MODEL QUESTION PAPER

M.Sc., MATHEMATICS (PREVIOUS)

PAPER I - ALGEBRA

Answer any ***THREE*** *questions*
All questions carry equal marks

1. a) If a permutation is a product of s transpositions and also a product of t transpositions, show that both s and t are even or odd.

   b) Express the following as the product of disjoint cycles.
   i) $(1,2,3)(4,5)(1,6,7,8,9)(1,5)$ ii) $((1,2)(1,2,3)(1,2)$

2. a) If $O(G) = p^2$ and p is a prime number, show that G is abelian.

   b) If G is finite group and if a prime number p divides $O(G)$, then show that G has an element of order p.

3. a) Define an integral domain and show that a finite integral domain is a field.

   b) State and prove the fundamental theorem of homomorphism for rings.

4. a) Define a maximal ideal in a ring. Determine all maximal ideals in the ring $(Z, +, .)$ of integers.

   b) Show that every integral domain can be imbedded in a field.

5. a) Define a Euclidean ring. Show that every ideal in a Euclidean ring is principal ideal.

   b) Show that every pair of elements in a Euclidean ring have a greatest common divisor.

6. a) Find all units in $J[i]$.

P.T.O.

b) If p is a prime number of the form $4n+1$, show that $p = a^2 + b^2$ for some integers a and b.

7. a) Show that the polynomial ring, over a field F is a Euclidean ring.

b) State and prove Gauss lemma for primitive polynomials.

8. a) Prove the following:

i) If $v_1, v_2, ..., v_n$ in a vector space V over a field F are linearly independent over F, then show that every element in their linear span has a unique representation of the form $\lambda_1 v + \lambda_2 v_2 + ... + \lambda_n v_n$, with $\lambda_{i \in F}$

ii) If $v_1, v_2, ..., v_n$ are in V, show that either they are linearly independent or some v_R is a linear combination of $v_1, v_2, ..., v_{R-1}$s

b) If V is finite dimensional and T is an isomorphism of V into V, prove that T must map V onto V.

9. a) If V is finite-dimensional over F then $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial of T is not O.

b) Define the rank $r(T)$ of a linear transformation T on a finite dimensional vector space V over F. Show that

i) $r(ST) \leq r(T)$; $r(TS) \leq r(T)$ and ii) $r(ST) = r(TS) = r(T)$ for some regular S in $A(V)$

10. a) If $A \in F$ is a characteristic roof of $T \in A(V)$, show that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.

b) If $\lambda_1, \lambda_2, ..., \lambda_k$ is F are distinct characteristic roots of $T \in A(V)$ and if $v_1, v_2, ..., v_k$ are characteristic vectors of T belonging to $\lambda_1, \lambda_2, ..., \lambda_k$ respectively, show that $v_1, v_2, ..., v_k$ are linearly independent.

MODEL QUESTION PAPER

M.Sc., MATHEMATICS (PREVIOUS)

PAPER II – REAL ANALYSIS

Answer any ***THREE*** *questions*
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1. a) Show that every k-cell is compact.

   b) Prove that every closed subset of a compact set is compact.

2. a) Let f be a continuous mapping of a compact metric space x into a metric space y. Then prove that f is uniformly continuous on x.

   b) Let f be a function defined on R^1 by $f(x) = \begin{cases} 0 & (x \text{ is irrational}) \\ 1/n & (x = m/n) \end{cases}$

   then prove that f is continuous at every irrational point and that f has a simple discontinuity at every rational point.

3. a) Let f be defined by $f(x) = \begin{cases} x^2 \ Sin\, 1/x & (x \neq 0) \\ 0 & (x = 0) \end{cases}$.

   Then show that f is differentiable at all points x. Verify the continuity of f^1 i.e. $f^1(0)$ exists or not

   b) State and prove Taylor's theorem.

4. a) If $\overline{f}$ maps on $[a,b]$ in to R^K and if $\left|\overline{f}\right| \in R(\alpha)$ for some monotonically increasing function α on $[a, b]$ then prove that $\left|\overline{f}\right| \in R(\alpha)$ and $\left|\int_a^b \overline{f}\, d\alpha\right| \leq \int_a^b \left|\overline{f}\right| d\alpha$.

   b) State and prove fundamental theorem of calculus.

5. a) State and prove Cauchy criterion for uniform convergence of functions.

P.T.O.

b) Suppose k is compact and

i) $\{f_n\}$ is a sequence of continuous functions on k.

ii) $\{f_n\}$ converges point wise to a continuous function f on k.

iii) $f_n(x) \geq f_{n+1}(x) \quad \forall x \in K,\ n = 1, 2 \ldots\ldots$ then prove that $f_n \to f$ uniformly on k.

6. a) Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in R(\alpha)$ on $[a, b]$ for $n = 1, 2 \ldots\ldots$ and suppose $f_n \to f$ uniformly on $[a, b]$. Then show that $f \in R(\alpha)$ on $[a, b]$ and $\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n\, d\alpha$.

   b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

7. Define finite μ-measurable f_n. Show that $m(\mu)$ is a σ-ring and μ^* is countably additive on $m(\mu)$.

8. a) Define a measurable function. Show that if f is measurable then $|f|$ is measurable.

   b) Let $\{f_n\}$ be a sequence of measurable functions. For $x \in X$ put $g(x) = \sup f_n(x)\ (n = 1, 2 \ldots\ldots)$, $h(x) = \limsup_{n\to\infty} f_n(x)$. Then show that g and h are measurable.

9. a) If $f \in \mathcal{L}(\mu)$ on E then prove that $|f| \in \mathcal{L}(\mu)$ on E and $\left| \int_E f\, d\mu \right| \leq \int_E |f|\, d\mu$.

   b) State and prove Fatou's theorem.

10. State and prove Lebesque dominated converging theorem.

MODEL QUESTION PAPER

M.Sc., MATHEMATICS (PREVIOUS)

PAPER III – DIFFERENTIAL EQUATIONS

Answer any ***THREE*** *questions*
All questions carry equal marks

All questions carry equal marks

1. a) Show that the particular solution to the differential equation $(1+x)y' = py,\ y(0)=1$ is $y=(1+x)^p$.
   b) Find the power series solution of $y''+y=0$.

2. Find power series solution of the Legendre equation $(1-x^2)y''-2x\ y'+p(p+1)y=0$ where p is constant.

3. a) Obtain $J_p(x)$ for the Bessel equation
   $$x^2y''+xy'+(x^2-p^2)y=0$$
   b) Prove that i) $\dfrac{d}{dx}\left[x^p\ J_p(x)\right]=x^p\ J_{p-1}(x)$
   ii) $\dfrac{d}{dx}\left[x^{-p}\ J_p(x)\right]=-x^{-p}\ J_{p+1}(x)$

4. a) Obtain the Orthogonality properties of Bessel equation.
   b) Express $J_2(x)$ and $J_3(x)$ interms of $J_0(x)$ and $J_1(x)$.

5. State and prove Picard's theorem.

6. Let $f(x,y)$be a continuous function that satisfies a Lipschitz condition $|\ f(x,y_1)-f(x,y_2)| \le K\ |y_1-y_2|$ on a strip defined by $a\le x\le b$and $-\infty<y<\infty$. If $(x_0,y_0)$is any point of the strip then prove that the $I.V.P$ $y'=f(x,y), y(x_0)=y_0$ has one and only one solution $y=y(x)$ on the interval $a\le x\le b$.

P.T.O.

7. a) If u is a function of x, y and z which satisfies the partial differential equation $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$. Show that u contains x, y and z only in combinations $x+y+z$ and $x^2+y^2+z^2$.

   b) Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1,\ z=1$.

8. a) Find the characteristics of the equation $pq=z$ and determine the integral surface which passes through the parabola $x=0,\ y^2=z$

   b) Find the complete integral of the equation $p^2x+q^2y=z$ by charpits method.

9. a) Find a particular integral of the equation $(D^2-D')z=e^{x+y}$.

   b) Solve the equation $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial y}-\frac{\partial^3 z}{\partial x\partial y^2}+2.\frac{\partial^3 z}{\partial y^3}=e^{x+y}$

10. Reduce the equation

$$y^2.\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x\partial y}+x^2.\frac{\partial^2 z}{\partial y^2}=\frac{y^2}{x}\frac{\partial z}{\partial x}+\frac{x^2}{y}\frac{\partial z}{\partial y}$$ to canonical

form and hence solve it.

MODEL QUESTION PAPER

M.Sc., MATHEMATICS (PREVIOUS)

PAPER IV – LINEAR PROGRAMMING

Answer any ***THREE*** *questions*
All questions carry equal marks

1. a) Explain the origin and development of operations Research in brief.
   b) Detail the various steps involved in mathematical formulation of the problem.

2. a) Solve the following LPP graphically.

   Minimize $Z = 3x_1 + 5x_2$
   subject to $x_1 - x_2 \leq 1$
   $x_1 + x_2 \geq 3$ and $x_1 + x_2 \geq 0$

   b) Show that every extreme point of the set of all feasible solutions of all LPP is a basic feasible solution.

3. a) State and prove fundamental theorem of Linear Programming problem.
   b) Solve the following LPP by using Big *M*-method.

   Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$
   subject to $x_1 + 2x_2 + 3x_3 = 15$
   $2x_1 + x2 + 5x_3 = 20$
   $x_1 + 2x_2 + x_3 + x_4 = 10$
   $x_1, x_2, x_3, x_4 \geq 0$

4. Use Two-phase Simplex method to

   Maximize $Z = 5x_1 - 4x_2 + 3x_3$
   Subject to the constraints:
   $2x_1 + x_2 - 6x_3 = 20$
   $6x_1 + 5x_2 + 10x_3 \leq 76$
   $8x_1 + 3x_2 + 6x_3 \leq 50$
   $x_1, x_2, x_3, \geq 0$

   b) What is degeneracy? How you will resolve it?

P.T.O.

5. a) Define standard primal problem and also give various steps involved in the formulation of a primedual pair.
   b) Using the dual, solve the following LPP:

   Maximize $Z = 3x_1 - 2x_2$
   subject to $x_1 \leq 4$
   $x_2 \leq 6$
   $x_1 + x_2 \leq 5$
   $-x_2 \leq -1$
   $x_1, x_2 \geq 0$

6. a) State and prove complementary slackness theorem.
   b) Obtain dual for the following LPP:

   Maximize $Z = 2x_1 + 5x_2 + 6x_3$
   subject to $5x_1 + 6x_2 - x_3 \leq 3$
   $-2x_1 + x_2 + 4x_3 \leq 4$
   $x_1 - 5x_2 + 3x_3 \leq 1$
   $-3x_1 - 3x_2 + 7x_3 \leq 0$
   $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

7. a) Write dual simplex algorithm to solve the given LPP.
   b) Use dual simplex method to solve the LPP given below:

   Maximize $Z = x_1 + x_2$
   subject to $2x_1 + x_2 \geq 4$
   $x_1 + 7x_2 \geq 7$
   $x_1, x_2 \geq 0$

8. a) What are the major steps involved in Revised simplex algorithm?
   b) Use revised simplex method to solve the following LPP:

   Maximize $Z = 3x_1 + 2x_2 + 5x_3$
   Subject to the constraints:
   $x_1 + 2x_2 + x_3 \leq 430$
   $3x_1 + 2x_3 \leq 460$
   $x_1 + 4x_2 \leq 420$
   $x_1, x_2, x_3 \geq 0$

P.T.O.

9. a) Define Loops in transportation tables and give its remarks in detail.

   b) Find the optimal solution for the following transportation problem by using least-cost method.

	D_1	D_2	D_3	D_4	*Capacity*
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

   Where O_i and D_j denote the i[th] origin and j[th] destination separately.

10. a) Explain unbalanced transportation problem with all necessary details.

    b) The *XYZ* company has 5 Jobs *I, II, III, IV, V* to be done and 5 men *A, B, C, D, E* to do these jobs. The number of hours each man would take to accomplish each job is given by the following table:

	A	*B*	*C*	*D*	*E*
I	16	13	17	19	20
II	14	12	13	16	17
III	14	11	12	17	18
IV	5	5	8	8	11
V	5	3	8	8	10

    Work out the optimum assignment and the total minimum time taken.

MODEL QUESTION PAPER

M.Sc., MATHEMATICS (PREVIOUS)

PAPER V - TOPOLOGY

Answer any ***THREE*** *questions*
All questions carry equal marks

1. a) Let $X = \{a,b,c,d,e\}$. Test whether $\tau_1 = \{X, \phi, \{a,b,c\}, \{a,b,d\}, \{a,b,c,d\}\}$ is a topology on X.

   b) Prove that is A and B are subsets of a topological space (X, τ), then $(A \cup B)' = A' \cup B'$.

2. a) Prove that any class A of subsets of a nonempty X is the sub base for a unique topology on X.

   b) Show that every second countable space is also first countable.

3. a) Define a compact topological space. Show that continuous images of compact sets are compact.

   b) Show that every closed subset of a compact space is also compact.

4. a) Prove that every compact Hausdorff space is normal.

   b) If A is a compact subset of a Hausdorff space X and $p \notin A$ then there is an open set G such that $p \in G \subset A^C$.

5. a) Show that a complete regular space is also regular.

   b) Prove that every second countable normal T_1 -space is metrizable.

6. a) Show that a finite subset of T_1 -space X has no accumulation points.

P.T.O.

b) Let F_1 and F_2 be disprint closed subsets of a normal space X. Then show that there exists a continuous function $f : X \to [0,1]$ such that $f[F_1] = (0)$ and $f[F_2] = 1$.

7. a) Prove that the Euclidian space R^m is connected.

   b) Show that if $\lfloor A_i \rfloor$ is a class of connected subsets of X such that no two members of it one separated then $U A_i$ is connected.

8. a) Prove that the components of a totals disconnected space X are the singleton subset of X.

   b) Show that a totally disconnected space is Hausdorff.

9. a) State and prove Weierstrass approximation theorem.

   b) Prove that if the components of a compact space are open then there are only a finite number of them.

10. Establish the extended Stone-Weierstrass theorem