AIEEE – 2004 (MATHEMATICS)

Important Instructions:

i) *The test is of* $1\frac{1}{2}$ *hours duration.*

ii) *The test consists of 75 questions.*

iii) *The maximum marks are 225.*

iv) *For each correct answer you will get 3 marks and for a wrong answer you will get -1 mark.*

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1. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is
(1) a function
(2) reflexive
(3) not symmetric
(4) transitive

2. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is
(1) {1, 2, 3}
(2) {1, 2, 3, 4, 5}
(3) {1, 2, 3, 4}
(4) {1, 2, 3, 4, 5, 6}

3. Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg zw = π. Then arg z equals
(1) $\frac{\pi}{4}$
(2) $\frac{5\pi}{4}$
(3) $\frac{3\pi}{4}$
(4) $\frac{\pi}{2}$

4. If $z = x - i\,y$ and $z^{\frac{1}{3}} = p + iq$, then $\dfrac{\left(\frac{x}{p} + \frac{y}{q}\right)}{\left(p^2 + q^2\right)}$ is equal to
(1) 1
(2) -2
(3) 2
(4) -1

5. If $\left|z^2 - 1\right| = |z|^2 + 1$, then z lies on
(1) the real axis
(2) an ellipse
(3) a circle
(4) the imaginary axis.

6. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is
(1) A is a zero matrix
(2) $A^2 = I$
(3) A^{-1} does not exist
(4) $A = (-1)I$, where I is a unit matrix

7. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} (10) B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverse of matrix A, then α is

(1) -2 (2) 5
(3) 2 (4) -1

8. If $a_1, a_2, a_3, ..., a_n, ...$ are in G.P., then the value of the determinant

$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is

(1) 0 (2) -2
(3) 2 (4) 1

9. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation

(1) $x^2 + 18x + 16 = 0$ (2) $x^2 - 18x - 16 = 0$
(3) $x^2 + 18x - 16 = 0$ (4) $x^2 - 18x + 16 = 0$

10. If (1 – p) is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are

(1) 0, 1 (2) -1, 2
(3) 0, -1 (4) -1, 1

11. Let $S(K) = 1 + 3 + 5 + ... + (2K - 1) = 3 + K^2$. Then which of the following is true?

(1) S(1) is correct
(2) Principle of mathematical induction can be used to prove the formula
(3) $S(K) \not\Rightarrow S(K+1)$
(4) $S(K) \Rightarrow S(K+1)$

12. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order?

(1) 120 (2) 480
(3) 360 (4) 240

13. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is

(1) 5 (2) ${}^8C_3$
(3) 3^8 (4) 21

14. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is

(1) $\frac{49}{4}$ (2) 4
(3) 3 (4) 12

15. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals

(1) $-\frac{5}{3}$ (2) $\frac{3}{5}$

(3) $\frac{-3}{10}$ (4) $\frac{10}{3}$

16. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is

(1) (n – 1) (2) $(-1)^n(1-n)$

(3) $(-1)^{n-1}(n-1)^2$ (4) $(-1)^{n-1}n$

17. If $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to

(1) $\frac{1}{2}n$ (2) $\frac{1}{2}n-1$

(3) n – 1 (4) $\frac{2n-1}{2}$

18. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals

(1) 0 (2) 1

(3) $\frac{1}{mn}$ (4) $\frac{1}{m}+\frac{1}{n}$

19. The sum of the first n terms of the series $1^2 + 2\cdot 2^2 + 3^2 + 2\cdot 4^2 + 5^2 + 2\cdot 6^2 + \ldots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

(1) $\frac{3n(n+1)}{2}$ (2) $\frac{n^2(n+1)}{2}$

(3) $\frac{n(n+1)^2}{4}$ (4) $\left[\frac{n(n+1)}{2}\right]^2$

20. The sum of series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots$ is

(1) $\frac{(e^2-1)}{2}$ (2) $\frac{(e-1)^2}{2e}$

(3) $\frac{(e^2-1)}{2e}$ (4) $\frac{(e^2-2)}{e}$

21. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ is

(1) $-\frac{3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$
(3) $\frac{6}{65}$ (4) $-\frac{6}{65}$

22. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by

(1) $2(a^2+b^2)$ (2) $2\sqrt{a^2+b^2}$
(3) $(a+b)^2$ (4) $(a-b)^2$

23. The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is

(1) 60° (2) 90°
(3) 120° (4) 150°

24. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meter away from the tree the angle of elevation becomes 30°. The breadth of the river is

(1) 20 m (2) 30 m
(3) 40 m (4) 60 m

25. If $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is

(1) [0, 3] (2) [-1, 1]
(3) [0, 1] (4) [-1, 3]

26. The graph of the function y = f(x) is symmetrical about the line x = 2, then

(1) f(x + 2)= f(x – 2) (2) f(2 + x) = f(2 – x)
(3) f(x) = f(-x) (4) f(x) = - f(-x)

27. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(1) [2, 3] (2) [2, 3)
(3) [1, 2] (4) [1, 2)

28. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b, are

(1) $a \in \underline{\underline{R}}, b \in \underline{\underline{R}}$ (2) $a = 1,\ b \in \underline{\underline{R}}$
(3) $a \in \underline{\underline{R}}, b = 2$ (4) a = 1 and b = 2

29. Let $f(x)=\frac{1-\tan x}{4x-\pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$. If f(x) is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

(1) 1 (2) $\frac{1}{2}$

(3) $-\frac{1}{2}$ (4) -1

30. If $x = e^{y+e^{y+\ldots \text{to } \infty}}$, x > 0, then $\frac{dy}{dx}$ is

(1) $\frac{x}{1+x}$ (2) $\frac{1}{x}$

(3) $\frac{1-x}{x}$ (4) $\frac{1+x}{x}$

31. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(1) (2, 4) (2) (2, -4)

(3) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (4) $\left(\frac{9}{8}, \frac{9}{2}\right)$

32. A function y = f(x) has a second order derivative f″(x) = 6(x – 1). If its graph passes through the point (2, 1) and at that point the tangent to the graph is y = 3x – 5, then the function is

(1) $(x-1)^2$ (2) $(x-1)^3$

(3) $(x+1)^3$ (4) $(x+1)^2$

33. The normal to the curve x = a(1 + cosθ), y = asinθ at 'θ' always passes through the fixed point

(1) (a, 0) (2) (0, a)

(3) (0, 0) (4) (a, a)

34. If 2a + 3b + 6c =0, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

(1) (0, 1) (2) (1, 2)

(3) (2, 3) (4) (1, 3)

35. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is

(1) e (2) e – 1

(3) 1 – e (4) e + 1

36. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin(x-\alpha) + C$, then value of (A, B) is

(1) (sinα, cosα) (2) (cosα, sinα)

(3) (- sinα, cosα) (4) (- cosα, sinα)

37. $\int \frac{dx}{\cos x - \sin x}$ is equal to

(1) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

(2) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(3) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(4) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

38. The value of $\int_{-2}^{3}|1-x^2|dx$ is

(1) $\frac{28}{3}$

(2) $\frac{14}{3}$

(3) $\frac{7}{3}$

(4) $\frac{1}{3}$

39. The value of I = $\int_{0}^{\pi/2}\frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$ is

(1) 0

(2) 1

(3) 2

(4) 3

40. If $\int_{0}^{\pi}xf(\sin x)\,dx = A\int_{0}^{\pi/2}f(\sin x)\,dx$, then A is

(1) 0

(2) π

(3) $\frac{\pi}{4}$

(4) 2π

41. If f(x) = $\frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)}xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)}g\{x(1-x)\}dx$ then the value of $\frac{I_2}{I_1}$ is

(1) 2

(2) –3

(3) –1

(4) 1

42. The area of the region bounded by the curves y = |x – 2|, x = 1, x = 3 and the x-axis is

(1) 1

(2) 2

(3) 3

(4) 4

43. The differential equation for the family of curves $x^2+y^2-2ay=0$, where a is an arbitrary constant is

(1) $2(x^2-y^2)y'=xy$

(2) $2(x^2+y^2)y'=xy$

(3) $(x^2-y^2)y'=2xy$

(4) $(x^2+y^2)y'=2xy$

44. The solution of the differential equation y dx + (x + x^{2}y) dy = 0 is

(1) $-\frac{1}{xy}=C$

(2) $-\frac{1}{xy}+\log y=C$

(3) $\frac{1}{xy}+\log y=C$

(4) log y = Cx

45. Let A (2, –3) and B(–2, 1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line 2x + 3y = 1, then the locus of the vertex C is the line

(1) 2x + 3y = 9 (2) 2x – 3y = 7

(3) 3x + 2y = 5 (4) 3x – 2y = 3

46. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is

(1) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$ (2) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

(3) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{2}+\frac{y}{1}=1$ (4) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

47. If the sum of the slopes of the lines given by $x^2-2cxy-7y^2=0$ is four times their product, then c has the value

(1) 1 (2) –1

(3) 2 (4) –2

48. If one of the lines given by $6x^2-xy+4cy^2=0$ is 3x + 4y = 0, then c equals

(1) 1 (2) –1

(3) 3 (4) –3

49. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is

(1) $2ax+2by+(a^2+b^2+4)=0$ (2) $2ax+2by-(a^2+b^2+4)=0$

(3) $2ax-2by+(a^2+b^2+4)=0$ (4) $2ax-2by-(a^2+b^2+4)=0$

50. A variable circle passes through the fixed point A (p, q) and touches x-axis. The locus of the other end of the diameter through A is

(1) $(x-p)^2=4qy$ (2) $(x-q)^2=4py$

(3) $(y-p)^2=4qx$ (4) $(y-q)^2=4px$

51. If the lines 2x + 3y + 1 = 0 and 3x – y – 4 = 0 lie along diameters of a circle of circumference 10π, then the equation of the circle is

(1) $x^2+y^2-2x+2y-23=0$ (2) $x^2+y^2-2x-2y-23=0$

(3) $x^2+y^2+2x+2y-23=0$ (4) $x^2+y^2+2x-2y-23=0$

52. The intercept on the line y = x by the circle $x^2+y^2-2x=0$ is AB. Equation of the circle on AB as a diameter is

(1) $x^2+y^2-x-y=0$ (2) $x^2+y^2-x+y=0$

(3) $x^2+y^2+x+y=0$ (4) $x^2+y^2+x-y=0$

53. If a ≠ 0 and the line 2bx + 3cy + 4d = 0 passes through the points of intersection of the parabolas $y^2=4ax$ and $x^2=4ay$, then

(1) $d^2+(2b+3c)^2=0$ (2) $d^2+(3b+2c)^2=0$

(3) $d^2+(2b-3c)^2=0$ (4) $d^2+(3b-2c)^2=0$

54. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is x = 4, then the equation of the ellipse is

(1) $3x^2 + 4y^2 = 1$
(2) $3x^2 + 4y^2 = 12$
(3) $4x^2 + 3y^2 = 12$
(4) $4x^2 + 3y^2 = 1$

55. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals

(1) $\frac{2}{3}$
(2) $\frac{1}{5}$
(3) $\frac{3}{5}$
(4) $\frac{2}{5}$

56. Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y + 4z + 5 = 0 is

(1) $\frac{3}{2}$
(2) $\frac{5}{2}$
(3) $\frac{7}{2}$
(4) $\frac{9}{2}$

57. A line with direction cosines proportional to 2, 1, 2 meets each of the lines x = y + a = z and x + a = 2y = 2z. The co-ordinates of each of the point of intersection are given by

(1) (3a, 3a, 3a), (a, a, a)
(2) (3a, 2a, 3a), (a, a, a)
(3) (3a, 2a, 3a), (a, a, 2a)
(4) (2a, 3a, 3a), (2a, a, a)

58. If the straight lines x = 1 + s, y = –3 – λs, z = 1 + λs and x = $\frac{t}{2}$, y = 1 + t, z = 2 – t with parameters s and t respectively, are co-planar then λ equals

(1) –2
(2) –1
(3) $-\frac{1}{2}$
(4) 0

59. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane

(1) x – y – z = 1
(2) x – 2y – z = 1
(3) x – y – 2z = 1
(4) 2x – y – z = 1

60. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a} + 2\vec{b} + 6\vec{c}$ equals

(1) $\lambda\vec{a}$
(2) $\lambda\vec{b}$
(3) $\lambda\vec{c}$
(4) 0

61. A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ which displace it from a point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The work done in standard units by the forces is given by

(1) 40 (2) 30
(3) 25 (4) 15

62. If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\bar{a}+2\bar{b}+3\bar{c},\ \lambda\bar{b}+4\bar{c}$ and $(2\lambda-1)\bar{c}$ are non-coplanar for

(1) all values of λ
(2) all except one value of λ
(3) all except two values of λ
(4) no value of λ

63. Let $\bar{u}, \bar{v}, \bar{w}$ be such that $|\bar{u}|=1,\ |\bar{v}|=2,\ |\bar{w}|=3$. If the projection $\bar{v}$ along $\bar{u}$ is equal to that of $\bar{w}$ along $\bar{u}$ and $\bar{v}, \bar{w}$ are perpendicular to each other then $|\bar{u}-\bar{v}+\bar{w}|$ equals

(1) 2
(2) $\sqrt{7}$
(3) $\sqrt{14}$
(4) 14

64. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be non-zero vectors such that $(\bar{a}\times\bar{b})\times\bar{c}=\frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$. If θ is the acute angle between the vectors $\bar{b}$ and $\bar{c}$, then $\sin\theta$ equals

(1) $\frac{1}{3}$
(2) $\frac{\sqrt{2}}{3}$
(3) $\frac{2}{3}$
(4) $\frac{2\sqrt{2}}{3}$

65. Consider the following statements:
(a) Mode can be computed from histogram
(b) Median is not independent of change of scale
(c) Variance is independent of change of origin and scale.
Which of these is/are correct?

(1) only (a)
(2) only (b)
(3) only (a) and (b)
(4) (a), (b) and (c)

66. In a series of 2n observations, half of them equal a and remaining half equal –a. If the standard deviation of the observations is 2, then |a| equals

(1) $\frac{1}{n}$
(2) $\sqrt{2}$
(3) 2
(4) $\frac{\sqrt{2}}{n}$

67. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is

(1) $\frac{3}{20}$
(2) $\frac{1}{5}$
(3) $\frac{7}{20}$
(4) $\frac{4}{5}$

68. A random variable X has the probability distribution:

X:	1	2	3	4	5	6	7	8
p(X):	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and F = {X < 4}, the probability P (E $\cup$ F) is

(1) 0.87 (2) 0.77
(3) 0.35 (4) 0.50

69. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

(1) $\frac{37}{256}$ (2) $\frac{219}{256}$
(3) $\frac{128}{256}$ (4) $\frac{28}{256}$

70. With two forces acting at a point, the maximum effect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are

(1) $(2+\sqrt{2})$N and $(2-\sqrt{2})$N (2) $(2+\sqrt{3})$N and $(2-\sqrt{3})$N
(3) $\left(2+\frac{1}{2}\sqrt{2}\right)$N and $\left(2-\frac{1}{2}\sqrt{2}\right)$N (4) $\left(2+\frac{1}{2}\sqrt{3}\right)$N and $\left(2-\frac{1}{2}\sqrt{3}\right)$N

71. In a right angle ΔABC, $\angle$A = 90° and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force $\vec{F}$ has moments 0, 9 and 16 in N cm. units respectively about vertices A, B and C, then magnitude of $\vec{F}$ is

(1) 3 (2) 4
(3) 5 (4) 9

72. Three forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC, are in equilibrium. Then $\vec{P}:\vec{Q}:\vec{R}$ is

(1) $\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$ (2) $\sin\frac{A}{2}:\sin\frac{B}{2}:\sin\frac{C}{2}$
(3) $\sec\frac{A}{2}:\sec\frac{B}{2}:\sec\frac{C}{2}$ (4) $\operatorname{cosec}\frac{A}{2}:\operatorname{cosec}\frac{B}{2}:\operatorname{cosec}\frac{C}{2}$

73. A particle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5 km/h. If AB = 12 km and BC = 5 km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively

(1) $\frac{17}{4}$ km/h and $\frac{13}{4}$ km/h (2) $\frac{13}{4}$ km/h and $\frac{17}{4}$ km/h
(3) $\frac{17}{9}$ km/h and $\frac{13}{9}$ km/h (4) $\frac{13}{9}$ km/h and $\frac{17}{9}$ km/h

74. A velocity $\frac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is

(1) $\frac{1}{8}$ m/s (2) $\frac{1}{4}(\sqrt{3}-1)$ m/s
(3) $\frac{1}{4}$ m/s (4) $\frac{1}{8}(\sqrt{6}-\sqrt{2})$ m/s

75. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal, then $t_1^2 + t_2^2$ is equal to

(1) $\frac{u^2}{g}$ (2) $\frac{4u^2}{g^2}$

(3) $\frac{u^2}{2g}$ (4) 1

FIITJEE AIEEE – 2004 (MATHEMATICS)

ANSWERS

1.	**3**	16.	**2**	31.	**4**	46.	**4**	61.	**1**
2.	**1**	17.	**1**	32.	**2**	47.	**3**	62.	**3**
3.	**3**	18.	**1**	33.	**1**	48.	**4**	63.	**3**
4.	**2**	19.	**2**	34.	**1**	49.	**2**	64.	**4**
5.	**4**	20.	**2**	35.	**2**	50.	**1**	65.	**3**
6.	**2**	21.	**1**	36.	**2**	51.	**1**	66.	**3**
7.	**2**	22.	**4**	37.	**4**	52.	**1**	67.	**3**
8.	**1**	23.	**3**	38.	**1**	53.	**1**	68.	**2**
9.	**4**	24.	**1**	39.	**3**	54.	**2**	69.	**4**
10.	**3**	25.	**4**	40.	**2**	55.	**3**	70.	**3**
11.	**4**	26.	**2**	41.	**1**	56.	**3**	71.	**3**
12.	**3**	27.	**2**	42.	**1**	57.	**2**	72.	**1**
13.	**4**	28.	**2**	43.	**3**	58.	**1**	73.	**1**
14.	**1**	29.	**3**	44.	**2**	59.	**4**	74.	**4**
15.	**3**	30.	**3**	45.	**1**	60.	**4**	75.	**2**

AIEEE – 2004 (MATHEMATICS)

SOLUTIONS

1. $(2, 3) \in R$ but $(3, 2) \notin R$.
   Hence R is not symmetric.

2. $f(x) = {}^{7-x}P_{x-3}$
   $7 - x \geq 0 \quad \Rightarrow \quad x \leq 7$
   $x - 3 \geq 0 \quad \Rightarrow \quad x \geq 3$,
   and $7 - x \geq x - 3 \quad \Rightarrow \quad x \leq 5$
   $\Rightarrow 3 \leq x \leq 5 \Rightarrow$ x = 3, 4, 5 $\Rightarrow$ Range is {1, 2, 3}.

3. Here $\omega = \frac{z}{i} \Rightarrow \arg\left(z.\frac{z}{i}\right) = \pi \Rightarrow 2\arg(z) - \arg(i) = \pi \Rightarrow \arg(z) = \frac{3\pi}{4}$.

4. $z = (p + iq)^3 = p\left(p^2 - 3q^2\right) - iq\left(q^2 - 3p^2\right)$

   $\Rightarrow \quad \frac{x}{p} = p^2 - 3q^2 \;\; \& \;\; \frac{y}{q} = q^2 - 3p^2 \Rightarrow \frac{\frac{x}{p} + \frac{y}{q}}{\left(p^2 + q^2\right)} = -2$.

5. $\left|z^2 - 1\right|^2 = \left(|z|^2 + 1\right)^2 \Rightarrow \left(z^2 - 1\right)\left(\bar{z}^2 - 1\right) = |z|^4 + 2|z|^2 + 1$

   $\Rightarrow z^2 + \bar{z}^2 + 2z\bar{z} = 0 \Rightarrow z + \bar{z} = 0$
   $\Rightarrow$ R (z) = 0 $\Rightarrow$ z lies on the imaginary axis.

6. $A.A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$.

7. AB = I $\Rightarrow$ A(10 B) = 10 I

   $\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha - 5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ if $\alpha = 5$.

8. $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$

   $C_3 \to C_3 - C_2$, $C_2 \to C_3 - C_1$

   $= \begin{vmatrix} \log a_n & \log r & \log r \\ \log a_{n+3} & \log r & \log r \\ \log a_{n+6} & \log r & \log r \end{vmatrix} = 0$ (where r is a common ratio).

9. Let numbers be a, b $\Rightarrow a + b = 18, \; \sqrt{ab} = 4 \quad \Rightarrow \quad ab = 16$, a and b are roots of the equation
   $\Rightarrow \quad x^2 - 18x + 16 = 0$.

10. **(3)**

$(1-p)^2 + p(1-p) + (1-p) = 0$ (since (1 – p) is a root of the equation $x^2 + px + (1 - p) = 0$)

$\Rightarrow (1-p)(1-p+p+1) = 0$

$\Rightarrow 2(1-p) = 0 \Rightarrow (1 - p) = 0 \Rightarrow p = 1$

sum of root is $\alpha + \beta = -p$ and product $\alpha\beta = 1 - p = 0$ (where $\beta = 1 - p = 0$)

$\Rightarrow \alpha + 0 = -1 \Rightarrow \alpha = -1 \Rightarrow$ Roots are 0, –1

11. $S(k) = 1 + 3 + 5 + ... + (2k-1) = 3 + k^2$

S(k + 1)=1 + 3 + 5 +............ + (2k – 1) + (2k + 1)

$= (3 + k^2) + 2k + 1 = k^2 + 2k + 4$ [from $S(k) = 3 + k^2$]

$= 3 + (k^2 + 2k + 1) = 3 + (k + 1)^2 = S(k + 1)$.

Although S (k) in itself is not true but it considered true will always imply towards S (k + 1).

12. Since in half the arrangement A will be before E and other half E will be before A.

Hence total number of ways = $\frac{6!}{2}$ = 360.

13. Number of balls = 8

number of boxes = 3

Hence number of ways = ${}^7C_2$ = 21.

14. Since 4 is one of the root of $x^2 + px + 12 = 0 \Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$

and equation $x^2 + px + q = 0$ has equal roots

$\Rightarrow D = 49 - 4q = 0 \Rightarrow q = \frac{49}{4}$.

15. Coefficient of Middle term in $(1 + \alpha x)^4 = t_3 = {}^4C_2 \cdot \alpha^2$

Coefficient of Middle term in $(1 - \alpha x)^6 = t_4 = {}^6C_3 (-\alpha)^3$

${}^4C_2\alpha^2 = -{}^6C_3.\alpha^3 \Rightarrow -6 = 20\alpha \Rightarrow \alpha = \frac{-3}{10}$

16. Coefficient of x^n in $(1 + x)(1 - x)^n = (1 + x)({}^nC_0 - {}^nC_1x + …….. + (-1)^{n-1}\, {}^nC_{n-1}\, x^{n-1} + (-1)^n\, {}^nC_n\, x^n)$

$= (-1)^n\, {}^nC_n + (-1)^{n-1}\, {}^nC_{n-1} = (-1)^n (1-n)$.

17. $t = \sum_{r=0}^{n} \frac{r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} = \sum_{r=0}^{n} \frac{n-r}{{}^nC_r} \quad (\because {}^nC_r = {}^nC_{n-r})$

$2t_n = \sum_{r=0}^{n} \frac{r+n-r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n}{{}^nC_r} \Rightarrow t_n = \frac{n}{2}\sum_{r=0}^{n} \frac{1}{{}^nC_r} = \frac{n}{2}S_n \Rightarrow \frac{t_n}{S_n} = \frac{n}{2}$

18. $T_m = \frac{1}{n} = a + (m-1)d$ .....(1)

and $T_n = \frac{1}{m} = a + (n-1)d$ .....(2)

from (1) and (2) we get $a = \frac{1}{mn}$, $d = \frac{1}{mn}$

Hence a – d = 0

19. If n is odd then (n – 1) is even $\Rightarrow$ sum of odd terms $= \frac{(n-1)n^2}{2} + n^2 = \frac{n^2(n+1)}{2}$.

20. $$\frac{e^{\alpha}+e^{-\alpha}}{2} = 1 + \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \frac{\alpha^6}{6!} + \ldots\ldots..$$

$$\frac{e^{\alpha}+e^{-\alpha}}{2} - 1 = \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \frac{\alpha^6}{6!} + \ldots\ldots.$$

put $\alpha = 1$, we get

$$\frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\ldots\ldots.$$

21. $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$.

Squaring and adding, we get

$2 + 2\cos(\alpha - \beta) = \frac{1170}{(65)^2}$

$$\Rightarrow \cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{9}{130} \Rightarrow \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}} \left(\because \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2}\right).$$

22. $$u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$$

$$= \sqrt{\frac{a^2+b^2}{2} + \frac{a^2-b^2}{2}\cos 2\theta} + \sqrt{\frac{a^2+b^2}{2} + \frac{b^2-a^2}{2}\cos 2\theta}$$

$$\Rightarrow u^2 = a^2 + b^2 + 2\sqrt{\left(\frac{a^2+b^2}{2}\right)^2 - \left(\frac{a^2-b^2}{2}\right)^2\cos^2 2\theta}$$

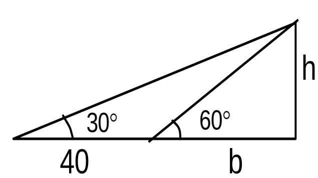
min value of $u^2 = a^2 + b^2 + 2ab$

max value of $u^2 = 2(a^2 + b^2)$

$\Rightarrow u^2_{max} - u^2_{min} = (a-b)^2$.

23. Greatest side is $\sqrt{1+\sin\alpha\cos\alpha}$, by applying cos rule we get greatest angle = 120^{o}.

24. $\tan 30^\circ = \frac{h}{40+b}$



$\Rightarrow \sqrt{3}h = 40 + b$ …..(1)

$\tan 60^\circ = h/b \Rightarrow h = \sqrt{3}\, b$ ….(2)

$\Rightarrow$ b = 20 m

25. $-2 \le \sin x - \sqrt{3}\cos x \le 2 \Rightarrow -1 \le \sin x - \sqrt{3}\cos x + 1 \le 3$

$\Rightarrow$ range of f(x) is [–1, 3].

Hence S is [–1, 3].

26. If y = f (x) is symmetric about the line x = 2 then f(2 + x) = f(2 – x).

27. $9 - x^2 > 0$ and $-1 \le x - 3 \le 1 \Rightarrow x \in [2, 3)$

28. $$\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = \lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{\left(\frac{1}{\frac{a}{x}+\frac{b}{x^2}}\right)\times 2x\times\left(\frac{a}{x}+\frac{b}{x^2}\right)} = e^{2a} \Rightarrow a = 1,\ b \in R$$

29. $$f(x) = \frac{1-\tan x}{4x-\pi} \Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{4x-\pi} = -\frac{1}{2}$$

30. $x = e^{y+e^{y+e^{y+\ldots\infty}}} \Rightarrow x = e^{y+x}$

$\Rightarrow \ln x - x = y \Rightarrow \frac{dy}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x}$.

31. Any point be $\left(\frac{9}{2}t^2,\ 9t\right)$; differentiating $y^2 = 18x$

$\Rightarrow \frac{dy}{dx} = \frac{9}{y} = \frac{1}{t} = 2$ (given) $\Rightarrow t = \frac{1}{2}$.

$\Rightarrow$ Point is $\left(\frac{9}{8},\ \frac{9}{2}\right)$

32. $f''(x) = 6(x - 1) \Rightarrow f'(x) = 3(x - 1)^2 + c$
and $f'(2) = 3 \Rightarrow c = 0$
$\Rightarrow f(x) = (x - 1)^3 + k$ and $f(2) = 1 \Rightarrow k = 0$
$\Rightarrow f(x) = (x - 1)^3$.

33. Eliminating θ, we get $(x - a)^2 + y^2 = a^2$.
Hence normal always pass through (a, 0).

34. Let $f'(x) = ax^2 + bx + c \Rightarrow f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$

$\Rightarrow f(x) = \frac{1}{6}\left(2ax^3 + 3bx^2 + 6cx + 6d\right)$, Now f(1) = f(0) = d, then according to Rolle's theorem

$\Rightarrow f'(x) = ax^2 + bx + c = 0$ has at least one root in (0, 1)

35. $$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}} = \int_0^1 e^x dx = (e-1)$$

36. Put $x - \alpha = t$

$\Rightarrow \int \frac{\sin(\alpha+t)}{\sin t}dt = \sin\alpha\int \cot t\,dt + \cos\alpha\int dt$

$= \cos\alpha(x-\alpha) + \sin\alpha\ln|\sin t| + c$

$A = \cos\alpha,\ B = \sin\alpha$

37. $\int \frac{dx}{\cos x - \sin x} = \frac{1}{\sqrt{2}}\int \frac{1}{\cos\left(x+\frac{\pi}{4}\right)}dx = \frac{1}{\sqrt{2}}\int \sec\left(x+\frac{\pi}{4}\right)dx = \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right| + C$

38. $\int_{-2}^{-1}\left(x^2-1\right)dx + \int_{-1}^{1}\left(1-x^2\right)dx + \int_{1}^{3}\left(x^2-1\right)dx = \left.\frac{x^3}{3}-x\right|_{-2}^{-1} + \left.x-\frac{x^3}{3}\right|_{-1}^{1} + \left.\frac{x^3}{3}-x\right|_{1}^{3} = \frac{28}{3}$.

39. $\int_0^{\frac{\pi}{2}} \frac{(\sin x+\cos x)^2}{\sqrt{(\sin x+\cos x)^2}}dx = \int_0^{\frac{\pi}{2}}(\sin x+\cos x)\,dx = \left|-\cos x+\sin x\right|_0^{\frac{\pi}{2}} = 2.$

40. Let I = $\int_0^{\pi} xf(\sin x)dx = \int_0^{\pi}(\pi-x)f(\sin x)dx = \pi\int_0^{\pi} f(\sin x)dx - I$ (since f (2a – x) = f (x))

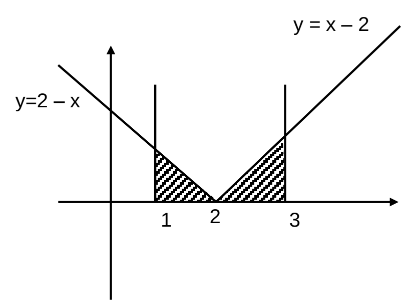
$\Rightarrow$ I = $\pi\int_0^{\pi/2} f(\sin x)dx \Rightarrow A = \pi$.

41. f(-a) + f(a) = 1

$I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx = \int_{f(-a)}^{f(a)} (1-x)g\{x(1-x)\}dx$ $\left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx\right)$

$2I_1 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx = I_2 \Rightarrow I_2 / I_1 = 2.$

42. Area = $\int_1^2 (2-x)dx + \int_2^3 (x-2)dx = 1.$



43. 2x + 2yy′ - 2ay′ = 0

a = $\frac{x+yy'}{y'}$ (eliminating a)

$\Rightarrow (x^2 - y^2)y' = 2xy.$

45. $y\,dx + x\,dy + x^2y\,dy = 0.$

$\frac{d(xy)}{x^2y^2} + \frac{1}{y}dy = 0 \Rightarrow -\frac{1}{xy} + \log y = C.$

45. If C be (h, k) then centroid is (h/3, (k – 2)/3) it lies on 2x + 3y = 1.

$\Rightarrow$ locus is 2x + 3y = 9.

46. $\frac{x}{a}+\frac{y}{b}=1$ where a + b = -1 and $\frac{4}{a}+\frac{3}{b}=1$

$\Rightarrow$ a = 2, b = -3 or a = -2, b = 1.

Hence $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$.

47. $m_1 + m_2 = -\frac{2c}{7}$ and $m_1 m_2 = -\frac{1}{7}$

$m_1 + m_2 = 4m_1m_2$ (given)

$\Rightarrow$ c = 2.

48. $m_1 + m_2 = \frac{1}{4c}$, $m_1m_2 = \frac{6}{4c}$ and $m_1 = -\frac{3}{4}$.

Hence c = -3.

49. Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0 \Rightarrow c = 4$ and it passes through (a, b)

$\Rightarrow a^2 + b^2 + 2ga + 2fb + 4 = 0$.

Hence locus of the centre is $2ax + 2by - (a^2 + b^2 + 4) = 0$.

50. Let the other end of diameter is (h, k) then equation of circle is

$(x - h)(x - p) + (y - k)(y - q) = 0$

Put y = 0, since x-axis touches the circle

$\Rightarrow x^2 - (h + p)x + (hp + kq) = 0 \Rightarrow (h + p)^2 = 4(hp + kq)$ (D = 0)

$\Rightarrow (x - p)^2 = 4qy$.

51. Intersection of given lines is the centre of the circle i.e. (1, $-$1)

Circumference = $10\pi \Rightarrow$ radius r = 5

$\Rightarrow$ equation of circle is $x^2 + y^2 - 2x + 2y - 23 = 0$.

52. Points of intersection of line y = x with $x^2 + y^2 - 2x = 0$ are (0, 0) and (1, 1)

hence equation of circle having end points of diameter (0, 0) and (1, 1) is

$x^2 + y^2 - x - y = 0$.

53. Points of intersection of given parabolas are (0, 0) and (4a, 4a)

$\Rightarrow$ equation of line passing through these points is y = x

On comparing this line with the given line 2bx + 3cy + 4d = 0, we get

d = 0 and $2b + 3c = 0 \Rightarrow (2b + 3c)^2 + d^2 = 0$.

54. Equation of directrix is $x = a/e = 4 \Rightarrow a = 2$

$b^2 = a^2 (1 - e^2) \Rightarrow b^2 = 3$

Hence equation of ellipse is $3x^2 + 4y^2 = 12$.

55. $l = \cos\theta$, $m = \cos\theta$, $n = \cos\beta$

$\cos^2\theta + \cos^2\theta + \cos^2\beta = 1 \Rightarrow 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$ (given)

$\cos^2\theta = 3/5$.

56. Given planes are

$2x + y + 2z - 8 = 0$, $4x + 2y + 4z + 5 = 0 \Rightarrow 2x + y + 2z + 5/2 = 0$

Distance between planes = $\frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}} = \frac{|-8-5/2|}{\sqrt{2^2+1^2+2^2}} = \frac{7}{2}$.

57. Any point on the line $\frac{x}{1}=\frac{y+a}{1}=\frac{z}{1}=t_1$ (say) is $(t_1, t_1 - a, t_1)$ and any point on the line $\frac{x+a}{2}=\frac{y}{1}=\frac{z}{1}=t_2$ (say) is $(2t_2 - a, t_2, t_2)$.
Now direction cosine of the lines intersecting the above lines is proportional to $(2t_2 - a - t_1, t_2 - t_1 + a, t_2 - t_1)$.
Hence $2t_2 - a - t_1 = 2k$, $t_2 - t_1 + a = k$ and $t_2 - t_1 = 2k$
On solving these, we get $t_1 = 3a$, $t_2 = a$.
Hence points are (3a, 2a, 3a) and (a, a, a).

58. Given lines $\frac{x-1}{1}=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s$ and $\frac{x}{1/2}=\frac{y-1}{1}=\frac{z-2}{-1}=t$ are coplanar then plan passing through these lines has normal perpendicular to these lines
$\Rightarrow a - b\lambda + c\lambda = 0$ and $\frac{a}{2}+b-c=0$ (where a, b, c are direction ratios of the normal to the plan)
On solving, we get $\lambda = -2$.

59. Required plane is $S_1 - S_2 = 0$
where $S_1 = x^2 + y^2 + z^2 + 7x - 2y - z - 13 = 0$ and
$S_2 = x^2 + y^2 + z^2 - 3x + 3y + 4z - 8 = 0$
$\Rightarrow 2x - y - z = 1$.

60. $\left(\vec{a}+2\vec{b}\right)=t_1\vec{c}$ ….(1)
and $\vec{b}+3\vec{c}=t_2\vec{a}$ ….(2)
(1) – 2×(2) $\Rightarrow$ $\vec{a}(1+2t_2)+\vec{c}(-t_1-6)=0 \Rightarrow 1+2t_2=0 \Rightarrow t_2=-1/2$ & $t_1=-6$.
Since $\vec{a}$ and $\vec{c}$ are non-collinear.
Putting the value of t_1 and t_2 in (1) and (2), we get $\vec{a}+2\vec{b}+6\vec{c}=\vec{0}$.

61. Work done by the forces $\vec{F}_1$ and $\vec{F}_2$ is $(\vec{F}_1+\vec{F}_2)\cdot\vec{d}$, where $\vec{d}$ is displacement
According to question $\vec{F}_1+\vec{F}_2 = (4\hat{i}+\hat{j}-3\hat{k})+(3\hat{i}+\hat{j}-\hat{k})=7\hat{i}+2\hat{j}-4\hat{k}$
and $\vec{d}=(5\hat{i}+4\hat{j}+\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=4\hat{i}+2\hat{j}-2\hat{k}$. Hence $(\vec{F}_1+\vec{F}_2)\cdot\vec{d}$ is 40.

63. Condition for given three vectors to be coplanar is $\begin{vmatrix}1 & 2 & 3\\ 0 & \lambda & 4\\ 0 & 0 & 2\lambda-1\end{vmatrix}=0 \Rightarrow \lambda = 0, 1/2$.
Hence given vectors will be non coplanar for all real values of λ except 0, 1/2.

63. Projection of $\bar{v}$ along $\bar{u}$ and $\bar{w}$ along $\bar{u}$ is $\frac{\bar{v}\cdot\bar{u}}{|\bar{u}|}$ and $\frac{\bar{w}\cdot\bar{u}}{|\bar{u}|}$ respectively
According to question $\frac{\bar{v}\cdot\bar{u}}{|\bar{u}|}=\frac{\bar{w}\cdot\bar{u}}{|\bar{u}|} \Rightarrow \bar{v}\cdot\bar{u}=\bar{w}\cdot\bar{u}$. and $\bar{v}\cdot\bar{w}=0$
$|\bar{u}-\bar{v}+\bar{w}|^2=|\bar{u}|^2+|\bar{v}|^2+|\bar{w}|^2-2\bar{u}\cdot\bar{v}+2\bar{u}\cdot\bar{w}-2\bar{v}\cdot\bar{w}=14 \Rightarrow |\bar{u}-\bar{v}+\bar{w}|=\sqrt{14}$.

64. $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a} \Rightarrow \left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{b}\cdot\vec{c}\right)\vec{a}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \left(\vec{a}\cdot\vec{c}\right)\vec{b}=\left(\frac{1}{3}|\vec{b}||\vec{c}|+\left(\vec{b}\cdot\vec{c}\right)\right)\vec{a} \Rightarrow \vec{a}\cdot\vec{c}=0$ and $\frac{1}{3}|\vec{b}||\vec{c}|+\left(\vec{b}\cdot\vec{c}\right)=0$

$\Rightarrow |\vec{b}||\vec{c}|\left(\frac{1}{3}+\cos\theta\right)=0 \Rightarrow \cos\theta = -1/3 \Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$.

65. Mode can be computed from histogram and median is dependent on the scale.
Hence statement (a) and (b) are correct.

66. $x_i = a$ for $i = 1, 2, \ldots, n$ and $x_i = -a$ for $i = n, \ldots, 2n$

S.D. = $\sqrt{\frac{1}{2n}\sum_{i=1}^{2n}\left(x_i-\bar{x}\right)^2} \Rightarrow 2 = \sqrt{\frac{1}{2n}\sum_{i=1}^{2n}x_i^2} \quad \left(\text{Since } \sum_{i=1}^{2n}x_i=0\right) \Rightarrow 2=\sqrt{\frac{1}{2n}\cdot 2na^2} \Rightarrow |a| = 2$

67. E_1 : event denoting that A speaks truth
E_2 : event denoting that B speaks truth

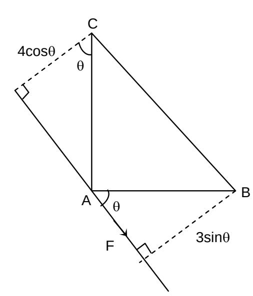
Probability that both contradicts each other = $P\left(E_1\cap\bar{E}_2\right)+P\left(\bar{E}_1\cap E_2\right) = \frac{4}{5}\cdot\frac{1}{4}+\frac{1}{5}\cdot\frac{3}{4}=\frac{7}{20}$

68. $P(E\cup F)=P(E)+P(F)-P\left(E\cap F\right) = 0.62 + 0.50 - 0.35 = 0.77$

69. Given that n p = 4, n p q = 2 $\Rightarrow$ q = 1/2 $\Rightarrow$ p = 1/2 , n = 8 $\Rightarrow$ p(x = 2) = ${}^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6=\frac{28}{256}$

70. P + Q = 4, $P^2 + Q^2 = 9 \Rightarrow P = \left(2+\frac{1}{2}\sqrt{2}\right)N$ and $Q=\left(2-\frac{1}{2}\sqrt{2}\right)N$.

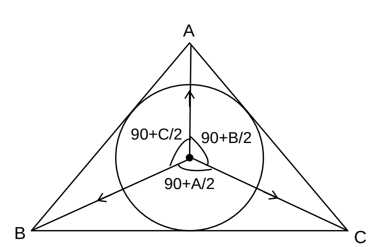
71. F . 3 sin θ = 9
F . 4 cos θ = 16
$\Rightarrow$ F = 5.



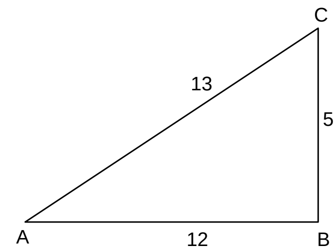
72. By Lami's theorem

$\vec{P}:\vec{Q}:\vec{R}=\sin\left(90^\circ+\frac{A}{2}\right):\sin\left(90^\circ+\frac{B}{2}\right):\sin\left(90^\circ+\frac{C}{2}\right)$

$\Rightarrow \cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$.



73. Time T_1 from A to B = $\frac{12}{4}$ = 3 hrs.



T_2 from B to C = $\frac{5}{5}$ = 1 hrs.

Total time = 4 hrs.

Average speed = $\frac{17}{4}$ km/ hr.

Resultant average velocity = $\frac{13}{4}$ km/hr.

74. Component along OB = $\dfrac{\frac{1}{4}\sin 30^\circ}{\sin(45^\circ + 30^\circ)} = \frac{1}{8}\left(\sqrt{6} - \sqrt{2}\right)$ m/s.

75. $t_1 = \dfrac{2u\sin\alpha}{g}$, $t_2 = \dfrac{2u\sin\beta}{g}$ where $\alpha + \beta = 90^0$

$\therefore t_1^2 + t_2^2 = \dfrac{4u^2}{g^2}$.