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Reg. No. : ....

D 1078 **Q.P. Code : [D 07 PMA 01]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

First Year

Part I – Mathematics

ALGEBRA

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each questions carries 20 marks.

$(5 \times 20 = 100)$

1. (a) If G is a finite group, then prove that the number of elements conjugate to a in G is the index of the normalizer of a in G.

   (b) If G is a group, Z its center, and if G/Z is cyclic, prove that G must be abelian.

2. (a) State and prove sylow's theorem for general group.

(b) Prove that G is the internal direct product of the normal subgroups $N_1, N_2 \ldots N_n$ if and only if

(i) $G = N_1 \cdots N_n$;

(ii) $N_i \cap (N_1 N_2 \cdots N_{i-1} N_{i+1} \cdots N_n) = (e)$ for $i = 1,2,\ldots n$.

3. (a) Prove that the ideal $A = (a_0)$ is a maximal ideal of Euclidean ring R if and only if a_0 is a prime element of R.

(b) Prove that Gaussian integer $J[i]$ is a Euclidean ring.

4. (a) Prove that $x^3 - 9$ is irreducible over the integers mod 31.

(b) If the primitive polynomial $f(x)$ can be factored as the product of two polynomials having rational coefficients, then prove that it can be factored as the product of two polynomials having inter co-efficients.

8. (a) Let $T \in A(V)$. Prove that there exists a sub space W of V, invariant under T, such that $V = V_1 \oplus W$.

(b) Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

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5. (a) Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

(b) If F is of characteristic O and if a,b are algebraic over F, then prove that there exists an element $c \in F(a,b)$ shuch that $F(a,b) = F(c)$.

6. (a) Prove that K is a normal extension of F if and only if K is the splitting field of same polynomial over F.

(b) Let K be a normal extension of F and let H be a subgroup of $G(K,F)$; let $K_H\{x \in K \mid \sigma(x) = x \text{ for all } \sigma \in H\}$ be the fixed field of H. then prove that

(i) $[K : K_H] = O(H)$; and

(ii) $H = G(K, K_H)$.

7. (a) IF $p(x) \in F[x]$ is solvable by radical over F, then prove that the Galois group over F of $p(x)$ is a solvable group.

(b) If V is n – dimensional over F and if $T \in A(V)$ has all its characteristics roots in F, then prove that T satisfies a polynomial of degree n over F.

Reg. No. : ....

D 1079 **Q.P. Code : [D 07 PMA 02]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

First Year

Mathematics

REAL ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each questions carries 20 marks.

$(5 \times 20 = 100)$

1. (a) If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$, then prove that $f \in R(\alpha)$.

(b) Suppose ϕ is a strictly increasing continuous function that maps an interval $[A, B]$ onto $[a, b]$. Suppose α is monotonically increasing on $[a, b]$ and $f \in R(\alpha)$ on $[a, b]$. Define β and g on $[A, B]$ by $\beta(y) = \alpha(\phi(y))$, $g(y) = f(\phi(y))$.

Then prove that $g \in R(\beta)$ and $\int_A^B g\, d\beta = \int_a^b f\, d\alpha$.

2. (a) State and prove fundamental theorem of calculus.

(b) If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

3. (a) Suppose K is compact and

(i) $\{f_n\}$ is a sequence of continuous functions on K,

(ii) $\{f_n\}$ converges point wise to a continuous function f on K,

(iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$, $n = 1, 1, 3, \ldots$ Then prove that $f_n \to f$ uniformly on K.

(b) If K is compact, if $f_n \in \mathcal{C}(k)$, for $n = 1, 2, 3, ..$ and if $\{f_n\}$ is point wise bounded and equicontinuous on K, then prove that $\{f_n\}$ is uniformly bounded on K and $\{f_n\}$ contains a uniformly convergent subsequence.

4. (a) Prove that if f is a continuous complex function on $[a, b]$, there exists a sequence of polynomials P_n such that $\lim_{n\to\infty} P_n(x) = f(x)$. Uniformly on $[a, b]$. Also if f is real, prove that P_n may be taken real.

(b). Suppose f maps an open set $E \subset R^n$ into R^m and f is differentiable at a point $x \in E$. Prove that the partial derivatives $(D_j f_i)(x)$ exist and $f'(x) e_j = \sum_{j=1}^{m} (D_j k_i)(x) u$; $(1 \le j \le n)$, where $\{e_1, e_2, \ldots e_n\}$ and $\{u_1, u_2, \ldots, u_m\}$ are the standard bases of R^n and R^m.

5. (a) Suppose f maps an open set $E \subset R^n$ into R^m. Prove that $f \in \mathscr{C}'(E)$ if and only if the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \le i \le m$, $1 \le j \le n$.

(b) Suppose

(i) $\phi(x, t)$ is defined for $a \le x \le b$, $c \le t \le d$;

(ii) α is an increasing function on $[a, b]$;

(iii) $\phi^t \in R(\alpha)$ for every $t \in [c, d]$;

(iv) $c < s < d$ and to every $\in > 0$ corresponds a $\delta > 0$ such that

$|(D_2 \phi)(x, t) - (D_2 \phi)(x, s)| < \in$ for all

$x \in [a, b]$ and for all $t \in (s - \delta, s + \delta)$. Define

$f(t) = \int_a^b \phi(x, t) d\alpha(x) \quad (c \le t \le d)$. Then

prove that $(D_2 \phi)^s \in R(\alpha)$, $f'(s)$ exists and

$$f'(s) = \int_a^b (D_2 \phi)(x, s)\, d\alpha(x).$$

6. (a) Let $< f_n >$ be a sequence of measurable functions. Prove that the functions sup $\{f_1, ..., f_n\}$, inf $\{f_1, f_2, ..., f_n\}$, $\sup_n f_n$ and lim f_n are all measurable.

(b) Let E be a measurable set of finite measure and $< f_n >$ a sequence of measurable functions defined on E. Let f be a real-valued function such that for each x in E we have $f_n(x) \to f(x)$. Prove that given $\in > 0$ and $\delta > 0$, there is a measurable set $A \subset E$ with $mA < \delta$ and an integer N such that for all $x \notin A$ and all $n \geq N$, $|f_n(x) - f(x)| < \in$.

7. (a) Let $< f_n >$ be a sequence of measurable functions that converges in measure of f. Prove that there is a sub sequence $< f_{nk} >$ that converges to f almost every where.

(b) Let f be an increasing real-valued function on the interval $[a, b]$. Prove that f is differentiable almost everywhere and the derivative f' is measurable, also

$$\int_a^b f'(x)dx \leq f(b) - f(a).$$

8. (a) Prove that a function f is of bounded variation on $[a, b]$ if and only if f is the difference of two monotone real-valued functions on $[a, b]$.

(b) Prove that the L^p spaces are complete.

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D 1080 **Q.P. Code : [D 07 PMA 03]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

First Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

$(5 \times 20 = 100)$

1. (a) If ϕ is a fundamental matrix for $x' = A(t)x$ $(-\infty < t < \infty)$, prove that so is ψ, where $\psi(t) = \phi(t + w)$ and corresponding to every such ϕ, there exist a periodic non singular matrix P with period w and a constant matrix R such that $\phi(t) = p(t)\, e^{tR}$.

   (b) State and prove Cauchy-Peono existence theorem.

2. (a) State and prove the Picard's theorem.

(b) Compete to first four successive approximation of to equation $y' = x^2 + y^2$, $y(0) = 0$.

3. (a) Solve the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{with} \quad u(x,0) = x^3 \quad \text{and}$$

$$\frac{\partial u}{\partial t}(x,0) = \sin x.$$

(b) Solve the boundary value problem

$$u_{tt} = c^2 u_{xx} = 0\ ; \qquad 0 \le x \le l, t \ge 0$$

$$u(0,t) = u(l,t) = 0; \qquad t \ge 0$$

$$u(x,0) = 10 \sin\left(\frac{\pi x}{l}\right); \quad 0 \le x \le l$$

$$u_t(x,0) = 0$$

4. (a) Solve $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}; x > 0, t > 0$

given $u(x,0) = 0, \qquad \dfrac{\partial u(x,0)}{\partial t} = 0, \quad x > 0$

$$u(0,t) = \sin t, \quad t > 0$$

(b) State and explain the Cauchy-Kowlalewsky theorem.

5. (a) Solve using the method of separation of variables.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \text{ subject to}$$

(i) $u(0,t) = u(2,t) = 0$

(ii) $u(x,0) = \sin^2 \frac{\pi x}{2}$

(iii) $\frac{\partial u}{\partial t}(x,0) = 0$.

(b) State and prove the maximum principle.

6. (a) Solve $r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$ with $u(r,0) = 0$ in $0 \le r \le a$, $u(r,\pi) = 0$ in $0 \le r \le a$, $u(a,\theta) = T$.

(b) Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with the conditions $u(0,t) = 0$, $u(2l,t) = 0$ for $t \ge 0$

$u(x,0) = 0$, $\frac{\partial u}{\partial t}(x,0) = K(2lx - x^2)$ for $0 \le x \le 2l$

7. (a) State the Dirichlet problem for a circle and obtain its solution.

(b) State the Neumann's problem for a rectangle and obtain its solution.

8. (a) State the Dirichlet problem for a circular annulus.

(b) Obtain the solution of Neumann's problem for a circle.

Reg. No. : ....

D 1081 **Q.P. Code : [D 07 PMA 04]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

First Year

Maths

NUMERICAL METHODS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

($5 \times 20 = 100$)

1. (a) Find an iterative formula to find $\sqrt{n}$ (where N is a positive number) and hence find $\sqrt{5}$

(b) Find the first and second derivative of the function tabulated below at $x = 0.6$.

x:	0.4	0.5	0.6	0.7	0.8
y:	1.5836	1.7974	2.0442	2.3275	2.6511

2. (a) Compute the value of $\int_{0.2}^{1.4}(\sin x - \log n + e^x)dx$.

Taking $h = 0.2$ and using Trapezoidal rule and Simpson's rule.

(b) From the following table, estimate $e^{0.644}$. Correct to 5 decimals using Bessel's formula.

x :	0.61	0.62	0.63	0.64
e^x :	1.840431	1.858928	1.877610	1.896481
x :	0.65	0.66	0.67	
e^x :	1.915541	1.934792	1.954237	

3. (a) Apply Gauss-Jordan method to find the solution of the following system :

$10x + y + 3 = 12;\ 2x + 10y + z = 13;$

$x + y + 5z = 7$.

(b) Solve the following system of equation by using Gauss-Seidal methods (Correct to 3 decimal places)

$8x - 3y + 2z = 20$

$4x + 11y - 3 = 33$

$6x + 3y + 12z = 35$

4. By the method of triangularization, solve the following system :

$$5x - 2y + 3 = 4, 7x + y - 5z = 8; \quad 3x + 7y + 4z = 10$$

5. (a) Solve $\frac{dy}{dx} = x + y$, given $y(1) = 0$ and get $y(1.1)$ by Taylor's method.

(b) Using R.K.method of Fourth order, find $y(0.7)$ correct to 4 decimal places if $y' = y - x^2, y(0.6) = 1.7379$.

6. (a) Using Euler's method, solve numerically the equation $y' = x + y, y(0) = 1$ for $x = 0$ to 1 and $h = 0.2$.

(b) Find $y(2)$ if $y(x)$ is the solution of $\frac{dy}{dx} = \frac{1}{2}(x + y)$ given $y(0) = 2$, $y(0.5) = 2.636$, $y(1) = 3.595$ and $y(1.5) = 4.968$ using Milne's method.

7. Using power method, find all the eigen values of

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

8. (a) Solve $\nabla^2 u = 8x^2y^2$ for square mesh given $u = 0$ on the 4 boundaries, dividing the square into 16 sub-squares of length 1 unit.

(b) Using Crank-Nicholson's scheme. Solve $u_{xx} = 16u_t, 0 < x < 1, t > 0$ given $u(x,0) = 0$, $u(0,t = 0, u(1,t) = 100t$. Compute u for one step in t direction taking $h = \frac{1}{4}$.

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D 1081 **Q.P. Code : [D 07 PMA 04]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

First Year

Maths

NUMERICAL METHODS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

$(5 \times 20 = 100)$

1. (a) Find an iterative formula to find $\sqrt{n}$ (where N is a positive number) and hence find $\sqrt{5}$

   (b) Find the first and second derivative of the function tabulated below at $x = 0.6$.

x:	0.4	0.5	0.6	0.7	0.8
y:	1.5836	1.7974	2.0442	2.3275	2.6511

2. (a) Compute the value of $\int_{0.2}^{1.4}(\sin x - \log n + e^{x})dx$.

Taking $h = 0.2$ and using Trapezoidal rule and Simpson's rule.

(b) From the following table, estimate $e^{0.644}$. Correct to 5 decimals using Bessel's formula.

x :	0.61	0.62	0.63	0.64
e^{x} :	1.840431	1.858928	1.877610	1.896481
x :	0.65	0.66	0.67	
e^{x} :	1.915541	1.934792	1.954237	

3. (a) Apply Gauss-Jordan method to find the solution of the following system :

$10x + y + 3 = 12$; $2x + 10y + z = 13$;

$x + y + 5z = 7$.

(b) Solve the following system of equation by using Gauss-Seidal methods (Correct to 3 decimal places)

$8x - 3y + 2z = 20$

$4x + 11y - 3 = 33$

$6x + 3y + 12z = 35$

4. By the method of triangularization, solve the following system :

$$5x - 2y + 3 = 4, 7x + y - 5z = 8; \quad 3x + 7y + 4z = 10$$

5. (a) Solve $\frac{dy}{dx} = x + y$, given $y(1) = 0$ and get $y(1.1)$ by Taylor's method.

(b) Using R.K.method of Fourth order, find $y(0.7)$ correct to 4 decimal places if $y' = y - x^2, y(0.6) = 1.7379$.

6. (a) Using Euler's method, solve numerically the equation $y' = x + y, y(0) = 1$ for $x = 0$ to 1 and $h = 0.2$.

(b) Find $y(2)$ if $y(x)$ is the solution of $\frac{dy}{dx} = \frac{1}{2}(x + y)$ given $y(0) = 2$, $y(0.5) = 2.636$, $y(1) = 3.595$ and $y(1.5) = 4.968$ using Milne's method.

7. Using power method, find all the eigen values of
$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

8. (a) Solve $\nabla^2 u = 8x^2y^2$ for square mesh given $u = 0$ on the 4 boundaries, dividing the square into 16 sub-squares of length 1 unit.

(b) Using Crank-Nicholson's scheme. Solve $u_{xx} = 16u_t, 0 < x < 1, t > 0$ given $u(x,0) = 0$, $u(0,t = 0, u(1,t) = 100t$. Compute u for one step in t direction taking $h = 1/4$.

Reg. No. : ....

D 1082 **Q.P. Code : [D 07 PMA 05]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2013.

First Year

Mathematics

COMPLEX ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) Prove that the function $f(z)$ defined by

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} \text{ if } z \neq 0$$

$= 0$ if $z = 0$ is a continuous and the cauchy's – Riemann equation are satisfied at the origin, but $f'(0)$ does not exist.

(b) State and prove Luca's theorem.

2. (a) State and prove Cauchy's theorem for a rectangular.

(b) Prove that the line integral $\int_{\gamma} pdx + qdy$ defined in a region Ω depends only on the end points of γ if and only if there exists a function $u(x, y)$ in Ω with $\frac{\partial u}{\partial x} = p$, $\frac{\partial u}{\partial y} = q$.

3. (a) State and prove Cauchy's integral formula.

(b) State and prove the maximum principle for analytical functions.

4. (a) State and prove the argument principle.

(b) Evaluate $\int_{0}^{\pi} \frac{d\theta}{a + \cos\theta}, a > 1$.

5. (a) Derive Poisson's formula.

(b) State and prove Schwary's theorem.

6. (a) State and prove Hurwitz's theorem.

(b) Prove that the infinite product $\prod_{n=1}^{\infty} (1 + a_n)$ converges if and only if $\sum_{n=1}^{\infty} \log(1 + a_n)$ converges.

7. (a) State and prove Riemann mapping theorem.

   (b) Derive Schwarz – Christoffel formula.

8. (a) Prove that a nonconstant elliptic function has equally many poles as it has zeros.

   (b) Derive Legendre's relation.
