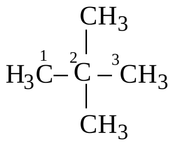
SOLUTIONS

CHEMISTRY

1. (a)

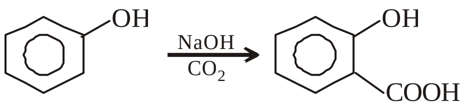


Neopentane
or2,2- Dimethylpropane

2. (d) The products of the concerned reaction react each other forming back the reactants.

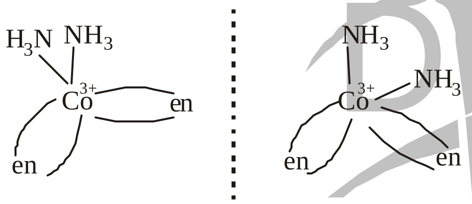
$$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$$

3. (b)



4. (c) Adsorption is an exothermic process, hence ΔH will always be negative

5. (c)



Enantiomers of *cis*-$\left[Co(en)_2(NH_3)_2\right]^{3+}$

6. (a) $\underset{1\times10^{-4}M}{Na_2CO_3} \longrightarrow \underset{1\times10^{-4}M}{2Na^+} + \underset{1\times10^{-4}M}{CO_3^{2-}}$

$$K_{SP(BaCO_3)} = [Ba^{2+}][CO_3^{2-}]$$

$$[Ba^{2+}] = \frac{5.1\times10^{-9}}{1\times10^{-4}} = 5.1\times10^{-5}M$$

7. (a) $\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{1.67\times10^{-27}\times1\times10^{3}}$

$= 3.97\times10^{-10}$ meter

$= 0.397$ nanometer

8. (a) Lower oxidation state of an element forms more basic oxide and hydroxide, while the higher oxidation state will form more acidic oxide/hydroxide. For example,

$\underbrace{\overset{+2}{MnO} \quad \overset{+3}{Mn_2O_3}}_{(basic)} \quad \underset{acidic}{\overset{+7}{Mn_2O_7}}$

9. (b) According to Heisenberg uncertainty principle.

$$\Delta x . m\Delta v = \frac{h}{4\pi}$$

$$\Delta x = \frac{h}{4\pi m\Delta v}$$

Here $\Delta v = \frac{600\times0.005}{100} = 0.03$

So, $\Delta x = \frac{6.6\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times0.03}$

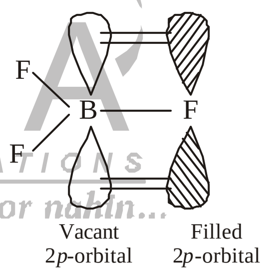
$= 1.92\times10^{-3}$ meter

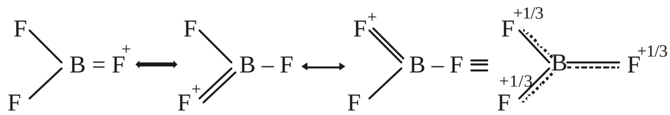
10. (a) The SCN^- ion can coordinate through S or N atom giving rise to linkage isomerism

$M \leftarrow SCN$ thiocyanato

$M \leftarrow NCS$ isothiocyanato.

11. (b) The delocalised $p\pi - p\pi$ bonding between filled *p*-orbital of F and vacant *p*-orbital of B leads to shortening of B–F bond length which results in higher bond dissociation energy of the B–F bond.





12. (d) Bond order

$$= \frac{\text{No.of bonding electrons} - \text{No.of antibonding electrons}}{2}$$

Bond order in $O_2^+ = \frac{10-5}{2} = 2.5$

Bond order in $O_2^- = \frac{10-7}{2} = 1.5$

Bond order in $O_2^{2-} = \frac{10-8}{2} = 1$

Bond order in $O_2^{2+} = \frac{10-4}{2} = 3$

Since Bond order $\propto \frac{1}{\text{Bond length}}$

$\therefore$ Bond length is shortest in O_2^{2+}.

13. (c) Fruity smell is due to ester formation which is formed between ethanol and acid.

$$CH_3COOH + C_2H_5OH \xrightarrow{Conc.H_2SO_4} CH_3COOC_2H_5 + H_2O.$$

14. (c) $CH_3CHCl_2 \xrightarrow{aq.KOH} CH_3CH(OH)_2 \xrightarrow{-H_2O} CH_3CHO$

15. (b) Buna – N is a copolymer of butadiene ($CH_2 = CH - CH = CH_2$) and acrylonitrile ($CH_2 = CHCN$).

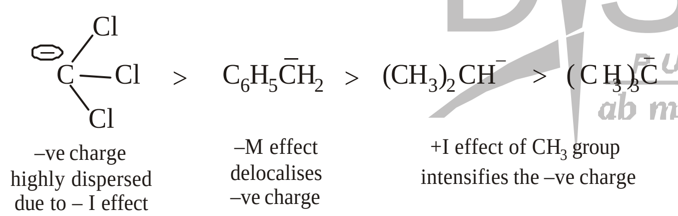
16. (c) Glucose is considered as a typical carbohydrate which contains –CHO and –OH group.

17. (b) In hydrides of 15th group elements, basic character decreases on descending the group i.e. $NH_3 > PH_3 > AsH_3 > SbH_3$.

18. (b) For this solution intermolecular interactions between *n*-heptane and ethanol aare weaker than *n*-heptane - *n*-heptane & ethanol-ethanol interactions hence the solution of *n*-heptane and ethanol is non-ideal and shows positive deviation from Raoult's law.

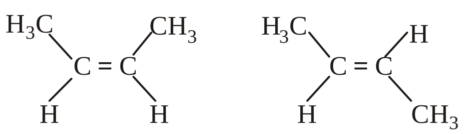
19. (a) Ionic radii increases on descending the group because of increase in the number of shell; however it decreases across the period due to increase in effective nuclear charge because electrons are added in the same shell. However, remember that Li^+ and Mg^{2+} are diagonally related.

20. (b)



21. (b) Most of the Ln^{3+} compounds except La^{3+} and Lu^{3+} are coloured due to the presence of *f*-electrons.

22. (b)

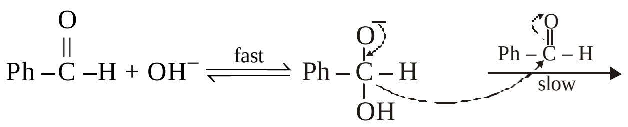


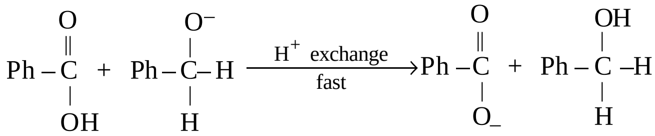
23. (b) $CH_3-CH=CH-\overset{*}{C}H(OH)CH_3$

exhibits both geometrical as well as optical isomerism.

cis - R　　*cis* - S
trans - R　　*trans* - S

24. (a)





25. (a) Given, for reaction

(i) $H_2O(\ell) \longrightarrow H^+(aq.) + OH^-(aq.)$; $\Delta H_r = 57.32$ kJ

(ii) $H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(\ell)$; $\Delta H_r = -286.20$ kJ

For reaction (i)

$$\Delta H_r = \Delta H^\circ_f(H^+.aq) + \Delta H^\circ_f(OH^-.aq) - \Delta H^\circ_f(H_2O, \ell)$$

$$57.32 = 0 + \Delta H^\circ_f(OH^-, aq) - \Delta H^\circ_f(H_2O, \ell) \quad \ldots(iii)$$

For reaction (ii)

$$\Delta H_r = \Delta H^\circ_f(H_2O, \ell) - \Delta H^\circ_f(H_2, g) - \frac{1}{2}\Delta H^\circ_f(O_2, g)$$

$$-286.20 = \Delta H^\circ_f(H_2O, \ell)$$

On replacing this value in equ. (iii) we have

$$57.32 = \Delta H^\circ_f(OH^-, aq) - (-286.20)$$

$$\Delta H^\circ_f = -286.20 + 57.32$$

$$= -228.88 \text{ kJ}$$

26. (a) For *fcc* unit cell,

$$4r = \sqrt{2}\,a$$

$$r = \frac{\sqrt{2} \times 361}{4} = 127 \text{ pm}$$

27. (c) $CH_3OH(l) + \frac{3}{2}O_2(g) \rightarrow CO_2(g) + 2H_2O(l)$

$$\Delta G_r = \Delta G_f(CO_2, g) + 2\Delta G_f(H_2O, \ell) - \Delta G_f(CH_3OH, \ell) - \frac{3}{2}\Delta G_f(O_2, g)$$

$= -394.4 + 2(-237.2) - (-166.2) - 0$
$= -394.4 - 474.4 + 166.2$
$= -702.6$ kJ

$$\% \text{ efficiency} = \frac{702.6}{726} \times 100 = 97\%$$

28. (b) $P_{total} = P^\circ_A X_A + P^\circ_B X_B$

$$550 = P^\circ_A \times \frac{1}{4} + P^\circ_B \times \frac{3}{4}$$

$P^\circ_A + 3P^\circ_B = 550 \times 4$ ...(i)

In second case

$$P_{total} = P^\circ_A \times \frac{1}{5} + P^\circ_B \times \frac{4}{5}$$

$P_A^\circ + 4P_B^\circ = 560 \times 5$ ...(ii)

Subtract (i) from (ii)

$\therefore P_B^\circ = 560 \times 5 - 550 \times 4 = 600$

$\because P_A^\circ = 400$

29. (b) Given

$Fe^{3+} + 3e^- \to Fe$, $E^\circ_{Fe^{3+}/Fe} = -0.036V$ … (i)

$Fe^{2+} + 2e^- \to Fe$, $E^\circ_{Fe^{2+}/Fe} = -0.439V$ … (ii)

we have to calculate

$Fe^{3+} + e^- \to Fe^{2+}$, $\Delta G = ?$

To obtain this equation subtract equ (ii) from (i) we get

$Fe^{3+} + e^- \to Fe^{2+}$ … (iii)

As we know that $\Delta G = -nFE$

Thus for reaction (iii)

$\Delta G = \Delta G_1 - \Delta G$

$-nFE = -nFE_1 - (-nFE_2)$

$-nFE = nFE_2 - nFE_1$

$-1FE^\circ = 2 \times 0.439F - 3 \times 0.036\ F$

$-1\ FE^\circ = 0.770\ F$

$\therefore$ $E^\circ = -0.770V$

30. (b) For first order reaction,

$$k = \frac{2.303}{t}\log\frac{100}{100-99}$$

$$\frac{0.693}{6.93} = \frac{2.303}{t}\log\frac{100}{1}$$

$$\frac{0.693}{6.93} = \frac{2.303 \times 2}{t}$$

$t = 46.06$ min

MATHS

31. (b) The truth table for the logical statements, involved in statement 1, is as follows :

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

We observe the columns for $\sim(p \leftrightarrow \sim q)$ and $p \leftrightarrow q$ are identical, therefore

$\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

But $\sim(p \leftrightarrow \sim q)$ is not a tautology as all entries in its column are not T.

$\therefore$ Statement-1 is true but statement-2 is false.

32. (a) We know that $|adj\,(adj\,A)| = |A|^{n-2} A$.

$= |A|^\circ A$

$= A$

Also $|adj\,A| = |A|^{n-1} = |A|^{2-1} = |A|$

$\therefore$ Both the statements are true but statement-2 is not a correct explantion for statement-1 .

33. (b) Given that $f(x) = (x+1)^2 - 1,\ x \geq -1$

Clearly $D_f = [-, \infty)$ but co-demain is not given. Therefore $f(x)$ need not be necessarily onto.

But if $f(x)$ is onto then as $f(x)$ is one one also, $(x + 1)$ being something +ve, $f^{-1}(x)$ will exist where

$(x+1)^2 - 1 = y$

$\Rightarrow x + 1 = \sqrt{y+1}$ (+ve square root as $x + 1 \geq 0$)

$\Rightarrow x = -1 + \sqrt{y+1}$

$\Rightarrow f^{-1}(x) = \sqrt{x+1} - 1$

Then $f(x) = f^{-1}(x)$

$\Rightarrow (x+1)^2 - 1 = \sqrt{x+1} - 1$

$\Rightarrow (x+1)^2 = \sqrt{x+1}$ $\Rightarrow$ $(x+1)^4 = (x+1)$

$\Rightarrow (x+1)[(x+1)^3 - 1] = 0$

$\Rightarrow x = -1, 0$

$\therefore$ The statement-1 is correct but statement-2 is false.

34. (c) For the numbers 2, 4, 6, 8, ..., $2n$

$$\bar{x} = \frac{2[n(n+1)]}{2n} = (n+1)$$

And $Var = \frac{\Sigma(x-\bar{x})^2}{2n} = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$$= \frac{4\Sigma n^2}{n} - (n+1)^2 = \frac{4n(n+1)(2n+1)}{6n} - (n+1)^2$$

$$= \frac{2(2n+1)(n+1)}{3} - (n+1)^2 = (n+1)\left[\frac{4n+2-3n-3}{3}\right]$$

$$= \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$$

$\therefore$ Statement-1 is false.

Clearly, statement - 2 is true.

35. (b) Given that $f(x) = x|x|$ and $g(x) = \sin x$

So that

$go\,f(x) = g(f(x)) = g(x|x|)$

$= \sin x|x|$

$$= \begin{cases} \sin(-x^2), & \text{if } x < 0 \\ \sin(x^2), & \text{if } x \geq 0 \end{cases}$$

$$= \begin{cases} -\sin x^2, & \text{if } x < 0 \\ \sin x^2, & \text{if } x \geq 0 \end{cases}$$

$$\therefore (go\,f)'(x) = \begin{cases} -2x\cos x^2, & \text{if } x < 0 \\ 2x\cos x^2, & \text{if } x \geq 0 \end{cases}$$

Here we observe

$L(go\,f)'(0) = 0 = R(go\,f)'(0)$

$\Rightarrow$ go f is differentiable at $x = 0$
and $(\text{go} f)'$ is continuous at $x = 0$

Now $(\text{go} f)''(x) = \begin{cases} -2\cos x^2 + 4x^2 \sin x^2, x < 0 \\ 2\cos x^2 - 4x^2 \sin x^2, x \geq 0 \end{cases}$

Here
$L(\text{go} f)''(0) = -2$ and $R(\text{go} f)''(0) = 2$

$\because$ $L(\text{go} f)''(0) \neq R(\text{go} f)''(0)$

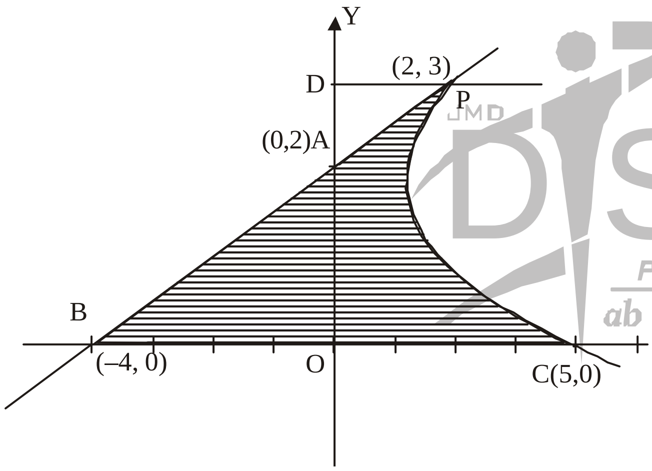
$\Rightarrow$ go $f(x)$ is not twice differentiable at $x = 0$.

$\therefore$ Statement - 1 is true but statement -2 is false.

36. (b) The given parabola is $(y-2)^2 = x - 1$
Vertex (1, 2) and it meets x –axis at (5, 0)
Also it gives $y^2 - 4y - x + 5 = 0$
So, that equation of tangent to the parabola at (2, 3) is

$y.3 - 2(y + 3) - \frac{1}{2}(x + 2) + 5 = 0$

or $x - 2y + 4 = 0$
which meets x-axis at (– 4, 0).
In the figure shaded area is the required area.
Let us draw PD perpendicular to y – axis.



Then required area = Ar ΔBOA + Ar (OCPD) – Ar (ΔAPD)

$= \frac{1}{2} \times 4 \times 2 + \int_0^3 x\,dy - \frac{1}{2} \times 2 \times 1$

$= 3 + \int_0^3 (y-2)^2 + 1\, dy$

$= 3 + \left[\frac{(y-2)^3}{3} + y\right]_0^3$

$= 3 + \left[\frac{1}{3} + 3 + \frac{8}{3}\right]$

$= 3 + 6 = 9$ Sq. units

37. (a) We have $P(x) = x^4 + ax^3 + bx^2 + cx + d$

$\Rightarrow$ $P'(x) = 4x^3 + 3ax^2 + 2bx + c$

But $P'(0) = 0 \Rightarrow c = 0$

\ $P(x) = x^4 + ax^3 + bx^2 + d$

As given that $P(-1) < P(a)$

$\Rightarrow$ $1 - a + b + d < 1 + a + b + d$

$\Rightarrow$ $a > 0$

Now $P'(x) = 4x^3 + 3ax^2 + 2bx$

$= x(4x^2 + 3ax + 2b)$

As $P'(x) = 0$, there is only one solution $x = 0$, therefore $4x^2 + 3ax + 2b = 0$ should not have any real roots i.e. $D < 0$

$\Rightarrow$ $9a^2 - 32b < 0$

$\Rightarrow$ $b > \frac{9a^2}{32} > 0$

Hence $a, b > 0 \Rightarrow P'(x) = 4x^3 + 3ax^2 + 2bx > 0$
$\forall x > 0$

$\therefore$ $P(x)$ is an increasing function on (0,1)

$\therefore$ $P(0) < P(a)$

Similarly we can prove $P(x)$ is decreasing on (– 1, 0)

$\therefore$ $P(-1) > P(0)$

So we can conclude that
Max $P(x) = P(1)$ and Min $P(x) = P(0)$

$\Rightarrow$ $P(-1)$ is not minimum but $P(1)$ is the maximum of P.

38. (d) Let (a^2, a) be the point of shortest distance on $x = y^2$
Then distance between (a^2, a) and line $x - y + 1 = 0$ is given by

$D = \frac{a^2 - a + 1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left[(a - \frac{1}{2})^2 + \frac{3}{4}\right]$

It is min when $a = \frac{1}{2}$ and $D_{min} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

39. (a) $\because$ The line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane
$x + 3y - a z + b = 0$

$\therefore$ Pt (2, 1, – 2) lies on the plane
i.e. $2 + 3 + 2\alpha + \beta = 0$

or $2\alpha + \beta + 5 = 0$ ....(i)

Also normal to plane will be perpendicular to line,

$\therefore$ $3 \times 1 - 5 \times 3 + 2 \times (-\alpha) = 0$

$\Rightarrow$ $a = -6$

From equation (i) then, $\beta = 7$

$\therefore$ $(\alpha, \beta) = (-6, 7)$

40. (c) 4 novels, out of 6 novels and 1 dictionary out of 3 can be selected in ${}^6C_4 \times {}^3C_1$ ways
Then 4 novels with one dictionary in the middle can be arranged in 4! ways.

$\therefore$ Total ways of arrangement $= {}^6C_4 \times {}^3C_1 \times 4!$
$= 1080$

41. (d) We have

$P(x \geq 1) \geq \frac{9}{10}$

$\Rightarrow\ 1 - P(x = 0) \ge \frac{9}{10}$

$\Rightarrow\ 1 - {}^nC_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^n \ge \frac{9}{10}$

$\Rightarrow\ 1 - \frac{9}{10} \ge \left(\frac{3}{4}\right)^n$

$\Rightarrow\ \left(\frac{3}{4}\right)^n \le \left(\frac{1}{10}\right)$

Taking log to the base 3/4, on both sides, we get

$n\log_{3/4}\left(\frac{3}{4}\right) \ge \log_{3/4}\left(\frac{1}{10}\right)$

$\Rightarrow\ n \ge -\log_{3/4} 10 = \frac{-\log_{10}10}{\log_{10}\left(\frac{3}{4}\right)} = \frac{-1}{\log_{10}3 - \log_{10}4}$

$\Rightarrow\ n \ge \frac{-1}{\log_{10}4 - \log_{10}3}$

42. (a) If the lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line then these lines must be parallel to each other,

$\therefore\ m_1 = m_2 \quad \Rightarrow\ -\frac{p(p^2+1)}{-1} = -\frac{(p^2+1)^2}{p^2+1}$

$\Rightarrow\ (p^2 + 1)\,(p + 1) = 0$

$\Rightarrow\ p = -1$

$\therefore$ p can have exactly one value.

43. (b) Let $x \in A$ and $x \in B \Leftrightarrow x \in A \cup B$

$\Leftrightarrow\ x \in A \cup C \quad (\because A \cup B = A \cup C)$

$\Leftrightarrow\ x \in C$

$\therefore\ B = C.$

Let $x \in A$ and $x \in B \Leftrightarrow x \in A \cap B$

$\Leftrightarrow x \in A \cap C \qquad (\because A \cap B = A \cap C)$

$\Leftrightarrow x \in C$

$\therefore B = C$

44. (b) Given that $f(x) = x^3 + 5x + 1$

$\therefore$ $f'(x) = 3x^2 + 5 > 0,\ \forall x \in R$

$\Rightarrow$ $f(x)$ is strictly increasing on R

$\Rightarrow$ $f(x)$ is one one

$\therefore$ Being a polynomial $f(x)$ is cont. and inc.

on R with $\lim_{x\to\infty} f(x) = -\infty$

and $\lim_{x\to\infty} f(x) = \infty$

$\therefore$ Range of $f = (-\infty, \infty) = R$

Hence f is onto also.

So, f is one one and onto R.

45.(c) We have $y = c_1 e^{c_2 x}$

$\Rightarrow\ y' = c_1 c_2 e^{c_2 x} = c_2 y$

$\Rightarrow\ \frac{y'}{y} = c_2$

$\Rightarrow\ \frac{y''y - (y')^2}{y^2} = O$

$\Rightarrow\ y''y = (y')^2$

46. (b) $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & a-1 & (-1)^{n+2}a \\ b+1 & b-1 & (-1)^{n+1}b \\ c-1 & c+1 & (-1)^n c \end{vmatrix} = 0$

(Taking transpose of second determineat)

$C_1 \Leftrightarrow C_3$

$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} - \begin{vmatrix} (-1)^{n+2}a & a-1 & a+1 \\ (-1)^{n+2}(-b) & b-1 & b+1 \\ (-1)^{n+2}c & c+1 & c-1 \end{vmatrix} = 0$

$C_2 \Leftrightarrow C_3$

$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2}\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$

$\Rightarrow \left[1 + (-1)^{n+2}\right]\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$

$C_2 - C_1,\ C_3 - C_1$

$\Rightarrow \left[1 + (-1)^{n+2}\right]\begin{vmatrix} a & 1 & -1 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$

$R_1 + R_3$

$\Rightarrow \left[1 + (-1)^{n+2}\right]\begin{vmatrix} a+c & 0 & 0 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow$ $[1 + (-1)^{n+2}](a + c)\,(2b + 1 + 2b - 1) = 0$

$\Rightarrow$ $4b\,(a + c)\,[1 + (-1)^{n+2}] = 0$

$\Rightarrow$ $1 + (-1)^{n+2} = 0$ as $b\,(a + c) \ne 0$

$\Rightarrow$ n should be an odd integer.

47. (a) $8^{2n} - (62)^{2n+1}$

$= (64)^n - (62)^{2n+1}$

$= (63+1)^n - (63-1)^{2n+1}$

$= \left[{}^nC_0 (63)^n + {}^nC_1 (63)^{n-1} + {}^nC_2 (63)^{n-2} + ... + {}^nC_{n-1} (63) + {}^nC_n\right]$

$= \left[{}^{2n+1}C_0 (63)^{2n+1} - {}^{2n+1}C_1 (63)^{2n} + {}^{2n+1}C_2 (63)^{2n-1} - .... + (-1)^{2n+1}\, {}^{2n+1}C_{2+1}\right]$

$= 63 \times \left[{}^nC_0 (63)^{n-1} + {}^nC_1 (63)^{n-2} + {}^nC_2 (63)^{n-3} + ...\right] + 1$

$- 63 \times \left[{}^{2n+1}C_0 (63)^{2n} - {}^{2n+1}C_1 (63)^{2n-1} + ...\right] + 1$

$\Rightarrow$ 63 × some integral value + 2

$\Rightarrow$ $8^{2n} - (62)^{2n+1}$ when divided by 9 leaves 2 as the remainder.

48. (d) $x^{2x} - 2x^x \cot y - 1 = 0$

$\Rightarrow 2 \cot y = x^x - x^{-x}$

$\Rightarrow 2 \cot y = u - \dfrac{1}{u}$ where $u = x^x$

Differentiating both sides with respect to x, we get

$\Rightarrow -2\operatorname{cosec}^2 y \dfrac{dy}{dx} = \left(1 + \dfrac{1}{u^2}\right)\dfrac{du}{dx}$

where $u = x^x \Rightarrow \log u = x \log x$

$\Rightarrow \dfrac{1}{u}\dfrac{du}{dx} = 1 + \log x$

$\Rightarrow \dfrac{du}{dx} = x^x (1 + \log x)$

$\therefore$ We get

$-2 \operatorname{cosec}^2 y \dfrac{dy}{dx} = (1 + x^{-2x}).x^x (1 + \log x)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\left(x^x + x^{-x}\right)(1 + \log x)}{-2(1 + \cot^2 y)}$ ....(i)

Now when $x = 1$, $x^{2x} - 2x^x \cot y - 1 = 0$, gives

$1 - 2 \cot y - 1 = 0$

$\Rightarrow \cot y = 0$

$\therefore$ From equation (i), at $x = 1$ and $\cot y = 0$, we get

$y'(1) = \dfrac{(1+1)(1+0)}{-2(1+0)} = -1$

49. (b) Given that roots of the equation $bx^2 + cx + a = 0$ are imaginary

$\therefore c^2 - 4ab < 0$ ....(i)

Let $y = 3b^2x^2 + 6\,bc\,x + 2c^2$

$\Rightarrow 3b^2x^2 + 6\,bc\,x + 2c^2 - y = 0$

As x is real, $D \geq 0$

$\Rightarrow 36\,b^2c^2 - 12\,b^2\,(2c^2 - y) \geq 0$

$\Rightarrow 12\,b^2\,(3\,c^2 - 2\,c^2 + y) \geq 0$

$\Rightarrow c^2 + y \geq 0$

$\Rightarrow y \geq -c^2$

But from eqn. (i), $c^2 < 4ab$ or $-c^2 > -4ab$

$\therefore$ we get $y \geq -c^2 > -4ab$

$\Rightarrow y > -4\,ab$

50. (a) We have

$S = 1 + \dfrac{2}{3} + \dfrac{6}{3^2} + \dfrac{10}{3^3} + \dfrac{14}{3^4} + ...\infty$ ....(1)

Multiplying both sides by $\dfrac{1}{3}$ we get

$\dfrac{1}{3} S = \dfrac{1}{3} + \dfrac{2}{3^2} + \dfrac{6}{3^3} + \dfrac{10}{3^4} + ...$ ....(2)

Subtracting eqn. (2) from eqn. (1) we get

$\dfrac{2}{3} S = 1 + \dfrac{1}{3} + \dfrac{4}{3^2} + \dfrac{4}{3^3} + \dfrac{4}{3^4} + ...$

$\Rightarrow \dfrac{2}{3} S = \dfrac{4}{3} + \dfrac{4}{3^2} + \dfrac{4}{3^3} + \dfrac{4}{3^4} + ...$

$\Rightarrow \dfrac{2}{3} S = \dfrac{\frac{4}{3}}{1 - \frac{1}{3}} = \dfrac{4}{3} \times \dfrac{3}{2}$

$\Rightarrow S = 3$

51. (b) Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the initial and final points of the vector whose projections on the three coordiante axes are 6, – 3, 2 then

$x_2 - x_1 = 6$; $y_2 - y_1 = -3$; $z_2 - z_1 = 2$

So that direction ratios of $\overrightarrow{PQ}$ are 6, – 3, 2

$\therefore$ Direction cosines of $\overrightarrow{PQ}$ are

$\dfrac{6}{\sqrt{6^2 + (-3)^2 + 2^2}}, \dfrac{-3}{\sqrt{6^2 + (-3)^2 + 2^2}}, \dfrac{2}{\sqrt{6^2 + (-3)^2 + 2^2}}$

$= \dfrac{6}{7}, \dfrac{-3}{7}, \dfrac{2}{7}$

52. (b) We have

$\cos(b - \gamma) + \cos(\gamma - a) + \cos(a - b) = -\dfrac{3}{2}$

$\Rightarrow 2\,[\cos(b - \gamma) + \cos(\gamma - a) + \cos(a - b)] + 3 = 0$

$\Rightarrow 2\,[\cos(b - \gamma) + \cos(\gamma - a) + \cos(a - b)] + \sin^2 a + \cos^2 a + \sin^2 b + \cos^2 b + \sin^2 \gamma + \cos^2 a = 0$

$\Rightarrow$ $[\sin^2 a + \sin^2 b + \sin^2 \gamma + 2\sin a \sin b + 2\sin b \sin\gamma + 2\sin\gamma \sin a]$
$+ [\cos^2 a + \cos^2 b + \cos^2\gamma + 2\cos a\cos b + 2\cos b\cos\gamma + 2\cos\gamma\cos a] = 0$

$\Rightarrow$ $[\sin a + \sin b + \sin\gamma]^2 + (\cos a + \cos b + \cos\gamma)^2 = 0$

$\Rightarrow$ $\sin a + \sin b + \sin\gamma = 0$ and $\cos a + \cos\beta + \cos\gamma = 0$

$\therefore$ A and B both are true.

53. (d) Let $A \equiv$ Sum of the digits is 8
$B \equiv$ Product of the digits is 0
Then A = {08, 17, 26, 35, 44}
B = {00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40,}
$A \cap B$ = { 08 }

$\therefore$ $P(A/B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{1/50}{14/50} = \dfrac{1}{14}$

54. (a) Given that
$P(1, 0), Q(-1, 0)$ and $\dfrac{AP}{AQ} = \dfrac{BP}{BQ} = \dfrac{CP}{CQ} = \dfrac{1}{3}$

$\Rightarrow$ $3AP = AQ$
Let $A = (x, y)$ then
$3AP = AQ \Rightarrow 9AP^2 = AQ^2$

$\Rightarrow$ $9(x-1)^2 + 9y^2 = (x+1)^2 + y^2$

$\Rightarrow$ $9x^2 - 18x + 9 + 9y^2 = x^2 + 2x + 1 + y^2$

$\Rightarrow$ $8x^2 - 20x + 8y^2 + 8 = 0$

$\Rightarrow$ $x^2 + y^2 - \dfrac{5}{3}x + 1 = 0$ ....(1)

$\therefore$ A lies on the circle given by eq (1). As B and C also follow the same condition, they must lie on the same circle.

$\therefore$ Centre of circumcircle of ΔABC
= Centre of circle given by (1) = $\left(\dfrac{5}{4}, 0\right)$

55. (b) Mean $= \dfrac{101 + d(1 + 2 + 3 + ...+100)}{101}$

$= 1 + \dfrac{d \times 100 \times 101}{101 \times 2}$

$= 1 + 50d$

$\because$ Mean deviation from the mean = 255

$\Rightarrow$ $\dfrac{1}{101}[|1-(1+50d)| + |(1+d)-(1+50d)| + |(1+2d)$
$-(1+50d)| + ... + |(1+100d)-(1+50d)|] = 255$

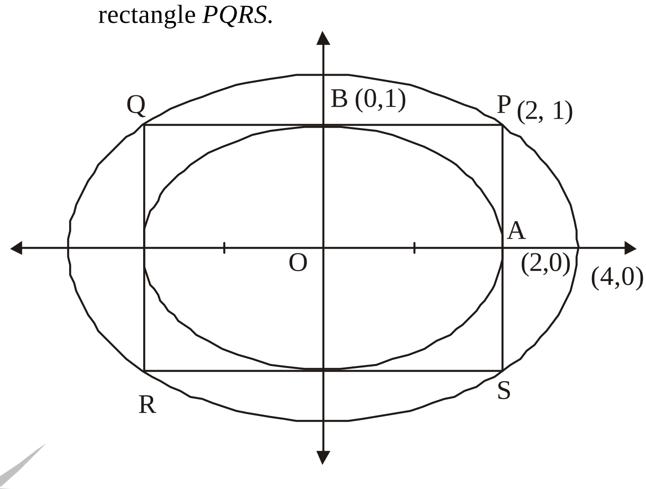
$\Rightarrow$ $2d[1+2+3+...+50] = 101 \times 255$

$\Rightarrow$ $2d \times \dfrac{50\times 51}{2} = 101 \times 255$

$\Rightarrow$ $d = \dfrac{101\times 255}{50\times 51} = 10.1$

56. (a) The given ellipse is $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$
So $A = (2, 0)$ and $B = (0, 1)$
If $PQRS$ is the rectangle in which it is inscribed, then $P = (2, 1)$.

Let $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ be the ellipse circumscribing the rectangle $PQRS$.



Then it passed through P (2,1)

$\therefore$ $\dfrac{4}{a^2} + \dfrac{1}{b^2} = 1$ ....(a)

Also, given that, it passes through (4, 0)

$\therefore$ $\dfrac{16}{a^2} + 0 = 1 \Rightarrow a^2 = 16$

$\Rightarrow$ $b^2 = 4/3$ [substituting $a^2 = 16$ in eq^n (a)]

$\therefore$ The required ellipse is $\dfrac{x^2}{16} + \dfrac{y^2}{4/3} = 1$ or $x^2 + 12y^2 = 16$

57. (a) Given that
$\left|z - \dfrac{4}{z}\right| = 2$

Now $|z| = \left|z - \dfrac{4}{z} + \dfrac{4}{-z}\right| \le \left|z - \dfrac{4}{z}\right| + \dfrac{4}{|z|}$

$\Rightarrow$ $|z| \le 2 + \dfrac{4}{|z|}$

$\Rightarrow$ $|z|^2 - 2|z| - 4 \le 0$

$\Rightarrow$ $\left(|z| - \dfrac{2+\sqrt{20}}{2}\right)\left(|z| - \dfrac{2-\sqrt{20}}{2}\right) \le 0$

$\Rightarrow$ $\left(|z| - (1+\sqrt{5})\right)\left(|z| - (1-\sqrt{5})\right) \le 0$

$\Rightarrow$ $\left(-\sqrt{5}+1\right) \le |z| \le \left(\sqrt{5}+1\right)$

$\Rightarrow$ $|z|_{max} = \sqrt{5} + 1$

58. (a) The given circles are

$S_1 \equiv x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ ....(1)

$S_2 \equiv x^2 + y^2 + 2x + 2y - p^2 = 0$ ....(2)

$\therefore$ Equation of common chord PQ is $S_1 - S_2 = 0$

$\Rightarrow L \equiv x + 5y + p^2 + 2p - 5 = 0$

$\Rightarrow$ Equation of circle passing through P and Q is $S_1 + \lambda L = 0$

$\Rightarrow (x^2 + y^2 + 3x + 7y + 2p - 5) + \lambda (x + 5y + p^2 + 2p - 5) = 0$

As it passes through (1, 1), therefore

$\Rightarrow (7 + 2p) + \lambda (2p + p^2 + 1) = 0$

$\Rightarrow \lambda = -\dfrac{2p+7}{(p+1)^2}$

which does not exist for p = – 1

59. (d) $\because \vec{u}, \vec{v}, \vec{w}$ are non coplanar vectors

$\therefore [\vec{u}, \vec{v}, \vec{w}] \neq 0$

Now,

$[3\vec{u}, p\vec{v}, p\vec{w}] - [p\vec{v}, p\vec{w}, q\vec{u}] - [2\vec{w}, q\vec{v}, q\vec{u}] = 0$

$\Rightarrow 3p^2 [\vec{u}, \vec{v}, \vec{w}] - pq [\vec{v}, \vec{w}, \vec{u}] - 2q^2 [\vec{w}, \vec{v}, \vec{u}] = 0$

$\Rightarrow 3p^2 [\vec{u}, \vec{v}, \vec{w}] - pq[\vec{u}, \vec{v}, \vec{w}] + 2q^2[\vec{u}, \vec{v}, \vec{w}]$

$\Rightarrow (3p^2 - pq + 2q^2) [\vec{u}, \vec{v}, \vec{w}] = 0$

$\Rightarrow 3p^2 - pq + 2q^2 = 0$

$\Rightarrow 2p^2 + p^2 - pq + \dfrac{q^2}{4} + \dfrac{7q^2}{4} = 0$

$\Rightarrow 2p^2 + \left(p - \dfrac{q}{2}\right)^2 + \dfrac{7}{4} q^2 = 0$

$\Rightarrow p = 0, q = 0, p = q/2$

This is possible only when $p = 0, q = 0$

$\therefore$ There is exactly one value of (p, q).

60. (c) Let $I = \int_0^{\pi} [\cot x]\, dx$ ....(1)

$= \int_0^{\pi} [\cot (\pi - x)]\, dx$

$= \int_0^{\pi} [-\cot x]\, dx$ ....(2)

Adding two values of I in eq^n s (1) & (2),

We get

$2I = \int_0^{\pi} ([\cot x] + [-\cot x])\, dx$

$= \int_0^{\pi} (-1) dx$ $[\because [x] + [-x] = -1$, if $x \notin z$ and $[x] + [-x] = 0$, if $x \in z]$

$= [-x]_0^{\pi} = -\pi$

$\Rightarrow I = -\dfrac{\pi}{2}$

PHYSICS

61. (b) For downward motion $v = -gt$

The velocity of the rubber ball increases in downward direction and we get a straight line between v and t with a negative slope.

Also applying $y - y_0 = ut + \frac{1}{2} at^2$

We get $y - h = -\frac{1}{2} gt^2 \Rightarrow y = h - \frac{1}{2} gt^2$

The graph between y and t is a parabola with $y = h$ at $t = 0$. As time increases y decreases.

For upward motion.

The ball suffer elastic collision with the horizontal elastic plate therefore the direction of velocity is reversed and the magnitude remains the same.

Here $v = u - gt$ where u is the velocity just after collision.

As t increases, v decreases. We get a straight line between v and t with negative slope.

Also $y = ut - \frac{1}{2} gt^2$

All these characteristics are represented by graph (b).

62. (d) We know that $\dfrac{g'}{g} = \dfrac{R^2}{(R+h)^2}$

$\therefore \dfrac{g/9}{g} = \left[\dfrac{R}{R+h}\right]^2$

$\therefore \dfrac{R}{R+h} = \dfrac{1}{3}$

$\therefore h = 2R$

63. (a) The heat flow rate is given by

$\dfrac{d\theta}{dt} = \dfrac{kA(\theta_1 - \theta)}{x}$

$\Rightarrow \theta_1 - \theta = \dfrac{x}{kA} \dfrac{d\theta}{dt}$

$\Rightarrow \theta = \theta_1 - \dfrac{x}{kA} \dfrac{d\theta}{dt}$

where θ_1 is the temperature of hot end and θ is temperature at a distance x from hot end.

The above equation can be graphically represented by option (a) .

64. (c) $\dfrac{W_{PQ}}{q} = (V_Q - V_P)$

$\Rightarrow W_{PQ} = q(V_Q - V_P)$

$= (-100 \times 1.6 \times 10^{-19})(-4 - 10)$

$= +2.24 \times 10^{-16}$ J

65. (a) The magnetic field at O due to current in DA is

$$B_1 = \frac{\mu_o I}{4\pi a} \times \frac{\pi}{6} \quad \text{(directed vertically upwards)}$$

The magnetic field at O due to current in BC is

$$B_2 = \frac{\mu_o I}{4\pi b} \times \frac{\pi}{6} \quad \text{(directed vertically downwards)}$$

The magnetic field due to current AB and CD at O is zero.

Therefore the net magnetic field is

$$B = B_1 - B_2 \quad \text{(directed vertically upwards)}$$

$$= \frac{\mu_o I}{4\pi a}\frac{\pi}{6} - \frac{\mu_o I}{4\pi b} \times \frac{\pi}{6}$$

$$= \frac{\mu_o I}{24}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\mu_o I}{24ab}(b - a)$$

66. (d) We know that $\vec{F} = I(\vec{\ell} \times \vec{B})$

The force on AD and BC due to current I_1 is zero. This is because the directions of current element $I\,\vec{d\ell}$ and magnetic field $\vec{B}$ are parallel.

67. (b) A to B is an isobaric process. The work done

$$W = nR(T_2 - T_1) = 2R(500 - 300) = 400R$$

68. (a) Work done by the system in the isothermal process DA is

$$W = 2.303\, nRT \log_{10}\frac{P_D}{P_A}$$

$$= 2.303 \times 2R \times 300 \log_{10}\frac{1 \times 10^5}{2 \times 10^5} = -414R$$

Therefore work done on the gas is + 414 R.

69. (a) The net work in the cycle $ABCDA$ is

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= 400R + 2.303\, nRT \log\frac{P_B}{P_C} + (-400R) - 414R$$

$$= 2.303 \times 2R \times 500 \log\frac{2 \times 10^5}{1 \times 10^5} - 414R$$

$= 693.2\,R - 414\,R$

$= 279.2\,R$

70. (d) 30 Divisions of vernier scale coincide with 29 divisions of main scales

Therefore 1 V.S.D = $\frac{29}{30}$ MSD

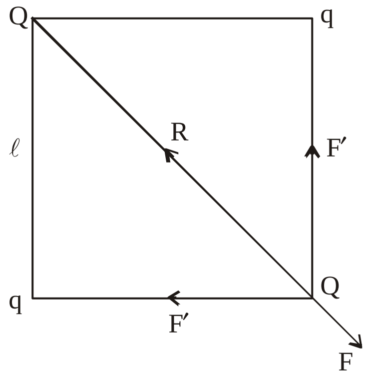
Least count = 1 MSD – 1VSD

$$= 1 \text{ MSD} - \frac{29}{30} \text{ MSD}$$

$$= \frac{1}{30} \text{MSD}$$

$$= \frac{1}{30} \times 0.5^\circ = 1 \text{ minute.}$$

71. (d) Let F be the force between Q and Q. The force between q and Q should be attractive for net force on Q to be zero. Let F' be the force between Q and q. The resultant of F' and F' is R. For equilibrium



$$\vec{R} + \vec{F} = 0$$

$$\sqrt{2}F' = -F$$

$$\sqrt{2} \times k\frac{Qq}{\ell^2} = -k\frac{Q^2}{(\sqrt{2}\,\ell)^2} \Rightarrow \frac{Q}{q} = -2\sqrt{2}$$

72. (a) Volume = $\frac{\text{mass}}{\text{density}} = \frac{1}{4}\,m^3$

$$\text{K.E} = \frac{5}{2}PV = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4\,J$$

Alternatively:

$$\text{K.E} = \frac{5}{2}nRT = \frac{5}{2}\frac{m}{M}RT = \frac{5}{2}\frac{m}{M} \times \frac{PM}{d} \quad [\because PM = dRT]$$

$$= \frac{5}{2}\frac{mP}{d} = \frac{5}{2} \times \frac{1 \times 8 \times 10^4}{4} = 5 \times 10^4\,J$$

73 (c) Growth in current in LR_2 branch when switch is closed is given by

$$i = \frac{E}{R_2}[1 - e^{-R_2 t/L}] \Rightarrow \frac{di}{dt} = \frac{E}{R_2}.\frac{R_2}{L}.e^{-R_2 t/L} = \frac{E}{L}e^{-\frac{R_2 t}{L}}$$

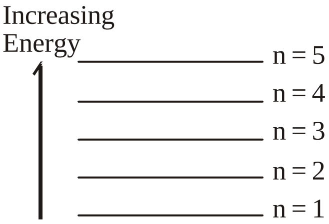
Hence, potential drop across L =

$$\left(\frac{E}{L}e^{-R_2 t/L}\right)L = Ee^{-R_2 t/L} = 12e^{-\frac{2t}{400 \times 10^{-3}}}$$

$= 12e^{-5t}$V

74. (c) (The relation $R = R_0(1 + \alpha\Delta t)$ is valid for small values of Δt and also $(R - R_0)$ should be much smaller than R_0. So, statement (1) is wrong but statement (2) is correct.

75 (c) It is given that transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom result in ultraviolet radiation. For infrared radiation the energy gap should be less. The only option is $5 \to 4$.



76. (b) Third bright fringe of known light coincides with the 4th bright fringe of the unknown light.

$$\therefore \frac{3(590)D}{d} = \frac{4\lambda D}{d}$$

$$\Rightarrow \lambda = \frac{3}{4} \times 590 = 442.5 nm$$

77. (a) Given $\vec{u} = 3\hat{i} + 4\hat{j}$, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$, $t = 10$ s

$$\vec{v} = \vec{u} + \vec{a}t = 3\hat{i} + 4\hat{j} + (0.4\hat{i} + 0.3\hat{j}) \times 10 = 7\hat{i} + 7\hat{j}$$

$$\therefore |\vec{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2} \text{ units}$$

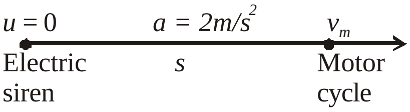
78. (a) $\lambda = 400$ nm, $hc = 1240$ eV.nm, K.E. = 1.68 eV

We know that $\frac{hc}{\lambda} - W = K.E \Rightarrow W = \frac{hc}{\lambda} - K.E$

$$\Rightarrow W = \frac{1240}{400} - 1.68 = 3.1 - 1.68 = 1.42 \text{ eV}$$

79. (b) Maximum number of beats = $(\nu + 1) - (\nu - 1) = 2$

80. (a)



$$v_m^2 - u^2 = 2as$$

$$\therefore v_m^2 = 2 \times 2 \times s$$

$$\therefore v_m = 2\sqrt{s}$$

According to Doppler's effect

$$v' = v\left[\frac{v - v_m}{v}\right]$$

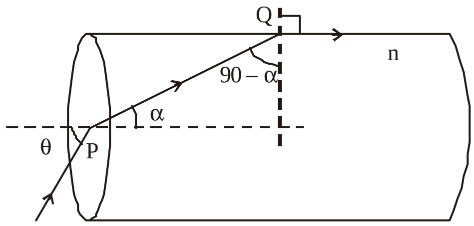
$$0.94v = v\left[\frac{330 - 2\sqrt{s}}{330}\right]$$

$$\Rightarrow s = 98.01\ m$$

81. (d) For $A + B \to C + \varepsilon$, ε is positive. This is because E_b for C is greater than the E_b for A and B.

Again for $F \to D + E + \varepsilon$, ε is positive. This is because E_b for D and E is greater than E_b for F.

82. (c)



Applying Snell's law at Q

$$n = \frac{\sin 90°}{\sin(90° - \alpha)} = \frac{1}{\cos\alpha}$$

$$\therefore \cos\alpha = \frac{1}{n}$$

$$\therefore \sin\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{1 - \frac{1}{n^2}} = \frac{\sqrt{n^2 - 1}}{n} \quad ...(1)$$

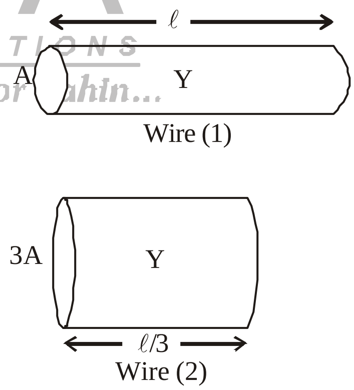
Applying Snell's Law at P

$$n = \frac{\sin\theta}{\sin\alpha} \Rightarrow \sin\theta = n \times \sin\alpha = \sqrt{n^2 - 1}\text{; from (1)}$$

$$\therefore \sin\theta = \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 - 1} = \sqrt{\frac{4}{3} - 1} = \frac{1}{\sqrt{3}}$$

or $\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

83. (c)



As shown in the figure, the wires will have the same Young's modulus (same material) and the length of the wire of area of cross-section 3A will be $\ell/3$ (same volume as wire 1).

For wire 1,

$$Y = \frac{F/A}{\Delta x/\ell} \quad ...(i)$$

For wire 2,

$$Y = \frac{F'/3A}{\Delta x/(\ell/3)} \quad ...(ii)$$

From (i) and (ii), $\frac{F}{A} \times \frac{\ell}{\Delta x} = \frac{F'}{3A} \times \frac{\ell}{3\Delta x} \Rightarrow F' = 9F$

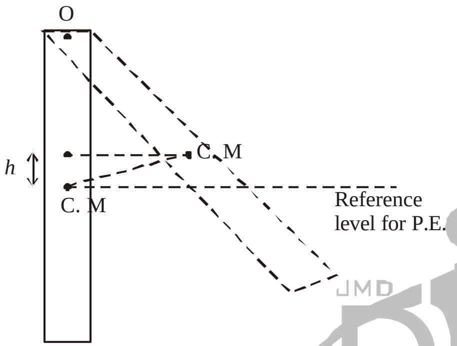
84. (a) Statement 1 is true.
Statement 2 is true and is the correct explanation of (1)

85. (d) Here $y = (\overline{\overline{A} + \overline{B}}) = \overline{\overline{A}}.\overline{\overline{B}} = A \cdot B$. Thus it is an AND gate for which truth table is

A	B	y
0	0	0
0	1	0
1	0	0
1	1	1

86. (a) For an SHM, the acceleration $a = -\omega^2 x$ where ω^2 is a constant. Therefore $\frac{a}{x}$ is a constant. The time period T is also constant. Therefore $\frac{aT}{x}$ is a constant.

87. (c)



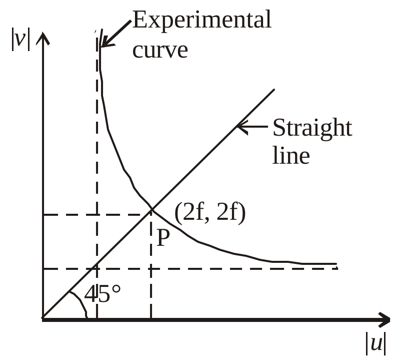
The moment of inertia of the rod about O is $\frac{1}{3}m\ell^2$.

The maximum angular speed of the rod is when the rod is instantaneously vertical. The energy of the rod in this condition is $\frac{1}{2}I\omega^2$ where I is the moment of inertia of the rod about O. When the rod is in its extreme portion, its angular velocity is zero momentarily. In this case, the energy of the rod is mgh where h is the maximum height to which the centre of mass (C.M) rises

$$\therefore \text{ mgh} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega^2$$

$$\Rightarrow h = \frac{\ell^2\omega^2}{6g}$$

88. (d)



For a convex lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

when $u = -\infty$, $v = +f$

when $u = -f$, $v = +\infty$

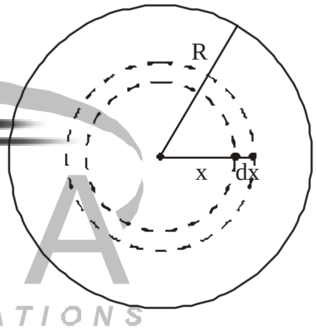
Then $u = -2f, v = 2f$

Also $v = \frac{f}{1 + \frac{f}{u}}$

As $|u|$ increases, v decreases for $|u| > f$. The graph between $|v|$ and $|u|$ is shown in the figure. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P $(2f, 2f)$.

89. (b) We know that a single p-n junction diode connected to an a-c source acts as a half wave rectifier [Forward biased in one half cycle and reverse biased in the other half cycle].

90. (b)



Let us consider a spherical shell of thickness dx and radius x. The volume of this spherical shell $= 4\pi x^2 dx$.
The charge enclosed within shell

$$= \left[\frac{Q.x}{\pi R^4}\right][4\pi x^2 dx]$$

$$= \frac{4Q}{R^4}x^3 dx$$

The charge enclosed in a sphere of radius r_1 is

$$= \frac{4Q}{R^4}\int_0^{r_1} x^3 dx = \frac{4Q}{R^4}\left[\frac{x^4}{4}\right]_0^{r_1} = \frac{Q}{R^4}r_1^4$$

$\therefore$ The electric field at point p inside the sphere at a distance r_1 from the centre of the sphere is

$$E = \frac{1}{4\pi\epsilon_0}\frac{\left[\frac{Q}{R^4}r_1^4\right]}{r_1^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{R^4}r_1^2$$