PART A – PHYSICS

Directions: Questions number 1 – 3 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is
   (1) Planar (2) Convex (3) Concave
   (4) Convex near the axis and concave near the periphery

1. **(1)** Wavefront is planar for parallel beam.

2. The speed of light in the medium is
   (1) Maximum on the axis of the beam (2) Minimum on the axis of the beam
   (3) The same everywhere in the beam (4) Directly proportional to the intensity I

2. **(2)** μ_{axis} is the maximum, $\therefore$ v_{axis} is minimum.

3. As the beam enters the medium, it will
   (1) Travel as a cylindrical beam (2) Diverge
   (3) Converge
   (4) Diverge near the axis and converge near the periphery

3. **(1)** All rays are entering normally (to interface), therefore in medium also they will move normally.

Directions: Questions number 4 – 5 are based on the following paragraph.

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $M/2$ each. Speed of light is c.

4. The speed of daughter nuclei is:

   (1) $c\sqrt{\dfrac{\Delta m}{M+\Delta m}}$ (2) $c\dfrac{\Delta m}{M+\Delta m}$ (3) $c\sqrt{\dfrac{2\Delta m}{M}}$ (4) $c\sqrt{\dfrac{\Delta m}{M}}$

4. **(3)** $\Delta m \cdot c^2 = 2\{(1/2)\cdot(M/2)\cdot v^2\}$ $\Rightarrow$ $v = c\sqrt{\dfrac{2\Delta m}{M}}$

5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then
   (1) $E_1 = 2E_2$ (2) $E_2 = 2E_1$ (3) $E_1 > E_2$ (4) $E_2 > E_1$

5. **(4)** $M + \Delta m \to \{(M/2) + (M/2) + E\}$, where, $E = \Delta m \cdot c^2$
   $\therefore$ Applying conservation of Energy on both sides, $-E_1 = -E_2 + E$ $\Rightarrow E_2 > E_1$

Directions: Questions number 6 – 7 contain Statement-1 and Statement - 2. Of the four choices given after the statements, choose the one that best describes tht two statements.

6. **Statement-1:** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement-2: Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

(1) Statement-1 is true, Statement-2 is false.
(2) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** the correct explanation of Statement-1.
(4) Statement-1 is false, Statement-2 is true.

6. **(1)** $hc/\lambda_{incident} = W + KE_{max}$, where $KE_{max} = eV_s$,
$\therefore$ On decreasing lincident, KE_{max} & therefore V_s increases.
Speeds of different photoelectrons are different due to electrons emitted from different orbits.

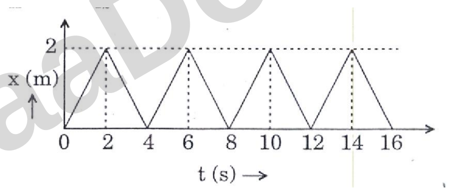
7. **Statement-1:** Two particles moving in the same direction do nto lose all their energy in a completely inelastic collision.

Statement-2: Principle of conservation of momentum holds true for all kinds of collisions.

(1) Statement-1 is true, Statement-2 is false.
(2) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** the correct explanation of Statement-1.
(4) Statement-1 is false, Statement-2 is true.

7. **(2)** $V_f = \{(m_1v_1 + m_2v_2) / (m_1+m_2)\}$
since collision is inclastic $\Rightarrow$ There is some final KE.

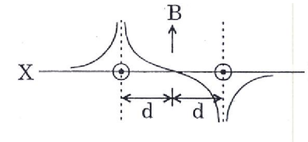
8. The figure shows the position – time ($x - t$) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is:

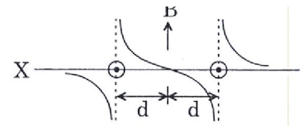


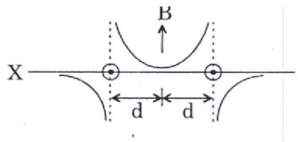
(1) 0.2 Ns (2) 0.4 Ns (3) 0.8 Ns (4) 1.6 Ns

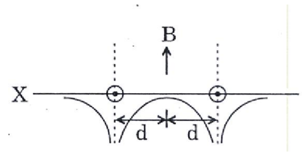
8. **(3)** Impulse **=** $m\Delta v$ **=** m × (–1 –1) **= 0.8 Ns**

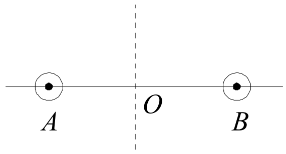
9. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field *B* along the line *XX'* is given by

(1) 

(2) 

(3) 

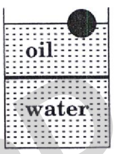
(4) 

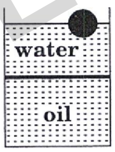
9. **(2)** 

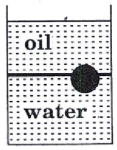
Magnetic field for infinite wire = $(\mu_0 I / 2\pi r)$

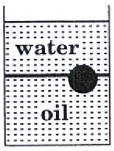
i. At mid point between *A* & *B* field is zero.
ii. In between *A* to *O* net field is upward.
iii. In between *O* to *B* net field is downward.
iv. Left of point *A* net field is downward
v. Right of point *B* net field is upward.

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?

(1) 

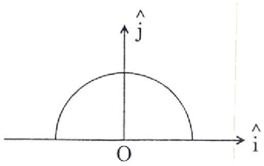
(2) 

(3) 

(4) 

10. **(3)** ***i.*** $\rho_{oil} < \rho_{water}$ $\Rightarrow$ oil will be above water
ii. $\rho_{oil} < \rho < \rho_{water}$ $\Rightarrow$ Ball will be in equilibrium at oil-water interface.

11. A thin semi-circular ring of radius *r* has a positive charge *q* distributed uniformly over it. The net field $\vec{E}$ at the centre *O* is:



(1) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (2) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (3) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (4) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

11. **(4)** $\bar{E} = \int dE \sin\theta(-\hat{j}) = \int_0^{\pi} k(rd\theta.\frac{q}{\pi r}).\frac{1}{r^2}\sin\theta(-\hat{j}) = \frac{2kq}{\pi r^2}(-\hat{j}) = \frac{q}{2\pi^2\varepsilon\quad 2}(-\hat{j})$

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is:

(1) 0.25 (2) 0.5 (3) 0.75 (4) 0.99

12. **(3)** $TV^{\gamma-1}$ = Const. $\Rightarrow T_1 V^{2/5} = T_2 (32V)^{2/5}$ $\Rightarrow T_1 / T_2 = 4$

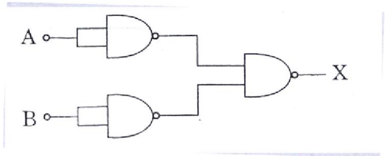
$\therefore \quad \eta = 1 - (T_2 / T_1) = 1 - (1/4) = 3/4 = 0.75$

13. The respective number of significant figure for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(1) 4, 4, 2 (2) 5, 1, 2 (3) 5, 1, 5 (4) 5, 5, 2

13. **(2)** Number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are 5, 1, 2 respectively.

14. The combination of gates shown below yields



(1) NAND gate (2) OR gate (3) NOT gate (4) XOR gate

14. **(2)** Each element in the circuit is NAND - gate

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

The truth table obtained is similar to OR-gate.

15. If a source of power 5 kW produces 10^{20} photons / second, the radiation belongs to a part of the spectrum called:

(1) γ-ray (2) X-rays (3) ultraviolet rays (4) microwaves

15. **(2)** Energy of one photon $= 4 \times 10^3 / 10^{20} = 4 \times 10^{-17}$ J

$\therefore \quad \lambda = hc / (4 \times 10^{-17}) = \{(6.67 \times 10^{-34} \times 3 \times 10^8) / (4 \times 10^{-17})\}$

$\approx 5 \times 10^{-9}$ m = 50Å $\Rightarrow$ X-rays

16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(1) $\frac{A-Z-4}{Z-2}$ (2) $\frac{A-Z-8}{Z-4}$ (3) $\frac{A-Z-4}{Z-8}$ (4) $\frac{A-Z-12}{Z-4}$

16. **(3)** If only 3 α particles emitted, No. of protons = $Z - 6$

No. of neutrons = $A - Z - 6$. Now, if again 2 positions emitted,

No. of protons = Z – 6 – 2 = Z – 8 & No. of neutons = A – Z – 6 + 2 = A – Z – 4.

17. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto r = R, and $\rho(r)$ = 0 for $r > R$, where r is the distance from the origin. The electric field at a distance $r(r < R)$ from the origin is given by

(1) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (2) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (3) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (4) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

17. **(3)** $q_{enclosed}(r) = \int_0^r \rho_0\left(\frac{5}{4} - \frac{x}{R}\right).4\pi x^2.dx$

$= \pi\rho_0 r^3 \{(5/3) - (r/R)\}$

$E(r) = (1/4\pi\varepsilon_0) . (q_{enclosed}/r^2) = (\rho_0 r/4\varepsilon_0) \{(5/3) - (r/R)\}$.

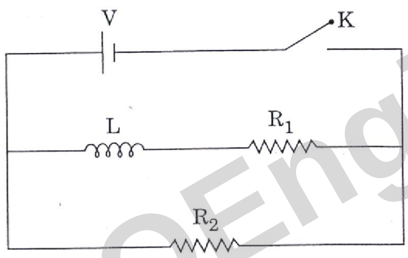
18. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the veltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is

(1) 242 W (2) 305 W (3) 210 W (4) Zero W

18. **(1)** $\tan 30° = X_L/R = X_C/R \quad \Rightarrow \quad X_L = X_C$

$\therefore \quad I = 220/200$ A $\therefore \quad P = I^2R = 242$ W

19. In the circuit shown below, the key K is closed at t = 0. The current through the battery is



(1) $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$ (2) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(3) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = \infty$ (4) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t = \infty$

19. **(3)** At $t = 0^+; i_L = 0 \quad \therefore \quad i_{battery} = V/R_2$

At $t = \infty; V_L = 0 \quad \therefore \quad i_{battery} = \{V(R_1+R_2)/R_1R_2\}$

20. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is

(1) $y^2 = x^2$ + constant (2) $y = x^2$ + constant

(3) $y^2 = x$ + constant (4) $xy =$ constant

20. **(1)** $V_x = Ky$, $V_y = xK$ $(dx/dt) = Ky$, $(dy/dt) = xK$; $x = Kyt + C_1$ $y = xKt + c_2$

$x = Ky.(y/xK) + c_3 \quad \Rightarrow x^2 - y^2 = c_3$

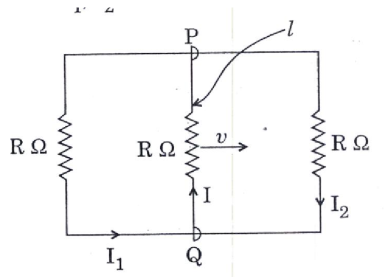
21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1 / t_2 will be

(1) 2 (2) 1 (3) 1 / 2 (4) 1 / 4

21. **(4)** $q = q_0\, e^{-t/CR}$ $U = 1/2\,(q^2/C) = U_0\, e^{-2t/CR}$

$q_0 / 4 = q_0\, e^{-t_2/CR}$ $U_0 / 2 = U_0 e^{-2t_1/CR}$ $\Rightarrow 2t_1 / t_2 = \ln 2 / \ln 4$ $\Rightarrow$ $t_1 / t_2 = 1/4$

22. A rectangular loop has sliding connector PQ of length l and resistance $R\,\Omega$ and it is moving with a speed v as shown. The set- up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(1) $I_1 = I_2 = \dfrac{Blv}{6R}, I = \dfrac{Blv}{3R}$ (2) $I_1 = -I_2 = \dfrac{Blv}{R}, I = \dfrac{2Blv}{R}$

(3) $I_1 = I_2 = \dfrac{Blv}{3R}, I = \dfrac{2Blv}{3R}$ (4) $I_1 = I_2 = I = \dfrac{Blv}{R}$

22. **(3)** Motional emf = Blv $I + I_2 = Blv / R$ …1

$I + I_1 = Blv / R$ …2

$I = I_1 + I_2$ …3

from 1, 2 and 3 $I_1 = I_2 = Blv / 3R$; $I = 2Blv / 3R$

23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$y = 0.02(m)\sin\left[2\pi\left(\dfrac{t}{0.04(s)} - \dfrac{x}{0.50(m)}\right)\right]$. The tension in the string is

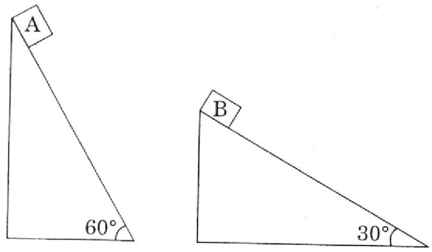
(1) 6.25 N (2) 4.0 N (3) 12.5 N (4) 0.5 N

23. **(1)** $y = 0.02(\mathrm{m})\sin\left[2\pi\left(\dfrac{t}{0.04} - \dfrac{x}{0.50}\right)\right]$

$\therefore$ v_{wave} = Cofficient of t / Coefficient of $x = 50/4\ \mathrm{ms}^{-1}$

$\therefore$ $T = v^2\mu = (50/4)^2 \times 0.04 = 6.25$ N

24. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks *A* and *B* are placed on the two planes. What is the relative vertical acceleration of *A* with respect to *B*?



(1) 4.9 ms^{-2} in vertical direction (2) 4.9 ms^{-2} in horizontal direction
(3) 9.8 ms^{-2} in vertical direction (4) Zero

24. **(1)** $\left|\vec{a}_{AB(v)}\right| = \left|\vec{a}_{A(v)} - \vec{a}_{B(v)}\right|_{=g(\sin^2 60° - \sin^2 30°)} = g/2 = 4.9 \text{ ms}^{-2}$

25. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the *x*-axis)

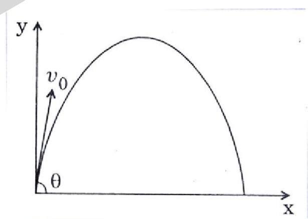
(1) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$ (2) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$

(3) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$ (4) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

25. **(4)** Resolving the V^2/R along x & y-axes at point $(R\cos\theta, R\cos\theta)$

$$\frac{V^2}{R}\cos\theta(-\hat{i}) + \frac{V^2}{R}(-\hat{j})$$

26. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the $x - y$ plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



(1) $\frac{1}{2}mg\,v_0\,t^2\cos\theta\,\hat{i}$ (2) $-mg\,v_0\,t^2\cos\theta\,\hat{j}$

(3) $mg\,v_0\,t\cos\theta\,\hat{k}$ (4) $-\frac{1}{2}mg\,v_0\,t^2\cos\theta\,\hat{k}$

Where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z - axis respectively.

26. **(4)** Only possible answer is last option since direction for angular momentum will be only in –ve z-direction.

27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is

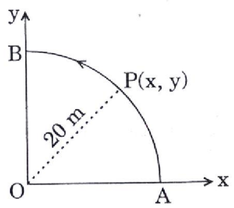
(1) 1 (2) 4 (3) 3 (4) 2

27. **(4)** $\tan 30° = V\rho g / F_e$

When system immersed in liquid $\tan 30° = \dfrac{\varepsilon_r V(\rho - \rho_{liq})g}{F_e}$

$\therefore \quad \varepsilon_r(\rho - \rho_{liq.}) = \rho \qquad \Rightarrow \varepsilon_r = \{\rho / (\rho - \rho_{liq.}) = 2$

28. A point *P* moves in counter-clockwise direction on a circular path as shown in the figure. The movement of *P* is such that it sweeps out a length $s = t^3 + 5$, where *s* is in metres and *t* is in seconds. The radius of the path is 20 m. The acceleration of *P* when *t* = 2 s is nearly



(1) 14 ms^{-2} (2) 13 ms^{-2} (3) 12 ms^{-2} (4) 7.2 ms^{-2}

28. **(1)** $a_T = d^2s / dt^2 = 6t$ & $a_N = v^2 / R = (3t^2)^2 / 20$

$\therefore$ At 2 sec, $a_T = 12$, $a_N = 7.2\ ms^{-2}$, $\therefore \quad a = \sqrt{(12^2 + 7.2^2)} \approx 14\ ms^{-2}$

29. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$, where *a* and *b* are constants and *x* is the distance between the atoms. If the dissociation energy of the molecule is $D = \left[U_{(x=\infty)} - U_{at\ equilibrium}\right] D$

(1) $\dfrac{b^2}{6a}$ (2) $\dfrac{b^2}{2a}$ (3) $\dfrac{b^2}{12a}$ (4) $\dfrac{b^2}{4a}$

29. **(4)** $U_{(x=\infty)} = 0$ At Equilibrium, $F = -dU/dx = -[(a.(-12)/x^{13}) - (b(-6)/x^7)] = 0$;
$x^6 = 2a/b$, $\therefore D = [0 - \{(a/(2a/b)^2) - (b/(2a/b))\}] = b^2/4a$

30. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(1) $\dfrac{\alpha_1 + \alpha_2}{2}, \dfrac{\alpha_1 + \alpha_2}{2}$ (2) $\dfrac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

(3) $\alpha_1 + \alpha_2, \dfrac{\alpha_1 + \alpha_2}{2}$ (4) $\alpha_1 + \alpha_2, \dfrac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$

30. **(1)** $\alpha_{eq} = \dfrac{R_{final} - R_{initial}}{R_{initial} \Delta\theta}$. In series : $R_{initial} = 2R$, $R_{final} = 2R + R(\alpha_1 + \alpha_2)\Delta\theta$.

$\therefore \alpha_{eq} = \dfrac{R(\alpha_1 + \alpha_2)\Delta\theta}{2R\Delta\theta} = \dfrac{\alpha_1 + \alpha_2}{2}$. In parallel $R_{initial} = R/2$

$\dfrac{R}{2}(1 + \alpha_{eq}.\Delta\theta) = \dfrac{R(1 + \alpha_1\Delta\theta).R(1 + \alpha_2.\Delta\theta)}{R(1 + \alpha_1\Delta\theta) + (1 + \alpha_2\Delta\theta)} \qquad \therefore \alpha_{eq} = \dfrac{\alpha_1 + \alpha_2}{2}$