Roll No. | | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilator

Question Booklet Series **Y**

PAPER–II

Question Booklet No.
(Identical with OMR Answer Sheet Number)

Subject Code : 15

MATHEMATICAL SCIENCES

Time : 2 Hours *Maximum Marks: 200*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set is to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D) , where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.**
12. **There is no negative marks for incorrect answer.**

MATHEMATICAL SCIENCES

Paper II

1. A fluid element has a velocity $\vec{v} = -y^2 x\hat{i} + 2yx^2\hat{j}$. The motion at $(x, y) = \left(\frac{1}{\sqrt{2}}, 1\right)$ is
   (A) rotational and incompressible.
   (B) rotational and compressible.
   (C) irrotational and compressible.
   (D) irrotational and incompressible.

2. If $\hat{\theta}$ is an estimator of $\theta + 1$ such that $P(\hat{\theta} = \theta + 1) = 1 - n^{-5/2}$ and $P(\hat{\theta} = \theta + n + 1) = n^{-5/2}$, then
   (A) $\hat{\theta}$ is not consistent
   (B) $\hat{\theta}$ is consistent and $MSE \to 0$ as $n \to \infty$.
   (C) $\hat{\theta}$ is consistent but $MSE \not\to 0$ as $n \to \infty$.
   (D) $\hat{\theta}$ is consistent but $MSE \to \infty$ as $n \to \infty$.

3. Force of mortality at age x, μ_x is
   (A) $\frac{1}{l_x} \cdot \frac{d\,l_x}{dx}$
   (B) $\frac{d\,l_x}{dx}$
   (C) $-\frac{d\,l_x}{dx}$
   (D) $-\frac{1}{l_x} \cdot \frac{dl_x}{dx}$

4. Consider the dihedral group
   $D_4 = \{e, r, r^2, r^3, f, rf, r^2 f, r^3 f\}$
   with $r^4 = e = f^2$ and $rf = f\,r^{-1}$
   Then the product $r^3 f\, r^{-1} f^{-1} r^3\, fr$ corresponds to
   (A) f
   (B) rf
   (C) $r^2 f$
   (D) $r^3 f$

5. If a function $f : (-a, a) \setminus \{0\} \to (0, \infty)$ satisfies $\lim_{x \to 0} \left(f(x) + \frac{1}{f(x)} \right) = 2$, then
   (A) $\lim_{x \to 0} f(x) = 0$.
   (B) $\lim_{x \to 0} f(x) = 1$.
   (C) $\lim_{x \to 0} f(x) = 2$.
   (D) $\lim_{x \to 0} f(x)$ does not exist.

6. Let $S = \{c_i\}$ be an infinite orthonormal set in an incomplete inner product space V. If S is complete, then
   (A) $\lim_{n \to \infty} \| x - \sum_{i=1}^{n} \langle x, c_i \rangle c_i \| = 0 \ \forall\, x \in V$
   (B) $\| x \|^2 = \sum_{i=1}^{\infty} | \langle x, c_i \rangle |^2 \ \forall\, x \in V$
   (C) for any $x, y \in V$, $\langle x, c_i \rangle = \langle y, c_i \rangle \ \forall\, i \Rightarrow x = y$
   (D) $\forall\, x, y \in V$, $\langle x, y \rangle = \sum_{i=1}^{\infty} \langle x, c_i \rangle \langle c_i, y \rangle$

7. Suppose $X_1, X_2, ..., X_n$ are iid Cauchy (0, 1) variables. Which of the following is not ancillary?
   (A) $Max\ (X_1, ..., X_n) - Min\ (X_1, X_2, ..., X_n)$
   (B) $Min\ (X_1, ..., X_n) - \bar{X}$
   (C) $\frac{Max\ (X_1, ..., X_n) - Min\ (X_1, X_2, ..., X_n)}{|Max\ (X_1, ..., X_n) - Min\ (X_1, X_2, ..., X_n)|}$
   (D) $X / |X|$

8. Trend value of a time series of each time point is not available in use of the method of

(A) graphical

(B) least squares

(C) moving averages

(D) None of the above

9. If $\rho_{X,Y}$ is the correlation between X and Y, the correlation between U and V, when $U = a + cX$ and $V = b - dY$ $(a, b, c, d > 0)$ is

(A) $\rho_{X,Y}$

(B) $\frac{ab}{cd}\rho_{X,Y}$

(C) $\frac{cd}{|c||d|}\rho_{X,Y}$

(D) $-\rho_{X,Y}$

10. Consider a one-way ANOVA set up with 5 treatments. However, after scrutiny, it was found that all observations were multiplied wrongly by 10 and obtained SS (Treatment) = 7·50 and SS(Error) = 3·25. If F_c and F_w are the F values based on the correct and wrong set of observations, respectively, then

(A) $F_w = 10F_c$

(B) $F_w = F_c$

(C) $F_w < F_c$

(D) $F_w = 10^2 F_c$

11. The zeros of the function $f(z) = \sin\left(\frac{1}{1-z}\right)$ are given by,

(A) $z_n = \frac{1}{n\pi}, n = \pm 1, \pm 2, ...$

(B) $z_n = 1 - \frac{1}{n\pi}, n = \pm 1, \pm 2, ...$

(C) $z_n = 1 + \frac{1}{n\pi}, n = \pm 1, \pm 2, ...$

(D) $z_n = n\pi + \frac{1}{n\pi}, n = \pm 1, \pm 2, ...$

12. The subspace $\{(x, y) \mid y = e^x\}$ of the usual topological space $\mathbb{R}^2$ is homeomorphic to

(A) Unit circle

(B) Q

(C) $\mathbb{R}$

(D) $\mathbb{R} - Q$

13. The number of positive integers between 1 and 1000 which are divisible neither by 2 nor by 5 is

(A) 100

(B) 300

(C) 200

(D) 400

14. A second degree polynomial regression of y on x was fitted based on A pairs of (x_i, y_i) values. The following actual and fitted values are obtained

i	1	2	3	4
y_i	3	7	9	11
$\hat{Y}_i$	5	6	7	*

However, $\hat{Y}_4$ was missing. Then

(A) nothing can be said about the value of $\hat{Y}_4$ based on these

(B) $\hat{Y}_4 = 10$

(C) $\hat{Y}_4 = 11$

(D) $\hat{Y}_4 = 12$

15. The equation of Cauchy stress quadratic at $P(x_1, x_2, x_3)$ for a state stress $(\tau_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ is

(A) $5x_1^2 + 2x_2^2 + 3x_3^2$ = constant

(B) $x_1^2 + 2x_2^2 + 2x_3^2$ = constant

(C) $2x_1^2 + 2x_2^2 + 3x_3^2$ = constant

(D) $x_1^2 + 2x_2^2 + 3x_3^2$ = constant

(where symbols have their usual meaning)

16. Consider literacy rate estimation in a certain locality. The investigator wants the estimation error to be at most 2% with at least 90% confidence. What will be the minimum sample size (rounded to the next integer) for the study?

(A) 625

(B) 1200

(C) 97

(D) 59

17. The relation between standarized death rate and crude death rate of a region A is $(\text{STDR})_A = \hat{C}\,(\text{CDR})_A$. Here the expression of the adjustment factor $\hat{C}$ with the usual notation is

(A) $\dfrac{\sum m_x^s P_x^s}{\sum P_x^s} \times \dfrac{\sum P_x^A}{\sum m_x^s P_x^A}$

(B) $\dfrac{\sum m_x^A P_x^s}{\sum P_x^s} \times \dfrac{\sum P_x^A}{\sum m_x^s P_x^A}$

(C) $\dfrac{\sum m_x^A P_x^s}{\sum P_x^s} \times \dfrac{\sum P_x^A}{\sum m_x^A P_x^A}$

(D) None of the above

18. Consider the following statements:

S_1 : Every monotone function on $[a, b]$ is of bounded variation on $[a, b]$.

S_2 : Every continuous function on $[a, b]$ is of bounded variation on $[a, b]$.

S_3 : Every function of bounded variation on $[a, b]$ is absolutely continuous on $[a, b]$.

S_4 : Every absolutely continuous function on $[a, b]$ is a function of bounded variation on $[a, b]$.

Then,

(A) only S_1 and S_2 are correct.

(B) only S_1 and S_3 are correct.

(C) only S_1 and S_4 are correct.

(D) only S_2 and S_4 are correct.

19. Consider the boundary value problem

$u_{xx} + u_{yy} = 0,\ x \in (0, \pi), y \in (0, \pi)$,

$u(x, 0) = u(x, \pi) = u(0, y) = 0$.

Any solution of this boundary value problem is of the form

(A) $\sum_{n=1}^{\infty} \mathrm{a}_n \sinh nx \sin ny$

(B) $\sum_{n=1}^{\infty} \mathrm{a}_n \cosh nx \sin ny$

(C) $\sum_{n=1}^{\infty} \mathrm{a}_n \sinh nx \cos ny$

(D) $\sum_{n=1}^{\infty} \mathrm{a}_n \cosh nx \cos ny$

20. If $X_i, i = 1(1)5$ are iid observation from a $N(\theta, 1)$ distribution, $\sum_{i=1}^{5} X_i$ is observed as 25. If it is known that $\theta \le 0$, then maximum likelihood estimate $\hat{\theta}$ satisfies

(A) $\hat{\theta} = 5$

(B) $\hat{\theta} = 2$

(C) $\hat{\theta} = -1$

(D) $\hat{\theta} = 0$

21. $x^3 \log\left(\dfrac{y}{x}\right)$ is a homogeneous function of x and y of degree

(A) 0

(B) 1

(C) 2

(D) 3

22. State which of the following is not correct.

(A) Point transformations are also canonical transformation.

(B) Contact transformations are canonical transformations in extended phase space.

(C) Gauge transformations are canonical transformation.

(D) Under canonical transformation Hamiltonian equations need not be invarient.

23. Suppose $E(y_1) = E(y_2) = \beta$,

$\text{Var}(y_1) = 5\sigma^2$, $\text{Var}(y_2) = 2\sigma^2$

$\text{Cov}(y_1, y_2) = \sigma^2$.

Which of the following is BLUE of β ?

(A) $\dfrac{y_1 + y_2}{2}$

(B) $\left(\dfrac{y_1}{5} + \dfrac{y_2}{2}\right) \Big/ \left(\dfrac{1}{5} + \dfrac{1}{2}\right)$

(C) $\dfrac{2y_1 + 3y_2}{5}$

(D) $\dfrac{y_1 + 4y_2}{5}$

24. In control chart for fraction defective for varying subgroup sizes, the control limits in standardized method is

(A) $LCL = p - 3\sqrt{\dfrac{p(1-p)}{n_i}}, UCL = p + 3\sqrt{\dfrac{p(1-p)}{n_i}}$

(B) $LCL = p - 3\sqrt{p(1-p)}, UCL = p + 3\sqrt{p(1-p)}$

(C) $LCL = -3$, $UCL = 3$

(D) None of the above

25. A nonconstant entire function

(A) has at least one zero in $\mathbb{C}$.

(B) cannot have finite number of real zeros.

(C) cannot have countable number of zeros in a bounded region of $\mathbb{C}$.

(D) cannot have uncountable number of zeros in $\mathbb{C}$.

26. Consider the optimization problem,

Maximize $z = 2x_1 + x_1x_2 + 3x_2$

subject to $x_1^2 + x_2 = 3$.

Then Global maximum of z

(A) is equal to $\dfrac{13}{2}$.

(B) is equal to $\dfrac{19}{43}$.

(C) occurs at more than one point.

(D) does not exist.

27. The equation of the circular hellix is

(A) $x = a \cos t,\ y = b \sin t,\ z = ct$

(B) $x = a \cos t,\ y = a \sin t,\ z = bt$

(C) $x = a \cos t,\ y = a \sin t,\ z = ct^2$

(D) $x = a \cos t,\ y = b \sin t,\ z = c$

where a, b, c are constants.

28. Among the designs CRD, RBD, LSD

(A) only CRD is orthogonal.

(B) only RBD is orthogonal.

(C) only LSD is orthogonal.

(D) all are orthogonal.

29. Suppose y_1, y_2, y_3 are iid observations from $N(\theta_1 + \theta_2, 1)$ distribution. Then

(A) ML estimator of θ_1 is Y_1.

(B) ML estimator of θ_2 is $\overline{Y}$.

(C) The family of distribution is not identifiable.

(D) θ_2 and θ_1 are separately estimable by the method of moments.

30. Let G be a commutative group of order 202. Then the number of element(s) of order 2 in G is

(A) 1

(B) 2

(C) 4

(D) 101

31. Let A be a 3×3 matrix with real entries such that $\det(A) = 6$ and the trace of A is 0. If $\det(A + I) = 0$ where I denotes the 3×3 identity matrix, then the eigenvalues of A are

(A) –1, 2, 3

(B) –1, 2, –3

(C) 1, 2, –3

(D) –1, –2, 3

32. Let E be a subset of $\mathbb{R}$. Then

(A) if E is Lebesgue measurable then E is a Borel set.

(B) if E is not a Borel set then E is Lebesgue measurable.

(C) if E is a Borel set then E is Lebesgue measurable.

(D) If E is a Borel set then E is not Lebesgue measurable.

33. The complete graph of n vertices contains two edge disjoint spanning tree if and only if

(A) $n < 3$

(B) $n \geq 4$

(C) $n < 4$

(D) $n \geq 3$

34. If N is the incidence matrix of a symmetric BIBD $(v = b, r = k, \lambda)$, which of the following is not true?

(A) $(r - \lambda)$ is always a perfect square.

(B) $N'N = (r - \lambda)I_v + \lambda \underset{\sim}{1}\, \underset{\sim}{1}'$

(C) $|N|$ is any positive real number.

(D) $NN' = (r - \lambda)I_v + \lambda \underset{\sim}{1}\, \underset{\sim}{1}'$

35. State which of the following is correct:

(A) Derived sets in a topological space are closed.

(B) Derived set of a subset of $\mathbb{R}$ (real line) under usual topology is closed.

(C) Closed subsets of a compact topological space are not compact.

(D) Compact subsets of a Hausdorff space are not closed.

36. The value of the integral $\int\limits_{|z|=1} \frac{\cos(e^z)}{z}\, dz$ is given by

(A) $2\pi i \cos(e)$

(B) $\pi i \cos(1)$

(C) $2\pi i \sin(e)$

(D) $\pi i \left(e^i + e^{-i}\right)$

37. Which of the following statements is not true $(z \in \mathbb{C})$?

(A) $\sum z^n$ and $\sum z^n/n^2$ have the same radius of convergence.

(B) $\sum z^n$ converges nowhere on the boundary of the disk of convergence.

(C) $\sum z^n/n^2$ converges everywhere on the boundary of the disk of convergence.

(D) $\sum z^n$ converges everywhere on the boundary of the disk of convergence.

38. Let x_1 and x_2 be two real numbers. Then which of the following is a convex set?

(A) $X_1 = \{(x_1, x_2) : x_1^2 + x_2^2 = 16\}$

(B) $X_2 = \{(x_1, x_2) : x_2^2 \geq 4x_1\}$

(C) $X_3 = \{(x_1, x_2) : x_1 x_2 \leq 4\}$

(D) $X_4 = \{(x_1, x_2) : x_1 \leq 5, x_2 \geq 3\}$

39. Let (X, S, λ) be an arbitrary signed measure space. Then

(A) (X, S, λ) may not admit a Hahn decomposition.

(B) (X, S, λ) admits a Hahn decomposition.

(C) λ may not admit a Jordan decomposition.

(D) λ admits a Jordan decomposition which is not necessarily unique.

40. Let X_1, X_2, X_3, X_4 be independent $N(0, 1)$ random variables. The distribution of $Y = X_1 X_2 - X_3 X_4$ is

(A) Logistic

(B) Cauchy

(C) Normal

(D) Laplace

41. Which of the following curve can give an extremum of the functional,

$$J(y(x)) = \int_0^{\pi} (y'^2 + 12xy)dx,$$

$$y(0) = 0,\ y(1) = 1,\ y' \equiv \frac{dy}{dx}?$$

(A) $y = x^2$

(B) $y = 2x^2$

(C) $y = x^3$

(D) $y = 2x^3$

42. A characteristic number λ and the corresponding characteristic function $\phi(x)$ of the homogeneous Fredholm integral equation with Kernel $K(x, t) = \begin{cases} x(t-1), & 0 \leq x \leq t \\ t(x-1), & t \leq x \leq 1 \end{cases}$ are

(A) $\lambda = -\pi^2, \phi(x) = \sin \pi x$

(B) $\lambda = -2\pi^2, \phi(x) = \sin 2\pi x$

(C) $\lambda = 2\pi^2, \phi(x) = \sqrt{\cos \pi x}$

(D) $\lambda = \pi^2, \phi(x) = \sqrt{\cos 2\pi x}$

43. In a primal problem, the 4th constraint is an equation and the 3rd variable is unrestricted in sign. Then the nature of the 4th dual variable and the 3rd dual constraint will be respectively

(A) unrestricted in sign and equation.

(B) non-negative and equation.

(C) non-negative and inequation.

(D) unrestricted in sign and inequation.

44. Total number of zeros of the function $f(z) = z^4 - 5z + 1$ within the annulus $1 < |z| < 2$ is

(A) 1

(B) 2

(C) 3

(D) 4

45. Which of the following is not correct?

(A) Bounded operators defined on normed linear spaces are continuous.

(B) Compact operators need not be a completely continuous operator.

(C) Identity operator is not always continuous.

(D) Identity operator is always continuous.

46. Consider a finite population $U = \{1, 2, 3\}$ with

$p(\{1, 2\}) = \frac{1}{2}$,

$p(\{1, 3\}) = \frac{1}{4}$, $p(\{2, 3\}) = \frac{1}{4}$.

Then which of the following is not true?

(A) $\pi_1 = 3/4$

(B) $\pi_2 = 3/4$

(C) $\pi_1 + \pi_2 + \pi_3 = 1$

(D) $\pi_1 + \pi_2 + \pi_3 = 2$

[π_k : first order inclusion probability of the k-th unit]

47. The partial differential equation of the set of all right circular cones whose axes coincide with z-axis is

(A) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$

(B) $y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$

(C) $x^2\frac{\partial^2 z}{\partial x^2} = y^2\frac{\partial^2 z}{\partial y^2}$

(D) $y^2\frac{\partial^2 z}{\partial x^2} = x^2\frac{\partial^2 z}{\partial y^2}$

48. If M and N be two smooth functions from $\mathbb{R}^2$ to $\mathbb{R}$, then the form "$Mdx + Ndy$" is exact if and only if which of the following is/are true?

(i) $\exists$ a smooth function f such that $Mdx + Ndy = \alpha f$

(ii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ for all x and y

(iii) curl $(M\hat{i} + N\hat{j}) = \vec{0}$

(A) (i) and (ii)

(B) (i) and (iii)

(C) (ii) and (iii)

(D) (i), (ii) and (iii)

49. On evaluating $\int_1^3\int_1^3 \frac{x^3}{y}dxdy$ numerically by Simpson's $\frac{1}{3}rd$ rule one would get the value

(A) $\frac{26}{3}$

(B) $\frac{100}{9}$

(C) $\frac{20}{9}$

(D) $\frac{200}{9}$

50. Let $\{A_n\}$ be a sequence of events such that

$$A_n = \begin{cases} \{a, b, c, d\}, & \text{if } n \text{ is odd} \\ \{b, d, e, f\}, & \text{if } n \text{ is even} \end{cases}.$$

Then which of the following is correct?

(A) $\overline{\lim} A_n = \underline{\lim} A_n$

(B) $\underline{\lim} A_n = \{b, c, d, e, f\}$

(C) $\underline{\lim} A_n = \{b, d\}$

(D) $\lim_{n\to\infty} A_n$ doesn't exist.

51. In the group of all invertible 4 × 4 matrices with entries in the field of three elements, any Sylow 3-subgroup has cardinality

(A) 3

(B) 81

(C) 243

(D) 729

52. Let α and β be two positive real numbers. If the number of optimal solutions of the LPP

$$\max\ z = \alpha x + \beta y$$

subject to

$$3x + 4y \geq 7$$
$$x + y \leq 20$$
$$x \geq 0,\ y \geq 0$$

is infinite, then which of the following is possible?

(A) α = 3, β = 4

(B) α = 4, β = 3

(C) α = 3, β = 3

(D) $\alpha = \frac{3}{2}$, β = 2

53. Suppose X is distributed with PDF $f(x) = \frac{e^{-x}}{(1+e^{-x})^2}$, $-\infty < x < \infty$.

Then $P\{X - |X| = 0\}$ equals

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

54. A three unit parallel system has independent components with reliabilities 0·2, 0·3 and 0·4 respectively. Then the reliability of the system is

(A) 0·024

(B) 0·336

(C) 0·664

(D) 0·886

55. Let $\{X_n = n \geq 0\}$ be a branching process with $X_0 = 1$ and $P(s)$ be the probability generating function of X_1. Let $Y_n = X_1 + X_2 + \ldots + X_n$ be the total number of individuals up to the nth generation and $H_n(s)$ be the probability generating function of Y_n. Then $H_{n+1}(s)$ is

(A) $sH_n(s)$

(B) $sP(H_n(s))$

(C) $\frac{P(H_n(s))}{s}$

(D) $\frac{H_n(s)}{s^2}$

56. Let *F(x)* be a distribution function (*d.f.*) given by

$$F(x) = \begin{cases} 0 & , \quad x < 0 \\ \frac{1}{2} + \frac{1}{2}(1 - e^{-x}), & x \geq 0. \end{cases}$$

Then which of the following is correct?

(A) *F* is a discrete *d.f.*

(B) *F* is a continuous *d.f.*

(C) *F* is a mixture of *d.f.*s

(D) None of the above statement is correct.

57. The number of subfields of a field of cardinality 3^{100} is

(A) 3

(B) 9

(C) 25

(D) 100

58. Which of the following statement is not true?

(A) Every Euclidean ring is a unique factorization domain.

(B) Every unique factorization domain is an Euclidean ring.

(C) Every integral domain can be embeded in a field.

(D) Ring of polynomials *F*[*x*], where *F* is a field, is a principal ideal ring.

59. The Lagrangion L of a dynamical system is $L = \overset{o}{q}_1^{\,2} + \overset{o}{q}_2^{\,2} + K_1 q_1^2$ and p_1, p_2 are the generalised momenta corresponding to the generalised coordinates q_1, q_2 (with K_1, a constant). Then the Hamiltonian H is given by

(A) $H = p_1^2 + p_2^2 - K_1 q_1^2$

(B) $H = \frac{1}{2}\left(p_1^2 + p_2^2\right) - K_1 q_1^2$

(C) $H = 4\left(p_1^2 + p_2^2\right) - K_1 q_1^2$

(D) $H = \frac{1}{4}\left(p_1^2 + p_2^2\right) - K_1 q_1^2$

60. If the P value of a test is 0·02, then

(A) it must be rejected at 1% level of significance.

(B) it cannot be rejected at 5% level of significance.

(C) it must be accepted at 1% level of significance.

(D) it is rejected at 5% level of significance.

61. For the first order auto-regressive series $U_{t+1} = a.U_t + \in_{t+1}, |a| < 1,$ where $\in_t$'s are independent with zero mean, then the correlogram is

(A) a^k

(B) a^{-k}

(C) $a^{\frac{1}{k}}$

(D) $a^{-\frac{1}{k}}$

62. For a symmetric distribution, which of the following is not necessarily true?

(A) Mean = Median

(B) Mean = Median = Mode

(C) First and third quartiles are equidistant from median.

(D) First and third quartiles are equidistant from mean.

63. Let $L\{y(t)\}$ denote the Laplace transformation of $y(t)$ and $\int_0^t y(x)\, y(t-x)\, dx = 16\sin(4t)$. Then $L\{y(t)\}$ is given by

(A) $\pm\frac{8}{\sqrt{p^2+4^2}}$

(B) $\frac{18}{\sqrt{p^2+4^2}}$

(C) $\pm\frac{4p}{\sqrt{p^2+4^2}}$

(D) $\pm\frac{4}{\sqrt{p^2+4^2}}$

where p is the transformed variable.

64. The rate of convergence of the interation process

$x_{n+1} = \frac{1}{8}x_n\left(6 + \frac{3a}{x_n^2} - \frac{x_n^2}{a}\right)$ if $x_n \to \sqrt{a}$ is

(A) 2

(B) 3

(C) 1

(D) 4

65. If $E \subset \mathbb{R}$ is uncountable, then

(A) E has no limit point.

(B) E has countably many limit points.

(C) E has uncountably many limit points.

(D) E has finitely many limit points.

66. Let $f:[0,1]\to(0,1)$ be a continuous function and $f_n(x)=\left(f(x)\right)^n$ for all $n\in\mathbb{N}$. Then

(A) $\{f_n\}$ converges to f pointwise but not uniformly.

(B) $\{f_n\}$ converges to f uniformly.

(C) $\lim_{n\to\infty}\int_0^1 f_n(x)dx=\int_0^1 f(x)dx$.

(D) $\lim_{n\to\infty}\int_0^1 f_n(x)dx=0$.

67. Let T_1 be the topology generated by the family of all open disks on $\mathbb{R}^2$ and let T_2 be the topology generated by the family of all open squares on $\mathbb{R}^2$. Then

(A) T_1 and T_2 are non-comparable.

(B) T_1 is strictly smaller than T_2.

(C) T_2 is strictly smaller than T_1

(D) $T_1 = T_2$

68. Suppose $X_1, ..., X_n$ are iid observations from Bernoulli (θ) distribution. If it is known apriori that $\frac{1}{3}\le\theta\le 2/3$, then expected Fisher information, $I(\theta)$ based on n observations is

(A) $I(\theta)=\frac{\theta(1-\theta)}{n}$

(B) $I(\theta)=\frac{n}{\theta(1-\theta)}$

(C) $I(\theta)=9n/2$

(D) not defined

69. Let $X_i, i=1(1)n$ be iid Poisson (θ) variables. If $T=\bar{X}^2-\frac{\bar{X}}{n}$, then

(A) T is MVUE of θ^2 and attains Crammer-Rao lower bound.

(B) T is not MVUE of θ^2 and attains Crammer-Rao lower bound.

(C) T is MVUE of $\theta^2-\theta$ and attains Crammer-Rao lower bound.

(D) T is MVUE of θ^2.

70. A solution (upto third approximation) of the equation $\frac{dy}{dx}=y+x$ such that $y=1$ when $x=0$ by Picard's process of successive approximation is

(A) $y=1+x+x^2+\frac{x^3}{3}+\frac{x^4}{12}+\frac{x^5}{120}$

(B) $y=1+x+x^2+\frac{x^3}{3}+\frac{x^4}{24}$

(C) $y=1+x+x^2+\frac{x^3}{6}$

(D) $y=1+x+\frac{x^2}{2}+\frac{x^3}{6}$

71. If $\hat{\mu}$ and $\hat{\Sigma}$ are the maximum likelihood estimators of μ and Σ based on a random sample of size N from $N_p(\mu,\Sigma)$, then

(A) $\hat{\mu}$ and $\hat{\Sigma}$ are unbiased estimators of μ and Σ respectively.

(B) $\hat{\mu}$ is unbiased for μ where as $\hat{\Sigma}$ is not unbiased for Σ.

(C) $\hat{\Sigma}$ is unbiased for Σ where as $\hat{\mu}$ is not unbiased for μ.

(D) None of the above statement is correct.

72. The eigenvalues λ of the integral equation $y(x)=\lambda \int_0^{2\pi} \sin(x+t)\, y(t)\, dt$ are

(A) $\frac{1}{2\pi}, -\frac{1}{2\pi}$

(B) $\frac{1}{\pi}, -\frac{1}{\pi}$

(C) $\pi, -\pi$

(D) $\frac{2}{\pi}, -\frac{2}{\pi}$

73. The solution of the initial value problem $\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2},\ t>0, -\infty<x<+\infty$ satisfying the conditions $u(x,0)=x, \frac{\partial u}{\partial t}(x,0)=0$ is

(A) x

(B) $2x$

(C) $x^2/2$

(D) $2t$

74. For what value of K, the function

$$f(x,y)=\begin{cases} \frac{\sin^{-1}(xy-2)}{\tan^{-1}(3xy-6)}, & (x,y)\neq(1,2) \\ K & (x,y)=(1,2) \end{cases}$$

is continuous at (1, 2)?

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{3}{4}$

75. Let $\{x_n\}$ be a sequence of real numbers. Then $\lim_{n\to\infty} x_n$ exists if and only if

(A) $\lim_{n\to\infty} x_{2n}$ and $\lim_{n\to\infty} x_{2n-1}$ exist.

(B) $\lim_{n\to\infty} x_{2n}$ and $\lim_{n\to\infty} x_{2n+2}$ exist.

(C) $\lim_{n\to\infty} x_{2n}$, $\lim_{n\to\infty} x_{2n+1}$ and $\lim_{n\to\infty} x_{3n}$ exist.

(D) $\lim_{n\to\infty} x_{3n}$ and $\lim_{n\to\infty} x_{2n}$ exist.

76. The value of $\int_C \frac{dz}{z(z+i\pi)}$, where $C: |z+3i|=1$, is

(A) πi

(B) $-\pi i$

(C) $2\pi i$

(D) 0

77. Zero opportunity cost in the optimal transportation table for a non-basic variable indicates

(A) unbounded solution.

(B) no feasible solution.

(C) degenerate solution.

(D) the existence of alternative optimal solution.

78. Suppose 6 observations 5·1, 5·6, 7·8, 8·1, 9·2 and 10·3 are available from a continuous population indexed by $F(x-\theta)$. A level 0·05 sign test for $H_0: \theta=0$ against $H_1: \theta\neq 0$ rejects the null hypothesis if number of positive observations is either at least 5 or at most 1. Then a confidence interval of θ with confidence coefficient at least 95% (but less than 100%) is

(A) [5·6, 9·2)

(B) [5·6, 10·3)

(C) [5·1, 9·2)

(D) [5·1, 10·3)

79. Let F_p denote the field Z/pZ, where p is a prime and Z is the set of integers. Let $F_p\,[x]$ be the associated polynomial ring. Then which of the following ring(s) is/are field?

(i) $F_5\,[x]/[x^2+x+1]$

(ii) $F_2\,[x]/[x^3+x+1]$

(iii) $F_3\,[x]/[x^3+x+1]$

(A) Both (i) and (ii)

(B) Both (ii) and (iii)

(C) Both (i) and (iii)

(D) All of the above

80. Which of the following matrices is a Jordan block?

(A) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 7 \end{pmatrix}$

(B) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$

(C) $\begin{pmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{pmatrix}$

(D) $\begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

81. The Fourier transform of $f(x)=e^{-|x|}$ is

(A) $\sqrt{\frac{2}{\pi}}\,\frac{1}{K^2+4}$

(B) $\sqrt{\frac{\pi}{2}}\,\frac{1}{K^2+4}$

(C) $\sqrt{\frac{\pi}{2}}\,\frac{1}{K^2+1}$

(D) $\sqrt{\frac{2}{\pi}}\,\frac{1}{K^2+1}$

(where K is the transform variable)

82. Let X_1, X_2, X_3, X_4 be independent and identically distributed random variables with probability density function

$$f(x)=\begin{cases} 1\,, & 0<x<1 \\ 0\,, & \text{otherwise.} \end{cases}$$

If $X_{(4)}$ and $X_{(1)}$ are order statistics, then $E\left(X_{(4)}-X_{(1)}\right)$

(A) $\frac{3}{5}$

(B) $\frac{1}{5}$

(C) $\frac{10}{7}$

(D) $\frac{5}{8}$

83. Based on a single observation X, consider testing $H_0 : X \sim p_0(x)$ against $H_1 : X \sim p_1(x)$, where

x	$p_0(x)$	$p_1(x)$
–1	0·02	0·03
0	0·03	0·07
1	0·45	0·03
2	0·03	0·07
3	0·02	0·50
4	0·02	0·10
5	0·03	0·05
6	0·02	0·05
7	0·03	0·03
8	0·30	0·02
9	0·05	0·05

Then the number of non-randomized size 0·05 tests for the above problem is

(A) ∞

(B) 12

(C) 17

(D) 10

84. Let $A \sim W_p(n, \Sigma)$, A and Σ are partitioned as $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ and $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$ respectively, where A_{11} and Σ_{11} are matrices of order $q \times q$, $q < p$. Which of the following statement is correct?

(A) $A_{12} \sim W_{p-q}(n, \Sigma_{12})$

(B) $A_{11\cdot 2} = A_{11} - A_{12} A_{22}^{-1} A_{21} \sim W_q(n, \Sigma_{11\cdot 2})$

where $\Sigma_{11\cdot 2} = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$

(C) $A_{11} \sim W_q(n, \Sigma_{22})$

(D) None of the above statement is correct.

85. If the Hamiltonian of a dynamical system is given by $H = p_1 q_1 - p_2 q_2 - a q_1^2 + b q_2^2$ where a and b are constants, then $\frac{p_2 - b q_2}{q_1}$ is

(A) a function of q_1 only.

(B) a function of q_1 and q_2.

(C) a constant.

(D) a function of p_1, q_1, p_2, q_2.

86. The norm of the linear functional f defined on $C[-1, 1]$ by $f(x) = \int_{-1}^{0} x(t)dt - \int_{0}^{1} x(t)dt$ is

(A) 0

(B) 1

(C) 2

(D) 3

where $C[-1, 1]$ denotes the Banach space of all real valued continuous functions $x(t)$ on $[-1, 1]$ with norm given by $\|x\| = \max_{t \in [-1, 1]} |x(t)|$.

87. Let $A = \{ z \in \mathbb{C} : |z - 2| + |z + 1| \geq 3 \}$. Then

(A) A is a bounded, closed subset of $\mathbb{C}$.

(B) A is an unbounded proper subset of $\mathbb{C}$.

(C) $A = \mathbb{C}$.

(D) A is an unbounded subset of $\mathbb{C}$ which is not closed.

88. If f is a real valued function which is continuous on $\mathbb{R}$ and satisfies

$$f\left(\frac{x+y}{2}\right) = \frac{f(x) + f(y)}{2} \quad \forall\, x, y \in \mathbb{R},\ f(0) = 1$$

and $f'(0) = 2$, then $f(2)$ is equal to

(A) 2

(B) 0

(C) 5

(D) 3

89. For a non-homogenous Poisson process $\{N(t), t \geq 0\}$, the correlation coefficient between $N(s)$ and $N(t)$ for $s<t$ is

(A) $\frac{s}{t}$

(B) $\sqrt{\frac{s}{t}}$

(C) $\sqrt{st}$

(D) st

90. Which of the following is/are correct?

(i) Centre of special linear group $SL_n(3)$ is a cyclic group.

(ii) Order of centre of complex special orthogonal group $SO_3(n)$ is 2.

(iii) $SL_n(3)/Z$, Z is the set of integers, is simple.

(iv) $SO_n(3)/Z$, Z is the set of integers, is simple.

(A) Only (i) and (ii)

(B) Only (iii) and (iv)

(C) Only (i) and (iv)

(D) All of the above

91. Which of the following is not a characteristic function?

(A) $\frac{1}{\cos ht}$

(B) $\cos t$

(C) $\frac{2}{1+\cos t}$

(D) $e^{-|t|^{\alpha}}, 0 < \alpha \leq 2.$

92. Let X be a non-negative integer-valued random variable satisfying the condition

$$P(X \geq m+1 \mid X \geq m) = P(X \geq 1)$$

for every non-negative integer m.

Then the distribution of X is

(A) Exponential

(B) Poisson

(C) Geometric

(D) Binomial

93. Solution of the Cauchy problem $u_x + xu_y = 0$ with $u(0, y) = \sin y$ is

(A) $u(x, y) = \sin\left(y - \frac{1}{2}x^2\right)$

(B) $u(x, y) = \sin\left(x - \frac{1}{2}y^2\right)$

(C) $u(x, y) = \sin\left(xy - \frac{1}{2}\right)$

(D) $u(x, y) = \sin\left(xy + \frac{1}{2}\right)$

94. Let f be a real-valued continuous function on $\mathbb{R}$. Let $A = \{x \in \mathbb{R} \mid 2 < f(x) < \sqrt{5}\}$, $B = Q$ and C be the Cantor ternary set. Then

(A) A, B, C all are Borel sets.

(B) only A is a Borel set, but B, C are not Borel sets.

(C) only A, B are Borel sets, but C is not a Borel set.

(D) none of the A, B, C is a Borel set.

95. Let A be a 2 × 2 real matrix such that trace of A is 5 and determinant of A is 6. Then the eigenvalues of the matrix $A^2 - 2A + I_2$ (where I_2 is 2×2 identity matrix) are

(A) 1, 4

(B) 2, 3

(C) 5, 6

(D) 11, 30

96. Let $R = \left\{\begin{pmatrix} a & a \\ a & a \end{pmatrix} \mid a \in \mathbb{R}\right\}$. Then with respect to usual addition and multiplication of matrices R forms a(an)

(A) Noncommutative Ring without identity.

(B) Commutative Ring with identity but not an integral domain.

(C) Integral domain but not a field.

(D) Field.

97. For a positive integer n, let P_n denote the space of all polynomials $p(x)$ with coefficients in $\mathbb{R}$ such that $\deg(p(x)) \leq n$ and let B_n denote the standard basis of P_n given by $B_n = \{1, x, x^2, ..., x_n\}$. If $T : P_3 \to P_4$ is the linear transformation defined by

$$T(p(x)) = x^2 p'(x) + \int_0^x p(t)dt \text{ and } A = (a_{ij})$$

is the 5 × 4 matrix of T with respect to the standard bases B_3 and B_4, then

(A) $a_{32} = \frac{3}{2}$ and $a_{33} = \frac{7}{3}$

(B) $a_{32} = \frac{3}{2}$ and $a_{33} = 0$

(C) $a_{32} = 0$ and $a_{33} = \frac{7}{3}$

(D) $a_{32} = 0$ and $a_{33} = 0$

98. The nature of $\frac{\partial^2 u}{\partial x^2} + 2y\frac{\partial^2 u}{\partial x \partial y} + 4x\frac{\partial^2 u}{\partial y^2} = 0$ is

(A) parabolic on the parabola $y^2 = 4x$ in the xy-plane

(B) elliptic outside the parabola $y^2 = 4x$ in the xy-plane.

(C) hyperbolic inside the parabola $y^2 = 4x$ in the xy-plane.

(D) parabolic everywhere in the xy-plane.

99. The degree of splitting field of $f(x) = x^4 - 2$ over Q(set of rationals) is

(A) 8

(B) 2

(C) 4

(D) 6

100. Let V be a vector space of all 4×4 real matrices and $T : V \to \mathbb{R}$ be a map, defined by

$T(A)$ = trace A, $A \in V$.

Then

(A) T is not a linear map.

(B) T is a linear map and dim ($Ker\ T$) = 8.

(C) T is a linear map and dim ($Im\ T$) = 1.

(D) T is a linear map and dim ($Ker\ T$) = 16.

ROUGH WORK

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