Notation

$\mathbb{N}$	The set of all natural numbers $\{1,2,3,\dots\}$
$\mathbb{Z}$	The set of all integers
$\mathbb{Q}$	The set of all rational numbers
$\mathbb{R}$	The set of all real numbers
S_n	The group of permutations of n distinct symbols
$\mathbb{Z}_n$	$\{0, 1, 2, \dots, n-1\}$ with addition and multiplication modulo n
ϕ	empty set
A^T	Transpose of A
i	$\sqrt{-1}$
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system
∇	$\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
I_n	Identity matrix of order n
ln	logarithm with base e

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The sequence $\{s_n\}$ of real numbers given by

$$s_n = \frac{\sin\frac{\pi}{2}}{1\cdot 2} + \frac{\sin\frac{\pi}{2^2}}{2\cdot 3} + \cdots + \frac{\sin\frac{\pi}{2^n}}{n\cdot(n+1)}$$

is

(A) a divergent sequence
(B) an oscillatory sequence
(C) not a Cauchy sequence
(D) a Cauchy sequence

Q.2 Let P be the vector space (over $\mathbb{R}$) of all polynomials of degree ≤ 3 with real coefficients. Consider the linear transformation $T: P \to P$ defined by

$$T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_3 + a_2x + a_1x^2 + a_0x^3.$$

Then the matrix representation M of T with respect to the ordered basis $\{1, x, x^2, x^3\}$ satisfies

(A) $M^2 + I_4 = 0$ (B) $M^2 - I_4 = 0$
(C) $M - I_4 = 0$ (D) $M + I_4 = 0$

Q.3 Let $f: [-1,1] \to \mathbb{R}$ be a continuous function. Then the integral

$$\int_0^{\pi} x\, f(\sin x)\, dx$$

is equivalent to

(A) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx$ (B) $\frac{\pi}{2}\int_0^{\pi} f(\cos x)\, dx$

(C) $\pi\int_0^{\pi} f(\cos x)\, dx$ (D) $\pi\int_0^{\pi} f(\sin x)\, dx$

Q.4 Let σ be an element of the permutation group S_5. Then the maximum possible order of σ is

(A) 5 (B) 6 (C) 10 (D) 15

Q.5 Let f be a strictly monotonic continuous real valued function defined on $[a,b]$ such that $f(a) < a$ and $f(b) > b$. Then which one of the following is TRUE?

(A) There exists exactly one $c \in (a,b)$ such that $f(c) = c$
(B) There exist exactly two points $c_1, c_2 \in (a,b)$ such that $f(c_i) = c_i,\ i = 1,2$
(C) There exists no $c \in (a,b)$ such that $f(c) = c$
(D) There exist infinitely many points $c \in (a,b)$ such that $f(c) = c$

Q.6 The value of $\lim_{(x,\ y)\to(2,-2)} \frac{\sqrt{(x-y)}-2}{x-y-4}$ is

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Q.7 Let $\overrightarrow{r} = \left(x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k} \right)$ and $r = |\overrightarrow{r}|$. If $f(r) = \ln r$ and $g(r) = \frac{1}{r}$, $r \neq 0$, satisfy $2\,\nabla f + h(r)\nabla g = \vec{0}$, then $h(r)$ is

(A) r (B) $\frac{1}{r}$ (C) $2r$ (D) $\frac{2}{r}$

Q.8 The nonzero value of n for which the differential equation

$$(3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)\,dy = 0, \quad x \neq 0,$$

becomes exact is

(A) -3 (B) -2 (C) 2 (D) 3

Q.9 One of the points which lies on the solution curve of the differential equation

$$(y - x)dx + (x + y)dy = 0,$$

with the given condition $y(0) = 1$, is

(A) $(1,\ -2)$ (B) $(2,\ -1)$ (C) $(2,\ 1)$ (D) $(-1,\ 2)$

Q.10 Let S be a closed subset of $\mathbb{R}$, T a compact subset of $\mathbb{R}$ such that $S \cap T \neq \phi$. Then $S \cap T$ is

(A) closed but not compact
(B) not closed
(C) compact
(D) neither closed nor compact

Q. 11 – Q. 30 carry two marks each.

Q.11 Let S be the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)\,2^{(2k-1)}}$$

and T be the series

$$\sum_{k=2}^{\infty} \left(\frac{3k-4}{3k+2}\right)^{\frac{(k+1)}{3}}$$

of real numbers. Then which one of the following is TRUE?

(A) Both the series S and T are convergent
(B) S is convergent and T is divergent
(C) S is divergent and T is convergent
(D) Both the series S and T are divergent

Q.12 Let $\{a_n\}$ be a sequence of positive real numbers satisfying

$$\frac{4}{a_{n+1}} = \frac{3}{a_n} + \frac{a_n^3}{81}, \qquad n \geq 1, \ a_1 = 1.$$

Then all the terms of the sequence lie in

(A) $\left[\frac{1}{2}, \ \frac{3}{2}\right]$ (B) $[0, \ 1]$ (C) $[1, \ 2]$ (D) $[1, \ 3]$

Q.13 The largest eigenvalue of the matrix $\begin{bmatrix} 1 & 4 & 16 \\ 4 & 16 & 1 \\ 16 & 1 & 4 \end{bmatrix}$ is

(A) 16 (B) 21

(C) 48 (D) 64

Q.14 The value of the integral

$$\frac{(2n)!}{2^{2n}\,(n!)} \int_{-1}^{1} (1-x^2)^n \ dx, \qquad n \in \mathbb{N}$$

is

(A) $\frac{2}{(2n+1)!}$ (B) $\frac{2n}{(2n+1)!}$

(C) $\frac{2\,(n!)}{2n+1}$ (D) $\frac{(n+1)!}{2n+1}$

Q.15 If the triple integral over the region bounded by the planes

$$2x + y + z = 4, \qquad x = 0, \qquad y = 0, \qquad z = 0$$

is given by

$$\int_0^2 \int_0^{\lambda(x)} \int_0^{\mu(x,y)} dz\, dy\, dx\,,$$

then the function $\lambda(x) - \mu(x,y)$ is

(A) $x + y$ (B) $x - y$ (C) x (D) y

Q.16 The surface area of the portion of the plane $y + 2z = 2$ within the cylinder $x^2 + y^2 = 3$ is

(A) $\frac{3\sqrt{5}}{2}\pi$ (B) $\frac{5\sqrt{5}}{2}\pi$ (C) $\frac{7\sqrt{5}}{2}\pi$ (D) $\frac{9\sqrt{5}}{2}\pi$

Q.17 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x+y} & \text{if } x+y \neq 0 \\ 0 & \text{if } x+y = 0 \end{cases}.$$

Then the value of $\left(\frac{\partial^2 f}{\partial x\, \partial y} + \frac{\partial^2 f}{\partial y\, \partial x}\right)$ at the point $(0,0)$ is

(A) 0 (B) 1 (C) 2 (D) 4

Q.18 The function $f(x,y) = 3x^2y + 4y^3 - 3x^2 - 12y^2 + 1$ has a saddle point at

(A) $(0,\ 0)$ (B) $(0,\ 2)$ (C) $(1,\ 1)$ (D) $(-2,\ 1)$

Q.19 Consider the vector field $\vec{F} = r^{\beta}(y\,\hat{\imath} - x\hat{\jmath}\,)$, where $\beta \in \mathbb{R}$, $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ and $r = |\vec{r}|$. If the absolute value of the line integral $\oint_C \vec{F} \cdot d\vec{r}$ along the closed curve $C: x^2 + y^2 = a^2$ (oriented counter clockwise) is 2π, then β is

(A) -2 (B) -1 (C) 1 (D) 2

Q.20 Let S be the surface of the cone $z = \sqrt{x^2 + y^2}$ bounded by the planes $z = 0$ and $z = 3$. Further, let C be the closed curve forming the boundary of the surface S. A vector field $\vec{F}$ is such that $\nabla \times \vec{F} = -x\hat{\imath} - y\hat{\jmath}$. The absolute value of the line integral $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$, is

(A) 0 (B) 9π (C) 15π (D) 18π

Q.21 Let $y(x)$ be the solution of the differential equation

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) = x; \quad y(1) = 0, \quad \left.\frac{dy}{dx}\right|_{x=1} = 0\,.$$

Then $y(2)$ is

(A) $\frac{3}{4} + \frac{1}{2}\ln 2$ (B) $\frac{3}{4} - \frac{1}{2}\ln 2$

(C) $\frac{3}{4} + \ln 2$ (D) $\frac{3}{4} - \ln 2$

Q.22 The general solution of the differential equation with constant coefficients

$$\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

approaches zero as $x \to \infty$, if

(A) b is negative and c is positive
(B) b is positive and c is negative
(C) both b and c are positive
(D) both b and c are negative

Q.23 Let $S \subset \mathbb{R}$ and ∂S denote the set of points x in $\mathbb{R}$ such that every neighbourhood of x contains some points of S as well as some points of complement of S. Further, let $\overline{S}$ denote the closure of S. Then which one of the following is FALSE?

(A) $\partial\mathbb{Q} = \mathbb{R}$
(B) $\partial(\mathbb{R} \setminus T) = \partial T, \quad T \subset \mathbb{R}$
(C) $\partial(T \cup V) = \partial T \cup \partial V, \quad T, V \subset \mathbb{R},\ T \cap V \neq \phi$
(D) $\partial T = \overline{T} \cap \overline{(\mathbb{R} \setminus T)}, \quad T \subset \mathbb{R}$

Q.24 The sum of the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+n-2}$$

is

(A) $\frac{1}{3}\ln 2 - \frac{5}{18}$ (B) $\frac{1}{3}\ln 2 - \frac{5}{6}$ (C) $\frac{2}{3}\ln 2 - \frac{5}{18}$ (D) $\frac{2}{3}\ln 2 - \frac{5}{6}$

Q.25 Let $f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-1|}$ for all $x \in [-1, 1]$. Then which one of the following is TRUE?

(A) Maximum value of $f(x)$ is $\frac{3}{2}$
(B) Minimum value of $f(x)$ is $\frac{1}{3}$
(C) Maximum of $f(x)$ occurs at $x = \frac{1}{2}$
(D) Minimum of $f(x)$ occurs at $x = 1$

Q.26 The matrix $M = \begin{bmatrix} \cos\alpha & \sin\alpha \\ i\sin\alpha & i\cos\alpha \end{bmatrix}$ is a unitary matrix when α is

(A) $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$ (B) $(3n+1)\frac{\pi}{3}, n \in \mathbb{Z}$

(C) $(4n+1)\frac{\pi}{4}, n \in \mathbb{Z}$ (D) $(5n+1)\frac{\pi}{5}, n \in \mathbb{Z}$

Q.27 Let $M = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & \alpha \\ 2 & -\alpha & 0 \end{bmatrix}$, $\alpha \in \mathbb{R} \setminus \{0\}$ and $\boldsymbol{b}$ a non-zero vector such that $M\boldsymbol{x} = \boldsymbol{b}$ for some $\boldsymbol{x} \in \mathbb{R}^3$. Then the value of $\boldsymbol{x}^T\boldsymbol{b}$ is

(A) $-\alpha$ (B) α (C) 0 (D) 1

Q.28 The number of group homomorphisms from the cyclic group $\mathbb{Z}_4$ to the cyclic group $\mathbb{Z}_7$ is

(A) 7 (B) 3 (C) 2 (D) 1

Q.29 In the permutation group S_n $(n \geq 5)$, if H is the smallest subgroup containing all the 3-cycles, then which one of the following is TRUE?

(A) Order of H is 2
(B) Index of H in S_n is 2
(C) H is abelian
(D) $H = S_n$

Q.30 Let $f: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x(1 + x^{\alpha} \sin(\ln x^2)) & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}.$$

Then, at $x = 0$, the function f is

(A) continuous and differentiable when $\alpha = 0$
(B) continuous and differentiable when $\alpha > 0$
(C) continuous and differentiable when $-1 < \alpha < 0$
(D) continuous and differentiable when $\alpha < -1$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let $\{s_n\}$ be a sequence of positive real numbers satisfying

$$2\, s_{n+1} = s_n^2 + \frac{3}{4}, \qquad n \geq 1.$$

If α and β are the roots of the equation $x^2 - 2x + \frac{3}{4} = 0$ and $\alpha < s_1 < \beta$, then which of the following statement(s) is(are) TRUE ?

(A) $\{s_n\}$ is monotonically decreasing
(B) $\{s_n\}$ is monotonically increasing
(C) $\lim_{n\to\infty} s_n = \alpha$
(D) $\lim_{n\to\infty} s_n = \beta$

Q.32 The value(s) of the integral

$$\int_{-\pi}^{\pi} |x| \cos nx \, dx \,, \quad n \geq 1$$

is (are)

(A) 0 when n is even
(B) 0 when n is odd
(C) $-\frac{4}{n^2}$ when n is even
(D) $-\frac{4}{n^2}$ when n is odd

Q.33 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{xy}{|x|} & \text{if } x \neq 0 \\ 0 & \text{elsewhere} \end{cases}.$$

Then at the point $(0,0)$, which of the following statement(s) is(are) TRUE ?

(A) f is not continuous
(B) f is continuous
(C) f is differentiable
(D) Both first order partial derivatives of f exist

Q.34 Consider the vector field $\vec{F} = x\hat{\imath} + y\hat{\jmath}$ on an open connected set $S \subset \mathbb{R}^2$. Then which of the following statement(s) is(are) TRUE ?

(A) Divergence of $\vec{F}$ is zero on S
(B) The line integral of $\vec{F}$ is independent of path in S
(C) $\vec{F}$ can be expressed as a gradient of a scalar function on S
(D) The line integral of $\vec{F}$ is zero around any piecewise smooth closed path in S

Q.35 Consider the differential equation

$$\sin 2x \ \frac{dy}{dx} = 2y + 2\cos x \ , \ \ y\left(\frac{\pi}{4}\right) = 1 - \sqrt{2}.$$

Then which of the following statement(s) is(are) TRUE?

(A) The solution is unbounded when $x \to 0$
(B) The solution is unbounded when $x \to \frac{\pi}{2}$
(C) The solution is bounded when $x \to 0$
(D) The solution is bounded when $x \to \frac{\pi}{2}$

Q.36 Which of the following statement(s) is(are) TRUE?

(A) There exists a connected set in $\mathbb{R}$ which is not compact
(B) Arbitrary union of closed intervals in $\mathbb{R}$ need not be compact
(C) Arbitrary union of closed intervals in $\mathbb{R}$ is always closed
(D) Every bounded infinite subset V of $\mathbb{R}$ has a limit point in V itself

Q.37 Let $P(x) = \left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x - 1$ for all $x \in \mathbb{R}$. Then which of the following statement(s) is(are) TRUE?

(A) The equation $P(x) = 0$ has exactly one solution in $\mathbb{R}$
(B) $P(x)$ is strictly increasing for all $x \in \mathbb{R}$
(C) The equation $P(x) = 0$ has exactly two solutions in $\mathbb{R}$
(D) $P(x)$ is strictly decreasing for all $x \in \mathbb{R}$

Q.38 Let G be a finite group and $o(G)$ denotes its order. Then which of the following statement(s) is(are) TRUE?

(A) G is abelian if $o(G) = pq$ where p and q are distinct primes
(B) G is abelian if every non identity element of G is of order 2
(C) G is abelian if the quotient group $\frac{G}{Z(G)}$ is cyclic, where $Z(G)$ is the center of G
(D) G is abelian if $o(G) = p^3$, where p is prime

Q.39 Consider the set $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \alpha x + \beta y + z = \gamma,\ \alpha, \beta, \gamma \in \mathbb{R} \right\}$. For which of the following choice(s) the set V becomes a two dimensional subspace of $\mathbb{R}^3$ over $\mathbb{R}$?

(A) $\alpha = 0,\ \beta = 1,\ \gamma = 0$
(B) $\alpha = 0,\ \beta = 1,\ \gamma = 1$
(C) $\alpha = 1,\ \beta = 0,\ \gamma = 0$
(D) $\alpha = 1,\ \beta = 1,\ \gamma = 0$

Q.40 Let $S = \left\{\frac{1}{3^n} + \frac{1}{7^m} \;\middle|\; n, m \in \mathbb{N}\right\}$. Then which of the following statement(s) is(are) TRUE?

(A) S is closed
(B) S is not open
(C) S is connected
(D) 0 is a limit point of S

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let $\{s_n\}$ be a sequence of real numbers given by

$$s_n = 2^{(-1)^n}\left(1 - \frac{1}{n}\right)\sin\frac{n\pi}{2}, \qquad n \in \mathbb{N}.$$

Then the least upper bound of the sequence $\{s_n\}$ is ____________

Q.42 Let $\{s_k\}$ be a sequence of real numbers, where

$$s_k = k^{\alpha/k}, \qquad k \geq 1, \qquad \alpha > 0.$$

Then

$$\lim_{n\to\infty}\left(s_1\ s_2\ \ldots\ s_n\right)^{1/n}$$

is ____________

Q.43 Let $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$ be a non-zero vector and $A = \dfrac{\boldsymbol{x}\,\boldsymbol{x}^T}{\boldsymbol{x}^T\boldsymbol{x}}$. Then the dimension of the vector space $\left\{ \boldsymbol{y} \in \mathbb{R}^3 \;\middle|\; A\boldsymbol{y} = \boldsymbol{0} \right\}$ over $\mathbb{R}$ is ____________

Q.44 Let f be a real valued function defined by

$$f(x, y) = 2\ln\left(x^2 y^2 e^{\frac{y}{x}}\right), \qquad x > 0, y > 0.$$

Then the value of $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}$ at any point (x, y), where $x > 0$, $y > 0$, is ____________

Q.45 Let $\vec{F} = \sqrt{x}\,\hat{\imath} + (x + y^3)\hat{\jmath}$ be a vector field for all (x, y) with $x \geq 0$ and $\vec{r} = x\hat{\imath} + y\hat{\jmath}$. Then the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$ from $(0, 0)$ to $(1, 1)$ along the path $C\colon\ x = t^2,\ y = t^3,\ 0 \leq t \leq 1$ is ____________

Q.46 If $f\colon(-1, \infty) \to \mathbb{R}$ defined by $f(x) = \frac{x}{1+x}$ is expressed as

$$f(x) = \frac{2}{3} + \frac{1}{9}(x - 2) + \frac{c(x-2)^2}{(1+\xi)^3},$$

where ξ lies between 2 and x, then the value of c is ____________

Q.47 Let $y_1(x)$, $y_2(x)$ and $y_3(x)$ be linearly independent solutions of the differential equation

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0.$$

If the Wronskian $W(y_1, y_2, y_3)$ is of the form ke^{bx} for some constant k, then the value of b is____________

Q.48 The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-4)^n}{n(n+1)}(x+2)^{2n} \quad \text{is} \quad \_\_\_\_\_\_\_\_\_\_$$

Q.49 Let $f:(0,\infty) \to \mathbb{R}$ be a continuous function such that

$$\int_0^x f(t)dt = -2 + \frac{x^2}{2} + 4x\sin 2x + 2\cos 2x.$$

Then the value of $\frac{1}{\pi} f\left(\frac{\pi}{4}\right)$ is ____________

Q.50 Let G be a cyclic group of order 12. Then the number of non-isomorphic subgroups of G is ____________

Q. 51 – Q. 60 carry two marks each.

Q.51 The value of $\lim_{n\to\infty}\left(8n - \frac{1}{n}\right)^{\frac{(-1)^n}{n^2}}$ is equal to _________

Q.52 Let R be the region enclosed by $x^2 + 4y^2 \geq 1$ and $x^2 + y^2 \leq 1$. Then the value of

$$\iint_R |xy|\, dx\, dy \quad \text{is } \_\_\_\_\_\_\_\_\_$$

Q.53 Let

$$M = \begin{bmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \gamma \end{bmatrix},\ \alpha\beta\gamma = 1,\ \alpha,\beta,\gamma \in \mathbb{R} \text{ and } \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3.$$

Then $M\boldsymbol{x} = \boldsymbol{0}$ has infinitely many solutions if $trace(M)$ is ____________

Q.54 Let C be the boundary of the region enclosed by $y = x^2$, $y = x + 2$, and $x = 0$. Then the value of the line integral

$$\oint_C (xy - y^2)dx - x^3 dy,$$

where C is traversed in the counter clockwise direction, is ____________

Q.55 Let S be the closed surface forming the boundary of the region V bounded by $x^2 + y^2 = 3$, $z = 0$, $z = 6$. A vector field $\vec{F}$ is defined over V with $\nabla \cdot \vec{F} = 2y + z + 1$. Then the value of

$$\frac{1}{\pi} \iint_S \vec{F} \cdot \hat{n} \, ds,$$

where $\hat{n}$ is the unit outward drawn normal to the surface S, is ____________,

Q.56 Let $y(x)$ be the solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0, \qquad y(0) = 1, \qquad \left.\frac{dy}{dx}\right|_{x=0} = -1.$$

Then $y(x)$ attains its maximum value at $x =$ ____________

Q.57 The value of the double integral

$$\int_0^{\pi} \int_0^{x} \frac{\sin y}{\pi - y} \, dy \, dx$$

is ____________

Q.58 Let H denote the group of all 2×2 invertible matrices over $\mathbb{Z}_5$ under usual matrix multiplication. Then the order of the matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ in H is ____________

Q.59 Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 1 \end{bmatrix}$, $N(A)$ the null space of A and $R(B)$ the range space of B. Then the dimension of $N(A) \cap R(B)$ over $\mathbb{R}$ is ____________

Q.60 The maximum value of $f(x, y) = x^2 + 2y^2$ subject to the constraint $y - x^2 + 1 = 0$ is ____________

END OF THE QUESTION PAPER