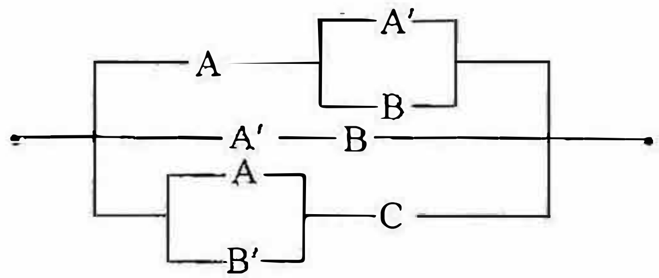
$$3x + y + z = 1,\ 2x + 2z = 0,\ 5x + y + 2z = 2$$

Question 3

(a) If $\sin^{-1}x + \tan^{-1}x = \frac{\pi}{2}$, prove that: **[5]**

$$2x^2 + 1 = \sqrt{5}$$

(b) Write the Boolean function corresponding to the switching circuit given below: **[5]**



A, B and C represent switches in 'on' position and A′, B′ and C′ represent them in 'off' position. Using Boolean algebra, simplify the function and construct an equivalent switching circuit.

Question 4

(a) Verify the conditions of Rolle's Theorem for the following function: **[5]**

$f(x) = \log(x^2 + 2) - \log 3$ on $[-1,1]$

Find a point in the interval, where the tangent to the curve is parallel to x-axis.

(b) Find the equation of the standard ellipse, taking its axes as the coordinate axes, whose minor axis is equal to the distance between the foci and whose length of latus rectum is 10. Also, find its eccentricity. **[5]**

Question 5

(a) If $\log y = \tan^{-1}x$, prove that: [5]

$$(1 + x^2)\ \frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx} = 0$$

(b) A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle to get maximum area. Also, find the maximum area. [5]

Question 6

(a) Evaluate: [5]

$$\int \frac{\sin x + \cos x}{\sqrt{9 + 16 \sin 2x}}\, dx$$

(b) Find the area of the region bound by the curves $y = 6x - x^2$ and $y = x^2 - 2x$. [5]

Question 7

(a) Calculate Karl Pearson's coefficient of correlation between x and y for the following data and interpret the result: [5]

(1, 6), (2, 5), (3,7), (4, 9), (5, 8), (6, 10), (7, 11), (8, 13), (9, 12)

(b) The marks obtained by 10 candidates in English and Mathematics are given below: [5]

Marks in English	20	13	18	21	11	12	17	14	19	15
Marks in Mathematics	17	12	23	25	14	8	19	21	22	19

Estimate the probable score for Mathematics if the marks obtained in English are 24.

Question 8

(a) A committee of 4 persons has to be chosen from 8 boys and 6 girls, consisting of at least one girl. Find the probability that the committee consists of more girls than boys. [5]

(b) An urn contains 10 white and 3 black balls while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the second urn. Find the probability that the ball drawn from the second urn is a white ball. [5]

Question 9

(a) Find the locus of a complex number, $z = x + iy$, satisfying the relation $\left|\frac{z-3i}{z+3i}\right| \leq \sqrt{2}$. Illustrate the locus of z in the Argand plane. **[5]**

(b) Solve the following differential equation: **[5]**

$$x^2\, dy + (xy + y^2)\, dx = 0, \text{ when } x = 1 \text{ and } y = 1.$$

SECTION B (20 Marks)

Question 10

(a) For any three vectors $\vec{a}, \vec{b}, \vec{c}$, show that $\vec{a} - \vec{b},\ \vec{b} - \vec{c}, \vec{c} - \vec{a}$ are coplanar. **[5]**

(b) Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ **[5]**

Question 11

(a) Find the image of the point $(2, -1, 5)$ in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$ **[5]**

Also, find the length of the perpendicular from the point $(2, -1, 5)$ to the line.

(b) Find the Cartesian equation of the plane, passing through the line of intersection of the planes: $\vec{r}.(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}) + 5 = 0$ and $\vec{r}.(\hat{\imath} - 5\hat{\jmath} + 7\hat{k}) + 2 = 0$ and intersecting y-axis at $(0, 3)$. **[5]**

Question 12

(a) In an automobile factory, certain parts are to be fixed into the chassis in a section before it moves into another section. On a given day, one of the three persons A, B and C carries out this task. A has 45% chance, B has 35% chance and C has 20% chance of doing the task. The probability that A, B and C will take more than the allotted time is $\frac{1}{6}, \frac{1}{10}$ *and* $\frac{1}{20}$ respectively. If it is found that the time taken is more than the allotted time, what is the probability that A has done the task? **[5]**

(b) The difference between mean and variance of a binomial distribution is 1 and the difference of their squares is 11. Find the distribution. **[5]**

SECTION C (20 Marks)

Question 13

(a) A man borrows ₹ 20,000 at 12% per annum, compounded semi-annually and agrees to pay it in 10 equal semi-annual installments. Find the value of each installment, if the first payment is due at the end of two years. **[5]**

(b) A company manufactures two types of products A and B. Each unit of A requires 3 grams of nickel and 1 gram of chromium, while each unit of B requires 1 gram of nickel and 2 grams of chromium. The firm can produce 9 grams of nickel and 8 grams of chromium. The profit is ₹ 40 on each unit of product of type A and ₹ 50 on each unit of type B. How many units of each type should the company manufacture so as to earn maximum profit? Use linear programming to find the solution. **[5]**

Question 14

(a) The demand function is $x = \frac{24 - 2p}{3}$ where x is the number of units demanded and p is the price per unit. Find: **[5]**

(i) The revenue function R in terms of p.

(ii) The price and the number of units demanded for which the revenue is maximum.

(b) A bill of ₹ 1,800 drawn on 10^{th} September, 2010 at 6 months was discounted for ₹ 1,782 at a bank. If the rate of interest was 5% per annum, on what date was the bill discounted? **[5]**

Question 15

(a) The index number by the method of aggregates for the year 2010, taking 2000 as the base year, was found to be 116. If sum of the prices in the year 2000 is ₹ 300, find the values of x and y in the data given below: **[5]**

Commodity	A	B	C	D	E	F
Price in the year 2000 (₹)	50	x	30	70	116	20
Price in the year 2010 (₹)	60	24	y	80	120	28

(b) From the details given below, calculate the five yearly moving averages of the number of students who have studied in a school. Also, plot these and original data on the same graph paper. **[5]**

Year	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Number of Students	332	317	357	392	402	405	410	427	405	438
