Section 4: Mathematics / Biology

Students will have to attempt either Mathematics/Biology as per the eligibility of the program applied.

Mathematics

66. The solution of the equation. $\log\left(\log_5\left(\sqrt{x+5}+\sqrt{x}\right)\right)=0$ is

(a) 2 (b) 4 (c) 3 (d) 8

67. Let $\frac{1}{q+r}, \frac{1}{r+p}$ and $\frac{1}{p+q}$ are in A.P. where $p, q, r, \neq 0$, then

(a) p, q, r are in A.P. (b) p^2, q^2, r^2 are in A.P.

(c) $\frac{1}{p}\cdot\frac{1}{q}\cdot\frac{1}{r}$ in A.P. (d) none of these

68. If $b \in R^+$ then the roots of the equation $(2+b)x^2+(3+b)x+(4+b)=0$ is

(a) real and distinct (b) real and equal (c) imaginar (d) cannot predicted

69. Solve for integral solutions $x_1 + x_2 + x_3 + \ldots + x_6 \leq 17$, where $1 \leq x_i \leq 6$, $i = 1, 2, \ldots 6$.
Number of solutions will be

(a) ${}^{17}C_6 - 6\,{}^{11}C_5$ (b) ${}^{17}C_{11} - 6\,{}^{11}C_5$ (c) ${}^{17}C_5 - 6\,{}^{11}C_5$ (d) ${}^{17}C_{11} - 5\,{}^{11}C_6$

70. The probability that a certain beginner at golf gets a good shot if he uses the correct club is $\frac{1}{3}$, and the probability of a good shot with an incorrect club is $\frac{1}{4}$. In his bag there are 5 different clubs, only one of which is correct for the shot in question. If he chooses a club at random and take a stroke, the probability that he gets a good shot is

(a) $\frac{1}{3}$ (b) $\frac{1}{12}$ (c) $\frac{4}{15}$ (d) $\frac{7}{12}$

71. OPQR is a square and M, N are the middle points of the side PQ and QR respectively. Then the ratio of the area of the square and the triangle OMN is

(a) 4 : 1 (b) 2 : 1 (c) 4 : 3 (d) 8 : 3

72. Two vertices of an equilateral triangle are (– 1, 0) and (1, 0) and its circumcircle is

(a) $x^2 + \left(y - \frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3}$ (b) $x^2 - \left(y + \frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3}$

(c) $x^2 + \left(y - \frac{1}{\sqrt{3}}\right)^2 = -\frac{4}{3}$ (d) none of these

73. If in a ΔABC, $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is always

(a) isosceles triangle (b) right angled (c) acute angled (d) obtuse angled

74. If the vertex and the focus of a parabola are (–1, 1) and (2, 3) respectively, then the equation of the directrix is

(a) $3x + 2y - 25 = 0$ (b) $x + 2y + 7 = 0$ (c) $2x - 3y + 10 = 0$ (d) $3x + 2y + 14 = 0$.

75. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having its centre at (0, 3) is

(a) 4 (b) 3 (c) $\sqrt{12}$ (d) 7/2

76. If $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$ are four concyclic points on the rectangular hyperbola $xy = c^2$, then the co-ordinates of the orthocentre of ΔPQR are

(a) $(x_4, -y_4)$ (b) (x_4, y_4) (c) $(-x_4, -y_4)$ (d) $(-x_4, y_4)$

77. The coefficient of $x^n y^n$ in the expansion of $[(1 + x)(1 + y)(x + y)]^n$ is

(a) $\sum_{r=0}^{n} C_r$ (b) $\sum_{r=0}^{n} C_r^{\,2}$ (c) $\sum_{r=0}^{n} C_r^{\,3}$ (d) none of these

78. z_0 is one of the roots of the equation $z^n \cos\theta_0 + z^{n-1}\cos\theta_1 + \ldots + \cos\theta_n = 2$, where $\theta_i \in R$, then

(a) $|z_0| < \frac{1}{2}$ (b) $|z_0| > \frac{1}{2}$ (c) $|z_0| = \frac{1}{2}$ (d) none of these

79. The second order differential equation is

(a) $y'^2 + x + y^2$ (b) $y'y'' + y = \sin x$ (c) $y''' + y'' + y = 0$ (d) $y' = 0$

80. $\int e^{3x}\left(\frac{1+3\sin x}{1+\cos x}\right)dx$ is equal to

(a) $e^{3x}\cot x + c$ (b) $e^{3x}\tan\frac{x}{2} + c$ (c) $e^{3x}\sin x + c$ (d) $e^{3x}\cos x + c$

81. If m and n are positive integers and $f(x) = \int_1^x (t-a)^{2n}(t-b)^{2m+1}\,dt$, $a \neq b$, then

(a) $x = b$ is a point of local minimum (b) $x = b$ is a point of local maximum

(c) $x = a$ is a point of local minimum (d) $x = a$ is a point of local maximum

82. If in a triangle ABC $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$, then the value of the angle A is

(a) 45° (b) 90° (c) 135° (d) 60°

83. The general solution of the equation $2^{\cos 2x} + 1 = 3.2^{-\sin^2 x}$ is

(a) $n\pi$ (b) $\left(n+\frac{1}{2}\right)\pi$ (c) $\left(n-\frac{1}{2}\right)\pi$ (d) all of the above.

84. Total number of positive real values of x satisfying $2[x] = x + \{x\}$, where [.] and {.} denote the greatest integer function and fractional part respectively is equal to

(a) 2 (b) 1 (c) 0 (d) 3

85. If $\lim_{x\to 0}\frac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$, where n is nonzero real number, then a is equal to

(a) 0 (b) $\frac{n+1}{n}$ (c) n (d) $n + \frac{1}{n}$

86. $f(x) = \begin{cases} 4x - x^3 + \ln\left(a^2 - 3a + 3\right), & 0 \le x < 3 \\ x - 18, & x \ge 3 \end{cases}$. Find the complete set of values of a such that $f(x)$ has a local minima at $x = 3$ is

(a) $[-1, 2]$ (b) $(-\infty, 1) \cup (2, \infty)$ (c) $[1, 2]$ (d) $(-\infty, -1) \cup (2, \infty)$

87. The number of values of k for the system of equations $(k + 1)x + 8y = 4k$ and $kx + (k + 3)y = 3k - 1$ has infinitely many solutions

(a) 0 (b) 1 (c) 2 (d) infinite

88. The matrix $\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is

(a) unitary (b) null matrix (c) symmetric (d) none of these

89. The area between the curves $y = xe^{x}$ and $y = xe^{-x}$ and the line $x = 1$ is

(a) $2e$ (b) e (c) $2/e$ (d) $1/e$

90. If the unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ and $|\vec{a} - \vec{b}| < 1$ then θ lies in the interval

(a) $\left[0, \frac{\pi}{6}\right)$ (b) $\left(\frac{5\pi}{6}, 2\pi\right]$ (c) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$