**Roll No.** | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

*Signature of Invigilators*

1. ______________________

2. ______________________

**1517**

**Question Booklet Series** **X**

**PAPER–III** **Question Booklet No.**

(Identical with OMR Answer Sheet Number)

**Subject Code : 15**

# MATHEMATICAL SCIENCES

*Time : 2 Hours 30 Minutes* *Maximum Marks: 150*

***Instructions for the Candidates***

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be get replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of seventy-five (75) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D). You have to darken the circle as indicated below on the correct response against each question.

   *Example:* (A) (B) ● (D), where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator or mobile phone etc. is strictly prohibited.**
12. **There are no negative marks for incorrect answers.**

**[Please Turn Over]**

## MATHEMATICAL SCIENCES

### Paper III

**1.** Let $f:[0,1] \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ n & \text{if } x \text{ is irrational, where } n \text{ is the total number of zeros immediately succeeding the decimal point in the decimal expression of } x. \end{cases}$$

Then $f$ is

(A) bounded and Riemann integrable over [0,1]

(B) bounded and Lebesgue integrable over [0,1]

(C) unbounded and Lebesgue integrabel over [0,1]

(D) continuous over $\left[\frac{1}{10^3}, \frac{1}{10^2}\right]$

**2.** Let $L[0,1]$, $R[0,1]$ respectively denote the families of Lebesgue integrable and Riemann integrable functions over [0,1] and $L_B[0,1]$ the family of all bounded Lebesgue measurable functions over [0,1]. Then

(A) $L_B[0,1] = L[0,1]$.

(B) $R[0,1] \subsetneqq L_B[0,1] \subsetneqq L[0,1]$.

(C) $L_B[0,1]$ is not dense in $L[0,1]$ in integral norm.

(D) each function in $L_B[0,1]$ is continuous almost everywhere on [0,1] with respect to Lebesgue measure.

**3.** The limit $\lim_{x \to 0} \frac{1}{x} \int_x^{2x} e^{-t^2} dt$

(A) does not exist.

(B) is infinite.

(C) exists and equals 1.

(D) exists and equals 0.

**4.** Let $\gamma$ be any simple closed curve in the complex plane. Then the set of all possible values of $\oint_\gamma \frac{dz}{z(1-z^2)}$ is

(A) $\{0\}$

(B) $\{0, \pm \pi i\}$

(C) $\{0, \pi i, 2\pi i\}$

(D) $\{0, \pm \pi i, \pm 2\pi i\}$

**5.** The function $f(z) = \frac{e^{z^5}(1-\cos z)^3}{e^z(\sin z)^3}$

(A) is analytic at z = 0.

(B) has a pole of order 2 at z = 0.

(C) has a pole of order 5 at z = 0.

(D) has an essential singularity at z = 0.

**6.** The value of $\int_\gamma \frac{dz}{z \sin z}$, where $\gamma$ is the anticlockwise unit circle $|z| = 1$, is

(A) 0

(B) $-\pi i$

(C) $\pi i$

(D) $2\pi i$

**7.** Let $f$: G $\to$ H be a homomorphism from a group G to a group H with kernal of order 15. If order of G = 75 and order of H = 45, then the order of $f$(G) is

(A) 3

(B) 5

(C) 9

(D) 15

**8.** The quadratic form
$6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$
in three variables is

(A) a positive definite

(B) a negative definite

(C) degenerate

(D) singular

**9.** Let $A \neq I_n$ be an $n \times n$ matrix such that $A^2 = A$, where $I_n$ is the $n \times n$ identity matrix. Which of the following is false?

(A) $(I_n - A)^2 = I_n - A$

(B) Trace(A) = Rank(A)

(C) Rank(A) + Rank($I_n$ – A) = $n$

(D) The eigenvalues of A are equal to 1

**10.** If K stands for the Cantor set, then

(A) every subset of K is a Borel set

(B) K is nowhere dense

(C) K has a subset which is not Lebesgue measureble

(D) the point $\frac{1}{4}$ is outside K

**11.** Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(\mathbb{Q}) \subseteq \mathbb{N}$. Then

(A) $f(\mathbb{R}) = \mathbb{N}$

(B) $f$ is not differentiable

(C) $f$ is a constant function

(D) $f$ is non-constant function but bounded

**12.** The element of order 5 in the group $\mathbb{Z}_{25} \oplus \mathbb{Z}_5$ is

(A) 5

(B) 10

(C) 24

(D) 25

**13.** Let G be a group of order 8. Then G is

(A) always abelian

(B) always non-abelian

(C) cyclic, if abelian

(D) generated by 2 elements if non-abelian

**14.** Which of the following quotient rings is a field?

(A) $\mathbb{Z}_2[x] / \langle x^2 + x + 1 \rangle$

(B) $\mathbb{Z}_3[x] / \langle x^2 + x + 1 \rangle$

(C) $\mathbb{Q}[x] / \langle x^2 + 2x + 1 \rangle$

(D) $\mathbb{R}[x] / \langle x^2 + 2x + 1 \rangle$

**15.** Let X be a normed linear space and $x_0 \in X$ such that for each bounded linear functional $f$ on $X$ with $\|f\| = 1$, $|f(x_0)| \leq K$. Then

(A) $\|x_0\| \leq K$

(B) $\|x_0\| = K$

(C) $\|x_0\| > K$

(D) $\|x_0\| = 1$

**16.** For any normed linear space X, which of the following is false?

(A) $X^*$ is a Banach space

(B) $X^{**}$ is a Banach space

(C) If X is a Hilbert space, then X is reflexive

(D) $X^*$ is a Banach space only if X is a Banach space

**17.** Let $X = \mathbb{N} \times \mathbb{Q}$ with the subspace topology of the usual topology of $\mathbb{R}^2$. Then the boundary of the set

$A = \left\{\left(n, \frac{1}{n}\right) : n \in \mathbb{N}\right\}$ is

(A) X

(B) A

(C) a singleton set

(D) the empty set

**18.** Let $S$ be a subset of $\mathbb{R}$ such that $\mathbb{R} \backslash S$ and $S$ both are dense in $\mathbb{R}$. Then

(A) $S$ is nowhere locally compact

(B) $S$ is connected

(C) $S$ is compact

(D) $S$ is never a $\sigma$-compact subset of $\mathbb{R}$

**19.** Any two constant maps $f, g : X \to Y$ (X,Y being topological spaces) are homotopic if

(A) X and Y are both connected

(B) the images of $f$ and $g$ lie in a path component of Y

(C) X is contractible

(D) X is simply connected

**20.** In the lattice $(D_{75}, /)$ the complement of 15 is

(A) 5

(B) 75

(C) 1

(D) does not exist

**21.** Every Hamiltonian graph is

(A) 3-connected

(B) 2-connected

(C) not 2-connceted

(D) not 3-connected

**22.** The general solution of $4u_{xx} - u_{yy} = 0$ is of the form

(A) $u = f(x + 2y) + g(x - 2y)$

(B) $u = f(x) + g(y)$

(C) $u = f(2x + y) + g(2x - y)$

(D) $u = f(x + 4y) + g(x - 4y)$

where $f$ and $g$ are twice differentiable functions.

**23.** The integral surface of the equation $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ ; $u(1, y) = y$ is given by

(A) $u = \frac{2y}{x+1}$

(B) $u = \frac{y}{x}$

(C) $u = y + x - 1$

(D) $u = \frac{y}{2-x}$

**24.** The diaphantine $x^2 + y^2 = z^2$ has primitive solution $(a, b, c)$ iff there exist $s, t \in \mathbb{N}$, $s > t$, $(s, t) = 1$ such that

(A) $a = s^2 - t^2, b = 2st, c = s^2 + t^2$

(B) $a = s^2 + t^2, b = 2st, c = s^2 - t^2$

(C) $a = 2st, b = s^2 - t^2, c = s^2 + t^2$

(D) None of the above

**25.** Given the transformation

$\mathrm{P} = q \cot p,\ Q = \log\left(\frac{\sin p}{q}\right)$, then

(A) $\dot{P} = -\frac{\partial H}{\partial P}$, (H is the Hamiltonian)

(B) $\dot{Q} = \frac{\partial H}{\partial Q}$

(C) The transformation is canonical

(D) The transformation is conditionally canonical

**26.** A particle is moving under central force about a fixed centre. Choose the correct statement.

(A) Its angular momentum is not conserved

(B) The motion of the particle is always on a circular path

(C) The motion of the particle is on a plane

(D) Its kinetic energy remains constant

**27.** Maximum limit up to which stress can be applied on a body without deformation is called

(A) Inelastic limit

(B) Strain-limit

(C) Elastic limit

(D) None of the above

**28.** If $(u(y), 0, 0)$ are the velocity components of an incompressible Newtonian fluid flow due to a pressure gradient in the $x$-direction, then $u(y)$ is a

(A) linear function of $y$

(B) quadratic function of $y$

(C) cubic function of $y$

(D) constant

**29.** $\frac{1}{P}\left(\frac{\partial P}{\partial t} + \vec{q}.\nabla P\right) + \nabla.\vec{q} = 0$ (P: density, $\vec{q}$ : velocity) is the equation of

(A) conservation of mass

(B) conservation of angular momentum

(C) conservation of linear momentum

(D) conservation of energy

**30.** Let $V$ be a space curve with curvature $k$ and torsion $\tau$ such that $\frac{k}{\tau} = \text{constant}$. Then $V$ is always a

(A) straight line

(B) twisted cubic

(C) helix

(D) plane curve

**31.** If $I[y(x)] = \int_a^b f(y, y')\,dx$, then a first integral of the Euler-Lagrange equation is

(A) $f - y'\dfrac{\partial f}{\partial y'} = c$

(B) $f - y\dfrac{\partial f}{\partial y} = c$

(C) $\dfrac{df}{dx} = 0$

(D) $\dfrac{\partial f}{\partial x} \neq 0$

**32.** The extremal of the functional $\int_0^1 \left[ y + x^2 + \dfrac{y'^2}{4} \right] dx$, $y(0) = 0$, $y(1) = 0$ is

(A) $4(x^2 - x)$

(B) $3(x^2 - x)$

(C) $5(x^2 - x)$

(D) $x^2 - x$

**33.** The resolvent kernal for the integral equation $u(x) = F(x) + \int_{\log 2}^{x} e^{(t-x)}\, u(t)\, dt$ is

(A) $\cos(x - t)$

(B) 1

(C) $e^{t-x}$

(D) $e^{2(t-x)}$

**34.** The integral equation $y(x) = \cos^2 x + \lambda \int_0^{\pi} \sin(x - t)\, y(t)\, dt$ is

(A) Voltera integral equation of first kind

(B) Voltera integral equation of second kind

(C) Fredholm integral equation of first kind

(D) Fredholm integral equation of second kind

**35.** Picard's method of solving $\dfrac{dy}{dx} = f(x, y(x))$, $y(x_0) = y_0$ gives us a/ an

(A) exact solution.

(B) numerical solution.

(C) sequence of power series solutions

(D) transcendental function

**36.** The interpolating polynomial for the function $f(x)$ given by the table

| $x$ | 2 | 1 | –1 |
|---|---|---|---|
| $f(x)$ | 12 | 16 | 24 |

is

(A) $4x - 20$

(B) $x^2 - 2x + 12$

(C) $-4x + 20$

(D) $3x^2 - 2x + 4$

**37.** If the points $x_1, x_2, \ldots, x_n$ are distinct, then for arbitrary real values $y_1, y_2, \ldots, y_n$, the degree of the unique interpolating polynomial $p(x)$ such that $p(x_i) = y_i$, $1 \leq i \leq n$, is

(A) $n$

(B) $n - 1$

(C) $\leq n - 1$

(D) $\leq n$

**38.** If $L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt = F(s)$, then $L\{tf(t)\}$ is

(A) $-\dfrac{d}{ds}\{F(s)\}$

(B) $\dfrac{s^2}{2}F(s)$

(C) $\dfrac{d}{ds}\{F(s)\}$

(D) $-\dfrac{s^2}{2}F(s)$

**39.** If $F_C\{f(t)\} = \int_0^\infty f(t)\cos(\omega t)\,dt = F_C(\omega)$, then $F_C\{f''(t)\} + \omega^2 F_C(\omega) + f'(0) = b$ implies,

(A) $b < 0$

(B) $b > 0$

(C) $b = 1$

(D) $b = 0$

**40.** In a quadratic programming problem, the optimal point is

(A) on the boundary of the feasible region

(B) inside the feasible region

(C) an extreme point

(D) either on the boundary or inside the feasible region

**41.** Consider the following LPP:

$Max\ Z = -5x_1 + 5x_2 + 13x_3$

Subject to $-x_1 + x_2 + 3x_3 \leq 20$

$12x_1 + 4x_2 + 10x_3 \leq 90$

$x_1, x_2, x_3 \geq 0.$

If the value of the upper bound 20 of the first constraint is changed to $20 + \delta$, then the range of $\delta$ for which the optimal solution remains same is

(A) [–10, 5/2]

(B) [0, 70]

(C) [–20, 5/2]

(D) [0, 20]

**42.** For $n \in \mathbb{N}$, let $f_n : [0, 1] \rightarrow \mathbb{R}$ be the function defined by $f_n(x) = \frac{1}{1+e^{nx^2}}$, then $\lim_{n\to\infty} \int_0^1 f_n(x)\,dx = 0$ by using

(A) Fatou's lemma

(B) Fubini's Theorem

(C) Positivity for each $f_n$

(D) Lebesgue dominated convergence theorem

**43.** Let $E$ be a measurable set in $\mathbb{R}$ and $f : E \rightarrow \mathbb{R}^*$ be a function on $E$ to extended real line. Which one of the following is not a sufficient condition for measurability of $f$?

(A) $f^{-1}(\alpha)$ is a measurable set $\forall \alpha \in \mathbb{R}$

(B) $f^{-1}(-\infty, \alpha)$ is a measurable set $\forall \alpha \in \mathbb{R}$

(C) $f^{-1}(\alpha, \infty)$ is a measurable set $\forall \alpha \in \mathbb{R}$

(D) $f^{-1}(G)$ is a measurable set for all open set $G$ in $\mathbb{R}$

**44.** An absent-minded person has to put five personal letters in five addressed envelops, and he does it at random. The probability that exactly three letters will be placed correctly is

(A) $\frac{1}{6}$

(B) $\frac{1}{12}$

(C) $\frac{1}{15}$

(D) None of the above

**45.** Suppose that 5% of men and 25% of women are colour-blind. A person is chosen at random and that person is colour-blind. What is the probability that the person is a male? (Assume that males and females to be in equal numbers).

(A) $\frac{1}{5}$

(B) $\frac{1}{3}$

(C) $\frac{2}{5}$

(D) $\frac{1}{6}$

**46.** Let $x_1$, $x_2$ be random samples from the exponential distribution $f(x) = e^{-x}$, $0 < x < \infty$.

Which of the following is true?

(A) $Z = x_1 + x_2$ and $x_1/x_2$ are independent

(B) $x_1\ x_2$ follows $F$-distribution

(C) $x_1 / x_2$ follows $t$-distribution

(D) $Z (= x_1 + x_2)$ follows $F$-distribution

**47.** If the joint distribution function of $X$ and $Y$ is given by

$$F(x, y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-(x+y)}, & x > 0, y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Then the random variables $X$ and $Y$ are

(A) independent

(B) dependent

(C) dependent and the correlation is 0·5

(D) None of the above

**48.** Let $x_1, x_2, ..., x_n$ $(n > 1)$ be a random sample from a Poisson($\theta$) population, $\theta > 0$ and $T = \sum_{i=1}^{n} x_i$ . Then the UMVUE of $\theta^2$ is

(A) $\frac{T(T-1)}{n^2}$

(B) $\frac{T(T-1)}{n(n-1)}$

(C) $\frac{T(T-1)}{n(n+1)}$

(D) $\frac{T^2}{n^2}$

**49.** Let $x_1, x_2, ..., x_n$ be $n$ observations which follow Bernoulli distribution with parameter $p$. $p$ also follows uniform (0, 1). What will be the Bayes estimate of $p$ under squarred error loss?

(A) $\frac{\sum x_i + 1}{n+2}$

(B) $\frac{\sum x_i + 1}{n+1}$

(C) $\frac{\sum x_i + 2}{n+1}$

(D) None of the above

**50.** Let $X$ be a random variable whose probability mass functions $f_0(x)$ (under the null hypothesis $H_0$) and $f_1(x)$ (under the alternative hypothesis) are given by

| $X = x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f_0(x)$ | 0·4 | 0·3 | 0·2 | 0·1 |
| $f_1(x)$ | 0·1 | 0·2 | 0·3 | 0·4 |

For testing the null hypothesis $H_0 : x \sim f_0$ against the alternative $H_1 : x \sim f_1$, consider the test given by : Reject $H_0$ if $x > \frac{3}{2}$. If $\alpha$ = size of the test and $\beta$ = power of the test, then

(A) $\alpha = 0·3$, $\beta = 0·3$

(B) $\alpha = 0·3$, $\beta = 0·7$

(C) $\alpha = 0·7$, $\beta = 0·3$

(D) $\alpha = 0·7$, $\beta = 0·7$

**51.** For the contingency table

| | $A_1$ | $A_2$ |
|---|---|---|
| $B_1$ | 10 | 5 |
| $B_2$ | 5 | 5 |

The value of Pearsonian $\chi^2$-statistic is

(A) $\frac{5}{36}$

(B) $\frac{5}{9}$

(C) $\frac{25}{9}$

(D) $\frac{25}{36}$

**52.** Let $U_1, U_2, ..., U_n$ be iid $U(0, 1)$. Then the asymptotic variance of $Y_n = \left(\prod_{i=1}^{n} U_i\right)^{-\frac{1}{n}}$ is

(A) $e$

(B) $e^2$

(C) $\frac{e}{n}$

(D) $\frac{e^2}{n}$

**53.** If $\underset{\sim}{X} \sim N(\underset{\sim}{\mu}, \Sigma)$, then the characteristic function of $\underset{\sim}{X}$ *is*

(A) $\exp\left(-it'\underset{\sim}{\mu} - \frac{1}{2}t'\sum t\right)$

(B) $\exp\left(it'\underset{\sim}{\mu} - \frac{1}{2}t'\sum t\right)$

(C) $\exp\left(it'\mu + \frac{1}{2}t'\sum t\right)$

(D) $\exp\left(-it'\mu + \frac{1}{2}t'\sum t\right)$

**54.** Let A ~ Wishart $(n, I_p)$ and Y be a p-component random vector. Then $\frac{Y'Y}{Y'A^{-1}Y}$ follows

(A) $x^2{}_{n-p+1}$

(B) $\frac{p}{n-p+1}.F_{p,n-p+1}$

(C) $Beta\left(\frac{n-p+1}{2}, \frac{p}{2}\right)$

(D) None of the above

**55.** The regression line of X on Y

(A) minimizes the total of squares of horizontal deviations

(B) minimizes the total of squares of vertical deviations

(C) minimizes the total of squares of both horizontal and vertical deviations

(D) minimizes the total of squares of perpendicular distances of the points from the regression lines

**56.** Consider a linear model $E(Y) = X\beta, \text{cov}(Y) = \sigma^2.I$, where $X$ is the design matrix of order $n \times p$ $(n > p)$ having rank $r \le p$. Then which of the following statements is necessarily true?

(A) The set of estimable linear functions form a vector space of dimension $r$

(B) If $E(C'Y) = 0$ for some non zero vector $C$, then there exists a function $l'\beta$ which is not estimable, for some non-zero vector $l$

(C) If all linear functions $l'\beta$ are estimable, then, $r \ne p$

(D) The set of functions $C'Y$ with $E(C'Y) = 0$ form a vector space with dimension $r$

**57.** A simple random sample of size $n$ is drawn with replacement (SRSWR) from a population of $N$ units. The expected number of distinct units in the sample is

(A) $n.\left[1-\left(\frac{N-1}{N}\right)^n\right]$

(B) $n.\left[1-\left(\frac{N-2}{N}\right)^n\right]$

(C) $N.\left[1-\left(\frac{N-1}{N}\right)^n\right]$

(D) $N.\left[1-\left(\frac{N-2}{N}\right)^n\right]$

**58.** Consider a population of eight households, say *a, b, c, d, e, f, g* and *h*. For the sample of size 3 and sampling interval 3 of circular systematic sampling, the possible systematic samples will be

(A) *adg, beh, cfa, dgb, ehc, fad, gbe* and *hcf*

(B) *aeh, bfa, cgb, dhg*

(C) *afc, chd*

(D) None of the above

**59.** A necessary condition for the existence of a BIBD with their usual notation

(A) $vr = bk$ and $b \geq v$

(B) $vr = bk$, $\lambda(v-1) = r(k-1)$

(C) $vr = bk$, $\lambda(v-1) = r(k-1)$ and $b \geq v$

(D) $vr = bk$, $\lambda v = r(k-1)$

**60.** Consider a randomised block design with $v = 4$ treatments and $b = 5$ blocks and for $1 \leq i \leq 4$, let $\tau_i$ denote the effect of the $i$ th treatment. Consider the following functions of treatment effects $\tau_1 - \tau_2$, $\tau_1 + \tau_2 - 2\tau_3$ and $\tau_1 + \tau_2 + \tau_3 - 3\tau_4$. The variance-covariance matrix of their BLUEs is $\frac{\sigma^2}{5}$ times.

(A) $I_3 - \frac{1}{3}J_3$

(B) $I_3$

(C) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 12 \end{pmatrix}$

(D) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 9 \end{pmatrix}$

**61.** For the first order Auto-regressive series $U_{t+1} = aU_t + \in_{t+1}, |a| < 1$, where $\in_t$'s are independent with zero mean, the kth correlogram is

(A) $a^k$

(B) $a^{-k}$

(C) $a^{\frac{1}{k}}$

(D) $a^{-\frac{1}{k}}$

**62.** The correlation between X($t$) and X($t + \tau$) is same as that between X($t$) and X($t - \tau$), then

(A) $\rho(\tau) = \rho(-\tau)$

(B) $\rho(\tau) \neq \rho(-\tau)$

(C) $\rho(\tau) = -\rho(-\tau)$

(D) None of the above

Where $\rho(\tau)$ represents the autocorrelation.

**63.** Consider a Markov chain with transition probability matrix $P = ((p_{ij}))$. Suppose $p_{ii} = 0 \ \forall$ states $i$. Then the Markov chain is

(A) always irreducible with period 1

(B) may be reducible and may have period 1

(C) may be reducible but period is always 1

(D) always irreducible but may have period >1

**64.** Consider a Markov chain with state space S = {0, 1, 2} and with transition probability matrix $p$ given by

$$\begin{pmatrix} \frac{2}{3} & 0 & \frac{1}{3} \\ 1 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}.$$

Then

(A) 1 is recurrent state

(B) 2 is recurrent state

(C) 0 is recurrent state

(D) None of the above

**65.** Net reproduction rate is computed by the formula with their usual notation

(A) $NRR = \sum_{W_1}^{W_2} f_{i_x} . f_{p_0}$

(B) $NRR = \sum_{W_1}^{W_2} f_{i_x} . f_{p_x}$

(C) $NRR = \sum_{W_1}^{W_2} f_{i_0} . f_{p_0}$

(D) $NRR = \sum_{W_1}^{W_2} f_{i_0} . f_{p_x}$

**66.** Force of mortality at age $x$, $\mu_x$ is

(A) $\frac{1}{l_x} . \frac{dlx}{dx}$

(B) $\frac{dlx}{dx}$

(C) $-\frac{dlx}{dx}$

(D) $-\frac{1}{l_x} . \frac{dlx}{dx}$

**67.** A system consisting of $n$ components will function, if and only if, at least one of the $n$ components functions. Suppose that all the $n$ components of the system function independently, each with probability $\frac{3}{4}$. If the probability of functioning of the system is $\frac{63}{64}$, then the value of $n$ is

(A) 2

(B) 4

(C) 3

(D) 5

**68.** The Average Sample Number (ASN) in a single sampling inspection plan for attribute is

(A) $n \cdot L(p) + N.[1 - L(p)]$

(B) $n \cdot L(p)$

(C) $n$

(D) $N.[1 - L(p)]$

**69.** In ($s$, $S$) inventory model, $s$ and $S$ stand for

(A) $s$ is the safety stock level and $S$ is the set up cost

(B) $s$ is the set up cost and $S$ is the inventory level

(C) $s$ is the set up cost and $S$ is the sum of all other costs

(D) $s$ is the safety stock level and $S$ is the maximum replenishment level

**70.** The consumption problem is defined as follows:

$$\max \sum_{t=0}^{T} b^t \log(C_t) \text{ such that}$$

$K_{t+1} = AK_t^a - C_t \geq 0,\ t = 0, 1, ..., T$, where $C_t$ is consumption in period $t$, $K_t$ is the capital in period $t$, $A > 0$, $0 < a < 1$, $0 < b < 1$ are constants. Then Bellman equation with same constraint is

(A) $V_t(K_t) = \max\{\log(C_t) + bV_{t+1}(K_{t+1})\}$

(B) $V_t(K_t) = \max\{a \log(C_t) bV_{t+1}(K_{t+1})\}$

(C) $V_t(K_t) = \max\{a \log(C_t) + bV_{t+1}(K_t)\}$

(D) $V_t(K_t) = \max\{\log(C_t) + K_t V_{t+1}(K_{t+1})\}$

**71.** A space curve is a plane curve if

(A) $\tau = 0$

(B) $k = 0$

(C) $\tau \neq 0$

(D) $k \neq 0$

($\tau$: torsion, $k$: curvature of the curve)

**72.** Let (X, $d$) be a metric space. Choose the false one.

(A) X is compact implies X is countably compact.

(B) X is countably compact implies X is compact.

(C) X is compact implies X is complete.

(D) X is Lindelöf implies X is compact.

**73.** Consider the matrix $A = \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$

where $a$, $b$, $c$ are non-zero real numbers. Then the matrix $A$ has.

(A) only one non-zero real eigenvalue.

(B) two non-zero real eigenvalues.

(C) three non-zero real eigenvalues.

(D) complex eigenvalues.

**74.** Let $f : X \to Y$ be a quotient map (X, Y being topological spaces). Then

(A) $f$ is always an open map

(B) $f$ is always a closed map

(C) $f$ is always an open as well as a closed map

(D) $f$ may be neither open nor closed

**75.** Let $f : \mathbb{R} \to \mathbb{R}$ be a monotonic function. Then

(A) $f$ has no discontinuities

(B) $f$ has only finitely many discontinuities

(C) $f$ can have at most countably many discontinuities

(D) $f$ can have uncountably many discontinuities

## ROUGH WORK

**ROUGH WORK**