Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Useful information

$\mathbb{N}$	set of all natural numbers $\{1, 2, 3, \dots\}$
$\mathbb{Z}$	set of all integers $\{0, \pm 1, \pm 2, \dots\}$
$\mathbb{Q}$	set of all rational numbers
$\mathbb{R}$	set of all real numbers
$\mathbb{C}$	set of all complex numbers
$\mathbb{R}^n$	n-dimensional Euclidean space $\{(x_1, x_2, \dots, x_n) \mid x_j \in \mathbb{R}, 1 \leq j \leq n\}$
S_n	group of all permutations of n distinct symbols
$\mathbb{Z}_n$	group of congruence classes of integers modulo n
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system
∇	$\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
$M_{m\times n}(\mathbb{R})$	real vector space of all matrices of order $m \times n$ with entries in $\mathbb{R}$
sup	supremum
inf	infimum

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Which one of the following is TRUE?

(A) $\mathbb{Z}_n$ is cyclic if and only if n is prime

(B) Every proper subgroup of $\mathbb{Z}_n$ is cyclic

(C) Every proper subgroup of S_4 is cyclic

(D) If every proper subgroup of a group is cyclic, then the group is cyclic

Q.2 Let $a_n = \frac{b_{n+1}}{b_n}$, where $b_1 = 1$, $b_2 = 1$ and $b_{n+2} = b_n + b_{n+1}$, $n \in \mathbb{N}$. Then $\lim_{n\to\infty} a_n$ is

(A) $\frac{1-\sqrt{5}}{2}$ (B) $\frac{1-\sqrt{3}}{2}$ (C) $\frac{1+\sqrt{3}}{2}$ (D) $\frac{1+\sqrt{5}}{2}$

Q.3 If $\{v_1, v_2, v_3\}$ is a linearly independent set of vectors in a vector space over $\mathbb{R}$, then which one of the following sets is also linearly independent?

(A) $\{v_1 + v_2 - v_3,\ 2v_1 + v_2 + 3v_3,\ 5v_1 + 4v_2\}$

(B) $\{v_1 - v_2,\ v_2 - v_3,\ v_3 - v_1\}$

(C) $\{v_1 + v_2 - v_3,\ v_2 + v_3 - v_1,\ v_3 + v_1 - v_2,\ v_1 + v_2 + v_3\}$

(D) $\{v_1 + v_2,\ v_2 + 2v_3,\ v_3 + 3v_1\}$

Q.4 Let a be a positive real number. If f is a continuous and even function defined on the interval $[-a, a]$, then $\int_{-a}^{a} \frac{f(x)}{1+e^x}\, dx$ is equal to

(A) $\int_0^a f(x)\, dx$

(B) $2\int_0^a \frac{f(x)}{1+e^x}\, dx$

(C) $2\int_0^a f(x)\, dx$

(D) $2a\int_0^a \frac{f(x)}{1+e^x}\, dx$

Q.5 The tangent plane to the surface $z = \sqrt{x^2 + 3y^2}$ at $(1, 1, 2)$ is given by

(A) $x - 3y + z = 0$

(B) $x + 3y - 2z = 0$

(C) $2x + 4y - 3z = 0$

(D) $3x - 7y + 2z = 0$

Q.6 In $\mathbb{R}^3$, the cosine of the acute angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z - x^2 - y^2 + 3 = 0$ at the point $(2, 1, 2)$ is

(A) $\frac{8}{5\sqrt{21}}$ (B) $\frac{10}{5\sqrt{21}}$ (C) $\frac{8}{3\sqrt{21}}$ (D) $\frac{10}{3\sqrt{21}}$

Q.7 Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a scalar field, $\vec{v}: \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field and let $\vec{a} \in \mathbb{R}^3$ be a constant vector. If $\vec{r}$ represents the position vector $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then which one of the following is FALSE?

(A) $curl(f\,\vec{v}) = grad(f) \times \vec{v} + f\,curl(\vec{v})$

(B) $div(grad(f)) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) f$

(C) $curl(\vec{a} \times \vec{r}) = 2\,|\vec{a}|\,\vec{r}$

(D) $div\left(\frac{\vec{r}}{|\vec{r}|^3}\right) = 0$, for $\vec{r} \neq \vec{0}$

Q.8 In $\mathbb{R}^2$, the family of trajectories orthogonal to the family of asteroids $x^{2/3} + y^{2/3} = a^{2/3}$ is given by

(A) $x^{4/3} + y^{4/3} = c^{4/3}$ (B) $x^{4/3} - y^{4/3} = c^{4/3}$

(C) $x^{5/3} - y^{5/3} = c^{5/3}$ (D) $x^{2/3} - y^{2/3} = c^{2/3}$

Q.9 Consider the vector space V over $\mathbb{R}$ of polynomial functions of degree less than or equal to 3 defined on $\mathbb{R}$. Let $T : V \to V$ be defined by $(Tf)(x) = f(x) - xf'(x)$. Then the rank of T is

(A) 1 (B) 2 (C) 3 (D) 4

Q.10 Let $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$ for $n \in \mathbb{N}$. Then which one of the following is TRUE for the sequence $\{s_n\}_{n=1}^{\infty}$

(A) $\{s_n\}_{n=1}^{\infty}$ converges in $\mathbb{Q}$

(B) $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence but does not converge in $\mathbb{Q}$

(C) the subsequence $\{s_{k^n}\}_{n=1}^{\infty}$ is convergent in $\mathbb{R}$, only when k is even natural number

(D) $\{s_n\}_{n=1}^{\infty}$ is not a Cauchy sequence

Q. 11 – Q. 30 carry two marks each.

Q.11 Let $a_n = \begin{cases} 2 + \frac{(-1)^{\frac{n-1}{2}}}{n} & , \quad if \; n \; is \; odd \\ 1 + \frac{1}{2^n} & , \quad if \; n \; is \; even \end{cases} \quad , \; n \in \mathbb{N}.$

Then which one of the following is TRUE?

(A) $\sup \{a_n \mid n \in \mathbb{N}\} = 3$ and $\inf \{a_n \mid n \in \mathbb{N}\} = 1$

(B) $\liminf (a_n) = \limsup (a_n) = \frac{3}{2}$

(C) $\sup \{a_n \mid n \in \mathbb{N}\} = 2$ and $\inf \{a_n \mid n \in \mathbb{N}\} = 1$

(D) $\liminf (a_n) = 1$ and $\limsup (a_n) = 3$

Q.12 Let $a, b, c \in \mathbb{R}$. Which of the following values of a, b, c do NOT result in the convergence of the series

$$\sum_{n=3}^{\infty} \frac{a^n}{n^b \, (\log_e n)^c} \quad ?$$

(A) $|a| < 1, \; b \in \mathbb{R}, c \in \mathbb{R}$

(B) $a = 1, \; b > 1, \; c \in \mathbb{R}$

(C) $a = 1, b \geq 0, c < 1$

(D) $a = -1, \; b \geq 0, c > 0$

Q.13 Let $a_n = n + \frac{1}{n}$, $n \in \mathbb{N}$. Then the sum of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{a_{n+1}}{n!}$ is

(A) $e^{-1} - 1$

(B) e^{-1}

(C) $1 - e^{-1}$

(D) $1 + e^{-1}$

Q.14 Let $a_n = \frac{(-1)^n}{\sqrt{1+n}}$ and let $c_n = \sum_{k=0}^{n} a_{n-k} a_k$, where $n \in \mathbb{N} \cup \{0\}$. Then which one of the following is TRUE?

(A) Both $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=1}^{\infty} c_n$ are convergent

(B) $\sum_{n=0}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} c_n$ is not convergent

(C) $\sum_{n=1}^{\infty} c_n$ is convergent but $\sum_{n=0}^{\infty} a_n$ is not convergent

(D) Neither $\sum_{n=0}^{\infty} a_n$ nor $\sum_{n=1}^{\infty} c_n$ is convergent

Q.15 Suppose that $f, g : \mathbb{R} \to \mathbb{R}$ are differentiable functions such that f is strictly increasing and g is strictly decreasing. Define $p(x) = f(g(x))$ and $q(x) = g\big(f(x)\big)$, $\forall x \in \mathbb{R}$. Then, for $t > 0$, the sign of $\int_0^t p'(x)\,(\,q'(x) - 3)dx$ is

(A) positive (B) negative (C) dependent on t (D) dependent on f and g

Q.16 For $x \in \mathbb{R}$, let $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then which one of the following is FALSE?

(A) $\lim_{x\to 0} \frac{f(x)}{x} = 0$

(B) $\lim_{x\to 0} \frac{f(x)}{x^2} = 0$

(C) $\frac{f(x)}{x^2}$ has infinitely many maxima and minima on the interval $(0,1)$

(D) $\frac{f(x)}{x^4}$ is continuous at $x = 0$ but not differentiable at $x = 0$

Q.17 Let $f(x, y) = \begin{cases} \frac{xy}{(x^2+y^2)^{\alpha}}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$

Then which one of the following is TRUE for f at the point $(0,0)$?

(A) For $\alpha = 1$, f is continuous but not differentiable

(B) For $\alpha = \frac{1}{2}$, f is continuous and differentiable

(C) For $\alpha = \frac{1}{4}$, f is continuous and differentiable

(D) For $\alpha = \frac{3}{4}$, f is neither continuous nor differentiable

Q.18 Let $a, b \in \mathbb{R}$ and let $f: \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function. If $z = e^u f(v)$, where $u = ax + by$ and $v = ax - by$, then which one of the following is TRUE?

(A) $b^2 z_{xx} - a^2 z_{yy} = 4a^2 b^2 e^u f'(v)$ (B) $b^2 z_{xx} - a^2 z_{yy} = -4e^u f'(v)$

(C) $bz_x + az_y = abz$ (D) $bz_x + az_y = -abz$

Q.19 Consider the region D in the yz plane bounded by the line $y = \frac{1}{2}$ and the curve $y^2 + z^2 = 1$, where $y \geq 0$. If the region D is revolved about the z-axis in $\mathbb{R}^3$, then the volume of the resulting solid is

(A) $\frac{\pi}{\sqrt{3}}$ (B) $\frac{2\pi}{\sqrt{3}}$ (C) $\frac{\pi\sqrt{3}}{2}$ (D) $\pi\sqrt{3}$

Q.20 If $\vec{F}(x,y) = (3x - 8y)\hat{\imath} + (4y - 6xy)\hat{\jmath}$ for $(x,y) \in \mathbb{R}^2$, then $\oint_C \vec{F} \cdot d\,\vec{r}$, where C is the boundary of the triangular region bounded by the lines $x = 0, y = 0$ and $x + y = 1$ oriented in the anti-clockwise direction, is

(A) $\frac{5}{2}$ (B) 3 (C) 4 (D) 5

Q.21 Let U, V and W be finite dimensional real vector spaces, $T: U \to V$, $S: V \to W$ and $P: W \to U$ be linear transformations. If range (ST) = nullspace (P), nullspace (ST) = range (P) and rank (T) = rank (S), then which one of the following is TRUE?

(A) nullity of T = nullity of S

(B) dimension of $U \neq$ dimension of W

(C) If dimension of $V = 3$, dimension of $U = 4$, then P is not identically zero

(D) If dimension of $V = 4$, dimension of $U = 3$ and T is one-one, then P is identically zero

Q.22 Let $y(x)$ be the solution of the differential equation $\frac{dy}{dx} + y = f(x)$, for $x \geq 0$, $y(0) = 0$, where $f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$. Then $y(x) =$

(A) $2(1 - e^{-x})$ when $0 \leq x < 1$ and $2(e - 1)e^{-x}$ when $x \geq 1$

(B) $2(1 - e^{-x})$ when $0 \leq x < 1$ and 0 when $x \geq 1$

(C) $2(1 - e^{-x})$ when $0 \leq x < 1$ and $2(1 - e^{-1})e^{-x}$ when $x \geq 1$

(D) $2(1 - e^{-x})$ when $0 \leq x < 1$ and $2\,e^{1-x}$ when $x \geq 1$

Q.23 An integrating factor of the differential equation $\left(y + \frac{1}{3}\,y^3 + \frac{1}{2}\,x^2\right) dx + \frac{1}{4}(x + x\,y^2)\,dy = 0$ is

(A) x^2 (B) $3\log_e x$ (C) x^3 (D) $2\log_e x$

Q.24 A particular integral of the differential equation $y'' + 3y' + 2y = e^{e^x}$ is

(A) $e^{e^x}e^{-x}$ (B) $e^{e^x}e^{-2x}$ (C) $e^{e^x}e^{2x}$ (D) $e^{e^x}e^{x}$

Q.25 Let G be a group satisfying the property that $f: G \to \mathbb{Z}_{221}$ is a homomorphism implies $f(g) = 0,\ \forall g \in G$. Then a possible group G is

(A) $\mathbb{Z}_{21}$ (B) $\mathbb{Z}_{51}$ (C) $\mathbb{Z}_{91}$ (D) $\mathbb{Z}_{119}$

Q.26 Let H be the quotient group $\mathbb{Q}/\,\mathbb{Z}$. Consider the following statements.

I. Every cyclic subgroup of H is finite.

II. Every finite cyclic group is isomorphic to a subgroup of H.

Which one of the following holds?

(A) I is TRUE but II is FALSE
(B) II is TRUE but I is FALSE
(C) both I and II are TRUE
(D) neither I nor II is TRUE

Q.27 Let I denote the 4×4 identity matrix. If the roots of the characteristic polynomial of a 4×4 matrix M are $\pm\sqrt{\frac{1\pm\sqrt{5}}{2}}$, then $M^8 =$

(A) $I + M^2$
(B) $2I + M^2$
(C) $2I + 3M^2$
(D) $3I + 2M^2$

Q.28 Consider the group $\mathbb{Z}^2 = \{(a,b) |\, a,b \in \mathbb{Z}\}$ under component-wise addition. Then which of the following is a subgroup of $\mathbb{Z}^2$?

(A) $\{(a,b) \in \mathbb{Z}^2 |\, ab = 0\}$
(B) $\{(a,b) \in \mathbb{Z}^2 |\, 3a + 2b = 15\}$
(C) $\{(a,b) \in \mathbb{Z}^2 |$ 7 divides $ab\}$
(D) $\{(a,b) \in \mathbb{Z}^2 |$ 2 divides a and 3 divides $b\}$

Q.29 Let $f: \mathbb{R} \to \mathbb{R}$ be a function and let J be a bounded open interval in $\mathbb{R}$. Define

$$W(f,J) = \sup\,\{f(x) \mid x \in J\} \;-\; \inf\,\{f(x) \mid x \in J\}\ .$$

Which one of the following is FALSE?

(A) $W(f,J_1) \le W(f,J_2)$ if $J_1 \subset J_2$

(B) If f is a bounded function in J and $J \supset J_1 \supset J_2 \cdots \supset J_n \supset \cdots$ such that the length of the interval J_n tends to 0 as $n \to \infty$, then $\lim_{n\to\infty} W(f,J_n) = 0$

(C) If f is discontinuous at a point $a \in J$, then $W(f,J) \neq 0$

(D) If f is continuous at a point $a \in J$, then for any given $\epsilon > 0$ there exists an interval $I \subset J$ such that $W(f,I) < \epsilon$

Q.30 For $x > \frac{-1}{2}$, let $f_1(x) = \frac{2x}{1+2x}$, $f_2(x) = \log_e(1+2x)$ and $f_3(x) = 2x$. Then which one of the following is TRUE?

(A) $f_3(x) < f_2(x) < f_1(x)$ for $0 < x < \frac{\sqrt{3}}{2}$

(B) $f_1(x) < f_3(x) < f_2(x)$ for $x > 0$

(C) $f_1(x) + f_2(x) < \frac{f_3(x)}{2}$ for $x > \frac{\sqrt{3}}{2}$

(D) $f_2(x) < f_1(x) < f_3(x)$ for $x > 0$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let $f: \mathbb{R} \setminus \{0\} \to \mathbb{R}$ be defined by $f(x) = x + \frac{1}{x^3}$. On which of the following interval(s) is f one-one?

(A) $(-\infty, -1)$ (B) $(0, 1)$ (C) $(0, 2)$ (D) $(0, \infty)$

Q.32 The solution(s) of the differential equation $\frac{dy}{dx} = (\sin 2x)\, y^{1/3}$ satisfying $y(0) = 0$ is (are)

(A) $y(x) = 0$

(B) $y(x) = -\sqrt{\frac{8}{27}}\, \sin^3 x$

(C) $y(x) = \sqrt{\frac{8}{27}}\, \sin^3 x$

(D) $y(x) = \sqrt{\frac{8}{27}}\, \cos^3 x$

Q.33 Suppose f, g, h are permutations of the set $\{\alpha, \beta, \gamma, \delta\}$, where

f interchanges α and β but fixes γ and δ,

g interchanges β and γ but fixes α and δ,

h interchanges γ and δ but fixes α and β.

Which of the following permutations interchange(s) α and δ but fix(es) β and γ?

(A) $f \circ g \circ h \circ g \circ f$ (B) $g \circ h \circ f \circ h \circ g$ (C) $g \circ f \circ h \circ f \circ g$ (D) $h \circ g \circ f \circ g \circ h$

Q.34 Let P and Q be two non-empty disjoint subsets of $\mathbb{R}$. Which of the following is (are) FALSE?

(A) If P and Q are compact, then $P \cup Q$ is also compact

(B) If P and Q are not connected, then $P \cup Q$ is also not connected

(C) If $P \cup Q$ and P are closed, then Q is closed

(D) If $P \cup Q$ and P are open, then Q is open

Q.35 Let $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ denote the group of non-zero complex numbers under multiplication. Suppose $Y_n = \{ z \in \mathbb{C} \mid z^n = 1\}$, $n \in \mathbb{N}$. Which of the following is (are) subgroup(s) of $\mathbb{C}^*$?

(A) $\cup_{n=1}^{100} Y_n$ (B) $\cup_{n=1}^{\infty} Y_{2^n}$ (C) $\cup_{n=100}^{\infty} Y_n$ (D) $\cup_{n=1}^{\infty} Y_n$

Q.36 Suppose $\alpha, \beta, \gamma \in \mathbb{R}$. Consider the following system of linear equations.
$x + y + z = \alpha,\ x + \beta y + z = \gamma,\ x + y + \alpha z = \beta$. If this system has at least one solution, then which of the following statements is (are) TRUE?

(A) If $\alpha = 1$ then $\gamma = 1$
(B) If $\beta = 1$ then $\gamma = \alpha$
(C) If $\beta \neq 1$ then $\alpha = 1$
(D) If $\gamma = 1$ then $\alpha = 1$

Q.37 Let $m, n \in \mathbb{N}$, $m < n$, $P \in M_{n\times m}(\mathbb{R})$, $Q \in M_{m\times n}(\mathbb{R})$. Then which of the following is (are) NOT possible?

(A) $rank\ (PQ) = n$
(B) $rank\ (QP) = m$
(C) $rank\ (PQ) = m$
(D) $rank\ (QP) = \left\lceil \frac{m+n}{2} \right\rceil$, the smallest integer larger than or equal to $\frac{m+n}{2}$

Q.38 If $\vec{F}(x,y,z) = (2x + 3yz)\hat{\imath} + (3xz + 2y)\hat{\jmath} + (3xy + 2z)\hat{k}$ for $(x,y,z) \in \mathbb{R}^3$, then which among the following is (are) TRUE?

(A) $\nabla \times \vec{F} = \vec{0}$
(B) $\oint_C \vec{F} \cdot d\vec{r} = 0$ along any simple closed curve C
(C) There exists a scalar function $\phi: \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla \cdot \vec{F} = \phi_{xx} + \phi_{yy} + \phi_{zz}$
(D) $\nabla \cdot \vec{F} = 0$

Q.39 Which of the following subsets of $\mathbb{R}$ is (are) connected?

(A) $\{ x \in \mathbb{R} \mid x^2 + x > 4\}$
(B) $\{ x \in \mathbb{R} \mid x^2 + x < 4\}$
(C) $\{ x \in \mathbb{R} \mid |x| < |x - 4|\}$
(D) $\{ x \in \mathbb{R} \mid |x| > |x - 4|\}$

Q.40 Let S be a subset of $\mathbb{R}$ such that 2018 is an interior point of S. Which of the following is (are) TRUE?

(A) S contains an interval

(B) There is a sequence in S which does not converge to 2018

(C) There is an element $y \in S$, $y \neq 2018$ such that y is also an interior point of S

(D) There is a point $z \in S$, such that $|z - 2018| = 0.002018$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The order of the element $(1\ 2\ 3)\ (2\ 4\ 5)\ (4\ 5\ 6)$ in the group S_6 is ________________

Q.42 Let $\phi(x, y, z) = 3y^2 + 3yz$ for $(x, y, z) \in \mathbb{R}^3$. Then the absolute value of the directional derivative of ϕ in the direction of the line $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z}{-2}$, at the point $(1, -2, 1)$ is ______________

Q.43 Let $f(x) = \sum_{n=0}^{\infty} (-1)^n x(x-1)^n$ for $0 < x < 2$. Then the value of $f\left(\frac{\pi}{4}\right)$ is ____________

Q.44 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x, y) = \begin{cases} \dfrac{x^2 y\,(x - y)}{x^2 + y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$$

Then $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$ at the point $(0,0)$ is ____________

Q.45 Let $f(x, y) = \sqrt{x^3 y}\ \sin\left(\frac{\pi}{2}\, e^{\left(\frac{y}{x} - 1\right)}\right) + xy\ \cos\left(\frac{\pi}{3}\, e^{\left(\frac{x}{y} - 1\right)}\right)$ for $(x, y) \in \mathbb{R}^2$, $x > 0$, $y > 0$.

Then $f_x(1,1) + f_y(1,1) =$ _______________

Q.46 Let $f: [0, \infty) \to [0, \infty)$ be continuous on $[0, \infty)$ and differentiable on $(0, \infty)$. If $f(x) = \int_0^x \sqrt{f(t)}\, dt$, then $f(6) =$ ___________

Q.47 Let $a_n = \frac{(1+(-1)^n)}{2^n} + \frac{(1+(-1)^{n-1})}{3^n}$. Then the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$ about $x = 0$ is __________

Q.48 Let A_6 be the group of even permutations of 6 distinct symbols. Then the number of elements of order 6 in A_6 is __________________

Q.49 Let W_1 be the real vector space of all 5×2 matrices such that the sum of the entries in each row is zero. Let W_2 be the real vector space of all 5×2 matrices such that the sum of the entries in each column is zero. Then the dimension of the space $W_1 \cap W_2$ is ______________

Q.50 The coefficient of x^4 in the power series expansion of $e^{\sin x}$ about $x = 0$ is ____________ (correct up to three decimal places).

Q. 51 – Q. 60 carry two marks each.

Q.51 Let $a_k = (-1)^{k-1}$, $s_n = a_1 + a_2 + \cdots + a_n$ and $\sigma_n = (s_1 + s_2 + \cdots + s_n)/n$, where $k, n \in \mathbb{N}$. Then $\lim_{n\to\infty} \sigma_n$ is _______________ (correct up to one decimal place).

Q.52 Let $f: \mathbb{R} \to \mathbb{R}$ be such that f'' is continuous on $\mathbb{R}$ and $f(0) = 1$, $f'(0) = 0$ and $f''(0) = -1$. Then $\lim_{x\to\infty} \left(f\left(\sqrt{\frac{2}{x}}\right)\right)^x$ is ________________ (correct up to three decimal places).

Q.53 Suppose x, y, z are positive real numbers such that $x + 2y + 3z = 1$. If M is the maximum value of xyz^2, then the value of $\frac{1}{M}$ is ________________

Q.54 If the volume of the solid in $\mathbb{R}^3$ bounded by the surfaces

$$x = -1, \quad x = 1, \quad y = -1, \quad y = 1, \quad z = 2, \quad y^2 + z^2 = 2$$

is $\alpha - \pi$, then $\alpha =$ ________________

Q.55 If $\alpha = \int_{\pi/6}^{\pi/3} \frac{\sin t + \cos t}{\sqrt{\sin 2t}}\, dt$, then the value of $\left(2 \sin\frac{\alpha}{2} + 1\right)^2$ is ______________

Q.56 The value of the integral

$$\int_0^1 \int_x^1 y^4 e^{xy^2}\, dy\, dx$$

is ______________ (correct up to three decimal places).

Q.57 Suppose $Q \in M_{3\times3}(\mathbb{R})$ is a matrix of rank 2. Let $T: M_{3\times3}(\mathbb{R}) \to M_{3\times3}(\mathbb{R})$ be the linear transformation defined by $T(P) = QP$. Then the rank of T is _______________

Q.58 The area of the parametrized surface

$S = \{((2 + \cos u) \cos v,\ (2 + \cos u) \sin v,\ \sin u) \in \mathbb{R}^3 \mid 0 \le u \le \frac{\pi}{2},\ 0 \le v \le \frac{\pi}{2}\}$

is ____________ (correct up to two decimal places).

Q.59 If $x(t)$ is the solution to the differential equation $\frac{dx}{dt} = x^2 t^3 + xt$, for $t > 0$, satisfying $x(0) = 1$, then the value of $x(\sqrt{2})$ is _____________ (correct up to two decimal places).

Q.60 If $y(x) = v(x) \sec x$ is the solution of $y'' - (2 \tan x)\, y' + 5y = 0$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, satisfying $y(0) = 0$ and $y'(0) = \sqrt{6}$, then $v\left(\frac{\pi}{6\sqrt{6}}\right)$ is _______________ (correct up to two decimal places).

END OF THE QUESTION PAPER