61212

ROLL No. [ ][ ][ ][ ][ ]

TEST BOOKLET No. **661**

TEST FOR POST GRADUATE PROGRAMMES

MATHEMATICS

Time: 2 Hours

Maximum Marks: 450

INSTRUCTIONS TO CANDIDATES

1. You are provided with a Test Booklet and an Optical Mark Reader (OMR) Answer Sheet to mark your responses. Do not soil the Answer Sheet. Read carefully all the instructions given on the Answer Sheet.
2. Write your Roll Number in the space provided on the top of **this page.**
3. Also write your Roll Number, Test Code, and Test Subject in the columns provided for the same on the **Answer Sheet**. Darken the appropriate bubbles with a **Ball Point Pen.**
4. The paper consists of 150 objective type questions. All questions carry equal marks.
5. Each question has four alternative responses marked **A, B, C** and **D** and you have to **darken** the bubble fully by a **Ball Point Pen** corresponding to the correct response as indicated in the example shown on the Answer Sheet.
6. Each correct answer carries **3** marks and each wrong answer carries **1** minus mark.
7. Please do your rough work only on the space provided for it at the end of this Test Booklet.
8. You should return the Answer Sheet to the Invigilator before you leave the examination hall. However, you can retain the Test Booklet.
9. Every precaution has been taken to avoid errors in the Test Booklet. In the event of such unforeseen happenings the same may be brought to the notice of the Observer/Chief Superintendent in writing. Suitable remedial measures will be taken at the time of evaluation, if necessary.

SEAL

MATHEMATICS

1. $\lim_{x \to \pm 2} f(x)$ where $f(x) = \lim_{t \to 7}\left(\frac{x^2 + 7t}{t + 7}\right)$ is

   (A) $\frac{53}{14}$
   (B) $\frac{51}{14}$
   (C) $\frac{-53}{14}$
   (D) $\frac{-51}{9}$

2. The number of prime factors of $(3 \times 5)^{12} (2 \times 7)^{10} (10)^{25}$ is

   (A) 47
   (B) 60
   (C) 72
   (D) 94

3. How many $\frac{1}{8}$s are there in $37\frac{1}{2}$?

   (A) 300
   (B) 350
   (C) 400
   (D) 440

4. If Ram gains 70 paise on Rs.70, then Ram's gain percent is

   (A) 0.1%
   (B) 1%
   (C) 7%
   (D) 10%

5. If $f(x) = x^3 + 5x^2 + 29x + 7$, then in $(-\infty, \infty)$

   (A) $f(x)$ is strictly increasing
   (B) $f(x)$ has a local maxima
   (C) $f(x)$ is strictly bounded
   (D) $f(x)$ is bounded

6. The smallest number to be added to 1000 so that 45 divides the sum exactly is

   (A) 10
   (B) 35
   (C) 20
   (D) 80

7. The HCF of $\frac{9}{10}, \frac{12}{25}, \frac{18}{35}$ and $\frac{21}{40}$ is

(A) $\frac{3}{5}$
(B) $\frac{5}{252}$
(C) $\frac{3}{2800}$
(D) $\frac{63}{700}$

8. If ω is a non-real cube root of unity and $(1+\omega^2)^n = (1+\omega^4)^n$, then the least positive value of n is

(A) 6
(B) 5
(C) 3
(D) 2

9. For every $x, x^2 + 2ax + (10-3a) > 0$. Then the interval in which 'a' lies is

(A) $2 < a < 5$
(B) $a > 5$
(C) $-5 < a < 2$
(D) $a < -5$

10. If $f(x) = \sin x + \cos x$; $g(x) = x^2 - 1$, then $g(f(x))$ is bijective in the domain

(A) $[0, \pi]$
(B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(C) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
(D) $\left[0, \frac{\pi}{2}\right]$

11. If $A = \begin{pmatrix} \alpha & 2 \\ 2 & \alpha \end{pmatrix}$ and $|A^3| = 125$, then the value of α is

(A) ± 5
(B) ± 3
(C) ± 2
(D) ± 1

12. If $f(x)$ is differentiable and strictly increasing, then the value of $\lim_{x \to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is

(A) 2
(B) 1
(C) 0
(D) -1

13. If $\frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1$ and $\frac{a}{p}+\frac{b}{q}+\frac{c}{r}=0$, then $\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}$ is equal to

(A) 0 (B) 2
(C) 1 (D) 3

14. The value of $\int_0^1 \sqrt{\frac{1-x}{1+x}}dx$ is

(A) -1 (B) 1
(C) $\frac{\pi}{2}-1$ (D) $\frac{\pi}{2}+1$

15. If the line $2x+\sqrt{6}y=2$ touches the hyperbola $x^2-2y^2=4$, then the point of contact is

(A) $(-2,6)$ (B) $(-5,2\sqrt{6})$
(C) $\left(\frac{1}{2},\frac{1}{\sqrt{6}}\right)$ (D) $(4,-\sqrt{6})$

16. The area enclosed between the curves $y=ax^2$ and $x=ay^2\,(a>0)$ is 1 square unit. Then the value of 'a' is

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$
(C) 1 (D) $\frac{1}{3}$

17. The sides of a triangle are in the ratio $1:\sqrt{3}:2$. Then the angles of the triangle are in the ratio

(A) 1 : 3 : 5 (B) 2 : 3 : 4
(C) 3 : 2 : 1 (D) 1 : 2 : 3

18. The area of the triangle formed by the lines $x+y=2$; x-axis and $x=1$ is

(A) 2
(B) 1
(C) $\frac{1}{2}$
(D) $\frac{1}{4}$

19. If $x>0$, then $\sqrt[3]{x\sqrt{x\sqrt{x....\text{to } \infty}}}$ is equal to

(A) $\sqrt{x}$
(B) $x^{2/3}$
(C) $x^{3/2}$
(D) x

20. The locus of a moving point originating from (1, 1) whose slope varies twice as the ordinate is

(A) $y=2x-1$
(B) $y=e^{2x-2}$
(C) $y=1+\log x$
(D) $y=1+\sin \pi x$

21. A function $f(x)$ satisfies $f(x+y)=f(x).f(y)$, $\forall\ x,y$ with $f(1)=3$ and $\sum_{x=1}^{n} f(x)=120$. Then the value of 'n' is

(A) 4
(B) 5
(C) 6
(D) 10

22. The equation $|z|^2-2|z|+1=0$ has

(A) no solution
(B) only one real solution
(C) two complex conjugate solutions
(D) infinite number of solutions

23. The value of $\left(\frac{1+i\sqrt{5}}{2}\right)^{17}+\left(\frac{1-i\sqrt{5}}{2}\right)^{17}$ is

(A) $6^{17/2}\cos\left(17\tan^{-1}\left(\sqrt{5}\right)\right)$

(B) $6^{17/2}\sin\left(17\tan^{-1}\left(\sqrt{5}\right)\right)$

(C) $6^{17/2}\tan\left(17\cos^{-1}\left(\sqrt{5}\right)\right)$

(D) $6^{17/2}\cos\left(17\sin^{-1}\left(\sqrt{5}\right)\right)$

24. The roots of the equation $(z-2)^2+49=0$ are

(A) $2+7i$ and $7+2i$

(B) $2-7i$ and $7+2i$

(C) $2+7i$ and $2-7i$

(D) $7+2i$ and $7-2i$

25. $\alpha+2i$ and $2+\beta i$ lie on the circle $x^2+y^2=\sqrt{53}$. Then the maximum value of $\alpha-\beta$ is

(A) 0

(B) 7

(C) 14

(D) 21

26. $\lim_{z\to 0}\left(\frac{\bar{z}}{z}\right)$ in the direction of the line $y=7x$ is

(A) $1-\frac{7}{24}i$

(B) $\frac{7}{24}-i$

(C) $\frac{7}{24}+i$

(D) $1+\frac{7}{24}i$

27. The value of $\lim_{z\to i}\left(\frac{z^{10}+1}{z^6+1}\right)$ is

(A) $\frac{5}{4}$

(B) $\frac{5}{3}$

(C) $\frac{5}{2}$

(D) 5

28. The value of the integral $\int_{|z|=\frac{1}{2}} \log(17z+29)\,dz$ is

(A) zero

(B) $\frac{17}{29}$

(C) $\frac{17}{29}i$

(D) $-\frac{17}{29}$

29. The residue of the function $\frac{\cos^2 z}{z^3}$ at its finite pole is

(A) 1

(B) $\frac{1}{2}$

(C) 0

(D) $-\frac{1}{2}$

30. The transformation $\omega = z^3 + z$ has

(A) three fixed points

(B) two fixed points

(C) one fixed point

(D) no fixed points

31. The fixed points of the transformation $z^2 - z$ are

(A) 0,0

(B) 0,2

(C) 1,0

(D) 1,2

32. A bi-linear transformation which has no finite fixed point is

(A) $\frac{z^2+1}{z^2-1}$

(B) $\frac{z+1}{z-1}$

(C) $\frac{1}{z}$

(D) $z+17$

33. A bi-linear transformation which has only one fixed point is

(A) $\frac{z+1}{z+3}$ (B) $\frac{z+3}{z+1}$

(C) $\frac{z-3}{z-1}$ (D) $\frac{z-1}{z+3}$

34. A bi-linear transformation which takes the points 2 and 7 respectively to 0 and ∞ is

(A) $\frac{z^2-4}{z-7}$ (B) $\frac{z-2}{z-7}$

(C) $\left(\frac{z-2}{z-7}\right)^2$ (D) $\frac{z-7}{z-2}$

35. The minimum value of $3\tan^2\theta+12\cot^2\theta$ is

(A) 6 (B) 15

(C) 243 (D) None of the above

36. For the function $e^{-\frac{1}{z}}$, $z=\infty$ is

(A) a pole (B) a removable

(C) an essential singularities (D) a regular point

37. The angle between the images of the lines $x=2$ and $y=7$ under the function $f(z)=z^2+3z$ at $f(2+7i)$ is

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

38. The locus of a complex number z such that $\operatorname{Re} z=|z-1|$ is

(A) parabola (B) hyperbola

(C) eclipse (D) circle

39. If $f(x)=(x+\sin x)^{\tan x}$, then the value of $\lim_{x\to 0} f(x)$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$
(C) 1 (D) 0

40. If $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$, then the value of $b-a$ is

(A) 2 (B) 1
(C) 0 (D) –2

41. If $\log\left(\frac{1+7x}{1+2x}\right)=\sum_{n=1}^{\infty}a_n x^n$, $|x|<\frac{1}{7}$, then a_3 is

(A) $\frac{335}{2}$ (B) $\frac{334}{2}$
(C) $\frac{331}{2}$ (D) 5

42. If $\sin x=\sum_{n=0}^{\infty}a_n\left(x-\frac{\pi}{2}\right)^n$ then the value of a_4-a_2 is

(A) $\frac{11}{24}$ (B) $\frac{13}{24}$
(C) $\frac{15}{24}$ (D) $\frac{17}{24}$

43. If $x^5+2x^4-x^2+x+1=\sum_{n=0}^{5}a_n(x+1)^n$, the value of $a_2+a_3+a_4$ is

(A) 3 (B) 2
(C) 1 (D) 0

44. The angle between the asymptotes of the hyperbola $(3x+y-1)(x-3y+3)+7=0$ is

(A) 30°
(B) 45°
(C) 60°
(D) None of the above

45. If $u(x,y)=\dfrac{x^5+xy^4-y^5}{3x^3-4x^2+5y^3}$, the value of $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}$ at the point $(1,1)$ is

(A) $\dfrac{1}{4}$
(B) $\dfrac{1}{2}$
(C) 0
(D) $-\dfrac{1}{2}$

46. If $f(z)=u+iv=z^3+3z^2-14$, then the value of $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{dy^2}$ is

(A) $3x^2$
(B) $3y^2$
(C) 3
(D) 0

47. If $A+B+C=\pi$, then $\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix} =$

(A) 0
(B) 2
(C) -1
(D) None of the above

48. The value of $\int\{(\cos x - x\cos y)\,dy - (\sin y + y\sin x)\,dx\}$ over a closed curve C is

(A) 0
(B) $x.\cos y$
(C) $y.\sin x$
(D) 3

49. A particular integral of the equation $y''+2y'+y=5$ is

(A) 0
(B) 1
(C) 2
(D) 5

50. If $\overline{a}+\overline{b}+\overline{c}=0$, then $\overline{a}\times\overline{b}=$

(A) $\overline{c}\times\overline{a}$
(B) $\overline{c}$
(C) $2\overline{b}\times\overline{c}$
(D) $\overline{a}\times\overline{c}$

51. If $\frac{dy}{dx}+y-x=0$ with $y(0)=1$, then $y(x)$ is

(A) $2e^{-x}+x-1$
(B) $2e^{-x}-x+1$
(C) $2e^{-x}+x+1$
(D) $2e^{-x}-x-1$

52. Given $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$, then the value of $\Gamma\left(\frac{5}{2}\right)$ is

(A) $\frac{5}{4}\sqrt{\pi}$
(B) $\frac{3}{4}\sqrt{\pi}$
(C) $\frac{\sqrt{\pi}}{2}$
(D) $\frac{\sqrt{\pi}}{4}$

53. The value of $\int_0^{\pi/2}\sin^8\theta d\theta$ is

(A) $\frac{35}{256}$
(B) $\frac{35\pi^{\circ}}{256}$
(C) $\frac{35}{128}$
(D) $\frac{35\pi}{128}$

54. The value of $\int_0^{\pi/2}\cos^7\theta d\theta$ is

(A) $\frac{16}{35}$
(B) $\frac{16\pi}{35}$
(C) $\frac{8}{35}$
(D) $\frac{8\pi}{35}$

55. If $y''+4y=0$, with $y(0)=0$ and $y\left(\pi/4\right)=1$, then $y(x)$ is

(A) $\cos 2x$
(B) $\sin 2x$
(C) $2\cos 2x$
(D) $2\sin 2x$

56. If $f(x)=\dfrac{x}{x-1}$, then $f(3x)$ is equal to

(A) $\dfrac{3f(x)}{3f(x)-1}$
(B) $\dfrac{3f(x)}{3f(x)-3}$
(C) $\dfrac{3f(x)}{2f(x)+1}$
(D) $3f(x)-1$

57. The radius of convergence of the series $\sum_{0}^{\infty}\left(\dfrac{n^2-1}{n^2+1}.x^n\right)$ is

(A) 1
(B) 2
(C) 3
(D) 4

58. A line makes angles 30° and 60° with the y and z axes respectively. Then the angle made by that line with the x-axis is

(A) 30°
(B) 45°
(C) 60°
(D) 90°

59. The angle between the line joining the points (8, 2, 0) and (4, 6, −7) and the line joining the points (−3, 1, 2) and (−9, −2, 4) is

(A) $\cos^{-1}\left(\dfrac{2}{63}\right)$
(B) $\sin^{-1}\left(\dfrac{2}{63}\right)$
(C) $\cos^{-1}\left(\dfrac{-2}{63}\right)$
(D) $\sin^{-1}\left(-\dfrac{2}{63}\right)$

60. The angle between the lines $\frac{x}{3}=\frac{y}{1}=\frac{z}{0}$ and the plane $x+2y-7=0$ is

(A) 0
(B) 45°
(C) 60°
(D) 90°

61. The lines $\frac{x}{1}=\frac{y+3}{2}=\frac{z+1}{3}$ and $\frac{x-3}{2}=\frac{y}{1}=\frac{z-1}{-1}$

(A) are parallel
(B) intersect at $(1,1,2)$
(C) intersect at $(1,-1,2)$
(D) are perpendicular

62. The lines $x+y-3z=0,\ 2x+3y-8z=1$ and $3x-y-z=3,\ x+y-3z=5$

(A) are parallel
(B) are perpendicular
(C) intersect
(D) intersect at $(1,2,1)$

63. The angle between any two diagonals of a cube is

(A) $\tan^{-1}\left(\frac{1}{3}\right)$
(B) $\sin^{-1}\left(\frac{1}{3}\right)$
(C) $\cos^{-1}\left(\frac{1}{3}\right)$
(D) $\cot^{-1}\left(\frac{1}{3}\right)$

64. The line joining the points $(-2,3,6)$, $(5,2,7)$ and the line joining points $(0,-1,-2)\ (14,-3,0)$ are

(A) perpendicular
(B) parallel
(C) skew
(D) same

65. The 4 points $(1,1,0),(1,2,1),(-2,2,-1)$ and $(1,-1,-2)$ lie on the plane

(A) $2x+3y-3z=5$
(B) $2x-3y+3z=5$
(C) $2x-3y-3z=5$
(D) $2x+3y+3z=5$

66. The equation to the plane through the intersection of the planes $x+y-z+3=0,\ 2x-y+z-1=0$ and passing through the point $(1,-1,2)$ is

(A) $2x+5y+5z+13=0$ (B) $2x+5y-5z+3=0$
(C) $2x-5y-5z+13=0$ (D) $2x-5y-5z-13=0$

67. A girl is looking at the mirror straight from the origin. Her image is at the point $(12,-4,-3)$. Then the equation of the mirror (taken as a plane) is

(A) $12x-4y-3z=169$ (B) $12x-4y-3z=144$
(C) $12x-4y-3z=-169$ (D) $12x-4y-3z+144=0$

68. The 10^{th} term of the series $1+\frac{1^2+2^2}{1+2}+\frac{1^2+2^2+3^2}{1+2+3}+....$ is

(A) 7 (B) 11
(C) 15 (D) $\frac{2}{3}$

69. If A and B are idempotent matrices, then $A+B$ will be idempotent if

(A) $AB=0$ (B) $BA=0$
(C) $AB=BA=0$ (D) None of the above

70. The unit vectors perpendicular to $a\hat{i}$ and $a\hat{j}$ are

(A) $\pm 2\hat{k}$ (B) $\pm\hat{k}$
(C) $a^2\hat{k}$ (D) $\pm a\hat{k}$

71. If $\left(1+\frac{1}{2!}+\frac{1}{4!}+....\right)^2=K+\left(1+\frac{1}{3!}+\frac{1}{5!}+....\right)^2$, then the value of K is

(A) 3 (B) 2
(C) 1 (D) 0

72. The coefficient of x^{100} in the expansion $1+\frac{(1-x)}{1!}+\frac{(1-x)^2}{2!}+\frac{(1-x)^3}{3!}+....$ is

(A) $\frac{e}{100!}$ (B) $\frac{2e}{100!}$

(C) $\frac{3e}{100!}$ (D) $\frac{4e}{100!}$

73. The coefficient of x^{29} in the expansion $\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+....\right)^2$ is

(A) $\frac{2^{28}}{29!}$ (B) $\frac{2^{29}}{29!}$

(C) 29 (D) 0

74. The sum of the series $1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}....$ is

(A) $\frac{5}{4}$ (B) $\frac{7}{4}$

(C) $\frac{9}{4}$ (D) $\frac{11}{4}$

75. If $k+\left[3-2+3.\frac{1}{3}-4\left(\frac{1}{3}\right)^2+5\left(\frac{1}{3}\right)^2-6\left(\frac{1}{3}\right)^3+....\right]=27$, then the value of k is

(A) 4 (B) 16

(C) 64 (D) 256

76. The series $1+\frac{(7x)}{1}+\frac{(7x)^2}{2!}+\frac{(7x)^3}{3!}+....$

(A) converges for all (B) converges if $|x|<1$

(C) diverges for all x (D) oscillates

77. The series $\frac{1}{3}+\frac{1}{3}\left(\frac{1}{3}\right)^3+\frac{1}{5}\left(\frac{1}{3}\right)^5+....$

(A) converges to $\log 2$

(B) converges to $\frac{1}{2\log 2}$

(C) converges to $\frac{1}{4}\log 2$

(D) diverges

78. $(\bar{a}\times\bar{b})\times\bar{c}$ lies on the plane containing the vertices

(A) $\bar{a}$ and $\bar{b}$

(B) $\bar{a}$ and $\bar{c}$

(C) $\bar{b}$ and $\bar{c}$

(D) $\bar{a},\bar{b}$ and $\bar{c}$

79. The value of $\frac{1}{2}-2.\frac{1}{3}+3.\frac{2}{3^2}-4.\frac{2^2}{3^3}+5.\frac{2^3}{3^4}-....$ is

(A) $\frac{3}{50}$

(B) $\frac{7}{50}$

(C) $\frac{9}{50}$

(D) $\frac{11}{50}$

80. The value of $\lim_{x\to 0}\left(\frac{1+x}{1-x}\right)^{\frac{1}{x}}$ is

(A) e

(B) e^2

(C) $\frac{1}{e}$

(D) $\frac{1}{e^2}$

81. If $\frac{1}{2x+7}=a+bx+....$ then $a+b$ is

(A) $\frac{2}{49}$

(B) $\frac{3}{49}$

(C) $\frac{4}{49}$

(D) $\frac{5}{49}$

82. $\lim_{x \to 0} \frac{\sinh x}{\log(1+x)}$ is

(A) 0 (B) 1
(C) 2 (D) 3

83. When x is large enough and if $\sqrt{x^2+16}+\sqrt{x^2+9}=1+ax+....$, then the value of '$a$' is

(A) $-\frac{7}{24}$ (B) 0
(C) $\frac{7}{24}$ (D) $\frac{1}{6}$

84. The curve $y=x^4-6x^3+12x^2-8x$ is convex upwards in

(A) $(-\infty,1)$ (B) $(-2,-1)$
(C) $(1,2)$ (D) $(2,\infty)$

85. If $x=e^{i\theta}$, then the value of $x^{17}+\frac{1}{x^{17}}$ is

(A) $2\sin 17\theta$ (B) $2\cos 17\theta$
(C) $2\cot 17\theta$ (D) $2\tan 17\theta$

86. If $\frac{\sin 5\theta}{\sin\theta}=\alpha\sin^4\theta+\beta\sin^2\theta+\gamma$, then the value of $\alpha+\beta+\gamma$ is

(A) 0 (B) 1
(C) 2 (D) -1

87. The value of $\lim_{x \to 0} \frac{3\sin x-\sin 3x}{x[\cos x-\cos 3x]}$ is

(A) 0 (B) 1
(C) 2 (D) -1

88. If $\tan^{-1}(\alpha)+\tan^{-1}(\beta)=\frac{\pi}{4}$, then the value of $\left(\frac{\alpha+\beta}{1-\alpha\beta}\right)^3$ is

(A) -1 (B) 0
(C) 1 (D) 2^3

89. If $\left((i)^i\right)^{\cdot^{\cdot^{\cdot \text{ to } \infty}}}=\alpha+i\beta$ then the value of $\alpha^2+\beta^2$ is

(A) $e^{-\pi\beta}$ (B) $\frac{1}{2}e^{-\pi\beta}$
(C) $e^{-\pi\alpha}$ (D) $\frac{1}{2}e^{-\pi\alpha}$

90. The rank of the matrix $\begin{pmatrix}1 & -2 & 3 & 4\\ 3 & 1 & 0 & 3\\ 5 & 4 & -3 & k\end{pmatrix}$ is 2. Then the value of 'k' is

(A) 3 (B) 2
(C) 1 (D) 0

91. The vectors $(2,3,k)$; $(1,2,0)$ and $(8,13,k)$ are linearly dependent. Then the value of k is

(A) 3 (B) $\frac{3}{2}$
(C) 1 (D) 0

92. The vectors $x_1=(2,-1,0)$; $x_2=(4,1,1)$; $x_3=(8,-1,1)$ are linearly dependent. Then the relationship between these vectors is

(A) $2x_1+x_2+x_3=0$ (B) $2x_1-x_2-x_3=0$
(C) $2x_1+x_2-x_3=0$ (D) $2x_1-x_2+x_3=0$

93. The system of equations $3x_1 + x_2 - \lambda x_3 = 0$, $4x_1 - 2x_2 - 3x_3 = 0$, $2x_1 + x_2 - x_3 = 0$ and $2\lambda x_1 + 4x_2 + \lambda x_3 = 0$ possesses non-trivial solutions. Then the values of λ are

(A) $-9,1$ (B) $-9,-1$
(C) $9,-1$ (D) $9,1$

94. The system of equations $x_1 - x_2 + x_3 = 0$, $x_1 + 2x_2 - x_3 = 0$ and $2x_1 + x_2 + 3x_3 = 0$ has

(A) a unique non-trivial solution (B) infinite numbers of solutions
(C) no non-trivial solutions (D) None of the above

95. The system of equations $2x + y + 3z = 1$; $x - y + 2z = -3$; $4x - y + 7z = k$ are inconsistent. Then the value of k is

(A) $= -5$ (B) $\neq -5$
(C) $= 0$ (D) $\neq 0$

96. The system of equations $x + y + z = a$; $x + 2y + 3z = b$ and $3x + 5y + 7z = c$ has a one parameter family solution, then

(A) $c - a - 2b = 0$ (B) $c + a - 2b = 0$
(C) $c - a + 2b = 0$ (D) $c + a + 2b = 0$

97. Two eigen values of the matrix $\begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}$ are 2 and 3. Then the cube of the third eigen value is

(A) 1 (B) 0
(C) -1 (D) -8

98. If the characteristic equation of a matrix is $\lambda^3 - 6\lambda^2 + 5\lambda + 12 = 0$, then the sum and product of its eigen values are

(A) 6, 12 (B) $6,-12$
(C) $-6,12$ (D) $-6,-12$

99. The sum of the squares of the eigen values of $\begin{pmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{pmatrix}$ is

(A) 87 (B) 74
(C) 64 (D) 50

100. If the square of one of the eigen values of a matrix is zero, then the matrix is

(A) singular (B) upper triangular
(C) lower triangular (D) invertible

101. The matrix corresponding to the quadratic form $2xy + 2yz + 2zx$ is

(A) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

102. The quadratic form corresponding to matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ is

(A) $x^2 + y^2 + z^2 + 2xy$ (B) $x^2 + y^2 + z^2 xy + yz + zx$
(C) $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ (D) $2x^2 + 2y^2 + 2z^2 + xy + yz + zx$

103. If λ is an eigen value of matrix A, then λ is also an eigen value of the matrix

(A) A^{-1}
(B) $Adj(A)$
(C) $B^{-1}AB$ for any invertible matrix B
(D) $B^T AB$ for any matrix B

104. For the matrix $A=\begin{pmatrix}2 & 1 & 5\\ 0 & 0 & 7\\ 0 & 0 & -2\end{pmatrix}$ $A^3-\alpha A=0$. Then the value of α is

(A) 4
(B) 3
(C) 2
(D) 1

105. The value of $\int_0^{\infty} e^{-2t}\cos 7t dt$ is

(A) $\frac{1}{53}$
(B) $\frac{2}{53}$
(C) $\frac{3}{53}$
(D) $\frac{4}{53}$

106. If $\int_0^{\infty} e^{-t}\cos kt\ dt=\frac{1}{65}$, then the non-negative value of k is

(A) 0
(B) 4
(C) 8
(D) 16

107. Let $\langle G,*\rangle$ be any group in which $(a*b)^2=a^2*b^2$ holds for all a,b then G is

(A) an associative group
(B) a commutative group
(C) a ring
(D) an associative ring

108. G is a group having 29 elements and S is a proper sub group of G. Then the number of elements of S is

(A) 1
(B) 2
(C) 9
(D) 29

109. For the group of 4$^{\text{th}}$ roots of unity a sub group consists of elements

(A) $\{1,i\}$
(B) $\{1,-i\}$
(C) $\{1,-1\}$
(D) $\{i,-i\}$

110. In a group of 4^{th} root of unity the order of the element -1 is

(A) 1 (B) 2
(C) 3 (D) 4

111. Let G be any group and $a \in G$. Then the set $\{a^n / n \in Z\}$ is

(A) a subset of G
(B) a sub group of G
(C) a subset but not a sub group of G
(D) neither a subset nor a subgroup of G

112. The value of $\int_c (y^2 dx - 2x^2 dy)$ along the parabola $y = x^2$ from $(0,0)$ to $(2,4)$ is

(A) $\frac{48}{5}$ (B) $\frac{24}{5}$
(C) $\frac{-24}{5}$ (D) $\frac{-48}{5}$

113. The field $\vec{F} = \left(y^2 \cos x + z^2\right)\vec{i} + (2y\sin x - 4)\vec{j} + (3xz^2 + 2)\vec{k}$ is

(A) solenoidal (B) conservative
(C) rotational (D) None of the above

114. If in the interval $0 < x < \pi$, $\sin x = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nx}{4n^2-1}$ the value of $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots$ is

(A) $\frac{\pi}{4}\left(1-\frac{2}{\pi}\right)$ (B) $\frac{\pi}{4}\left(1-\frac{4}{\pi}\right)$
(C) $\frac{\pi}{4}\left(1+\frac{2}{\pi}\right)$ (D) $\frac{\pi}{2}\left(1+\frac{4}{\pi}\right)$

115. If $xyz = \phi(x+y+z)$, then

(A) $z(x-y)\frac{\partial z}{\partial x}+x(y-z)\frac{\partial z}{\partial y}=z(z-x)$

(B) $y(z-x)\frac{\partial z}{\partial x}+z(x-y)\frac{\partial z}{\partial y}=x(y-z)$

(C) $x(y-z)\frac{\partial z}{\partial x}+y(z-x)\frac{\partial z}{\partial y}=z(x-y)$

(D) $x(y+z)\frac{\partial z}{\partial x}-y(z+x)\frac{\partial z}{\partial y}=z(x+y)$

116. The volume of the solid formed when the area between $y^2 = 4ax$ and $x^2 = 4ay$ is rotated about x-axis is

(A) $\frac{48\pi a^2}{5}$ (B) $\frac{96\pi a^3}{5}$

(C) $\frac{32\pi a^3}{5}$ (D) $\frac{16\pi a^3}{5}$

117. One of the solution of the equation $(y^2+z^2)\frac{\partial z}{\partial x}-xy\frac{\partial z}{\partial y}+zx=0$ is

(A) $x = cy$ (B) $x = c_1 z$

(C) $y = c_2 z$ (D) $y = c_3 x$

118. If the surface area of a cube is decreased by 19%, the volume of the cube decreases by

(A) 9% (B) 19%
(C) 27.1% (D) 38.2%

119. The smallest number with exactly 10 proper divisors is

(A) 52 (B) 60
(C) 72 (D) 80

120. The sum of all odd integers larger than 100 and less than 1000 is

(A) 247500
(B) 495000
(C) 990000
(D) 1100000

121. The value of α such that the planes $\alpha x + y + z = 0$; $x + 3z = 0$ and $5y + 6z = 0$ have a line in common is

(A) $\frac{11}{5}$
(B) $\frac{1}{15}$
(C) $\frac{11}{15}$
(D) $-\frac{11}{15}$

122. If 1 is added to the denominator of certain fraction it becomes $\frac{1}{3}$ and if 1 is subtracted from the denominator it becomes $\frac{1}{2}$. Then the fraction is

(A) $\frac{4}{9}$
(B) $\frac{3}{8}$
(C) $\frac{2}{5}$
(D) $\frac{1}{3}$

123. The sum of two integers is 88. If the greater number is divided by the smaller, quotient is 5 and the remainder is 10. Then the integers are

(A) 22, 66
(B) 13, 75
(C) 15, 73
(D) 14, 74

124. If $2\cos^2\alpha = \cos^2 45^\circ + \sin 30^\circ$ then $\sec^2\alpha$ is

(A) $\frac{1}{\sqrt{2}}$
(B) $\sqrt{2}$
(C) 2
(D) 1

125. The value of $\dfrac{\sin 44}{\cos 46}+\dfrac{\sin 46}{\cos 44}$ is

(A) 0
(B) 2
(C) $\sin 22^{\circ}$
(D) $\cos 22^{\circ}$

126. If $\dfrac{x^3+y^3}{x^3-y^3}=\dfrac{91}{37}$, then $\dfrac{x}{y}$ is

(A) $\dfrac{4}{3}$
(B) $\dfrac{5}{3}$
(C) $\dfrac{7}{3}$
(D) $\dfrac{8}{3}$

127. If a,b are the roots of the equation $4x^2-5x+4=0$ then the equation whose roots are a^2,b^2 is

(A) $16x^2-7x+16=0$
(B) $16x^2-7x-16=0$
(C) $16x^2+7x+16=0$
(D) $16x^2+7x-16=0$

128. The value of the integral $\int_0^1 \left(\log\dfrac{1}{y}\right)^{29} dy$ is

(A) 30!
(B) 29!
(C) $\dfrac{30!}{2}\sqrt{\pi}$
(D) $\dfrac{29!}{2}\sqrt{\pi}$

129. If the Laplace transform of the second derivative of a function $f(t)$ with $f(0)=\alpha$ and $f^{'}(0)=\beta$ is $\dfrac{s}{s^2+1}$, then the function is

(A) $\alpha+(\beta+1)t-\sin t$
(B) $\alpha-(\beta+1)t+\sin t$
(C) $\alpha+(\beta+1)t+\sin t$
(D) $\alpha-(\beta+1)t-\sin t$

130. If $\int_0^\infty \frac{\sin 2x}{x} dx = \frac{\pi}{2}$, then the value of $\int_0^\infty \frac{\sin 7x}{x} dx$ is

(A) $\frac{\pi}{7}$ (B) $\frac{\pi}{5}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

131. If the mean and variance of a random variable X are 50 and 4 respectively, then $Var(2X-3)$ is

(A) 16 (B) 4

(C) 1 (D) $\frac{1}{2}$

132. The value of the integral $\int_{|z-2|=7} \frac{dz}{(z-2)^2}$ is zero because $\frac{1}{(z-2)^2}$

(A) is not analytics at $z=2$ (B) has a residue zero at its pole

(C) is analytics inside $|z-2|=7$ (D) is a meromerphic function

133. If M and N are invertible matrices, then

(A) $M+N$ is invertible (B) $M-N$ is invertible

(C) MN is invertible (D) $M-N^{-1}$ is invertible

134. If the Laplace transform of the second derivative of $f(t)$ is $\frac{s^3}{s^2+1}$ and $f(0)=\alpha$ $f'(0)=\beta$, then $f(t)$ is

(A) $\alpha+\beta t+\cos t$

(B) $\alpha+\beta t+\sin t$

(C) $f(t)$ does not exist because $f(t)$ may not be continuous

(D) $f(t)$ does not exist because $f''(t)$ has no Laplace inverse

135. If a die is rolled three times the probability that each roll will result in a number larger than the previous roll is

(A) $\frac{12}{216}$ (B) $\frac{18}{216}$

(C) $\frac{20}{216}$ (D) $\frac{24}{216}$

136. The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. Then $P(X \geq 1) =$

(A) $\frac{727}{728}$ (B) $\frac{728}{729}$

(C) $\frac{726}{728}$ (D) $\frac{726}{729}$

137. If X has uniform distribution in $(-1,3)$ and Y has exponential distribution with parameter λ, the value of λ such that $Var(X) = Var(Y)$ is

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$

(C) $\frac{2}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$

138. For Bernoulli distribution with probability p of a success and q of a failure, the relation between mean and variance is

(A) mean < variance (B) mean > variance

(C) mean = variance (D) mean ≤ variance

139. If a point is selected at random inside an equilateral triangle of side length 3, then the probability that its distance to any corner is greater than 1 is given by

(A) $1+\frac{\pi}{5\sqrt{3}}$ (B) $1-\frac{2\pi}{9\sqrt{3}}$

(C) $1+\frac{2\pi}{\sqrt{3}}$ (D) None of these

140. A sample space consists of 3 points with associated probabilities given by $2p, p^2$ and $4p-1$. The value of p is

(A) $\sqrt{11}-3$ (B) $-\sqrt{11}-3$

(C) $\sqrt{11}+3$ (D) $-\sqrt{11}+3$

141. A coin is tossed till head appears for the first time. The probability that the number of required toss is 1 is given by

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

142. The diameter of the electric cable X is a continuous random variable with the *pdf* $f(x)=kx(1-x), 0<x<1$. The value of k is

(A) $\frac{1}{6}$ (B) 6

(C) 5 (D) $\frac{6}{5}$

143. The point of inflection of the curve $y=x^{\frac{1}{3}}$ is

(A) $(0,0)$ (B) $(1,1)$

(C) $\left(2,2^{\frac{1}{3}}\right)$ (D) $\left(3,3^{\frac{1}{3}}\right)$

144. The limit of the function $f(x,y)=\dfrac{2xy}{x^2+y^2}$ is $\dfrac{4}{5}$ along the path

(A) $y=x$
(B) $y=2x$
(C) $y=x^2$
(D) $5y-4x=0$

145. Which of the following is NOT an assumption of the Binomial distribution?

(A) All trials must be identical.
(B) All trials must be independent.
(C) Each trial must be classified as a success or a failure.
(D) The probability of success is equal to 5 in all trials.

146. The probability that a certain machine will produce a defective item is $\dfrac{1}{4}$. If a random sample of 6 items is taken from the output of this machine, what is the probability that there will be 5 or more defectives in the sample?

(A) $\dfrac{1}{4096}$
(B) $\dfrac{3}{4096}$
(C) $\dfrac{4}{4096}$
(D) $\dfrac{19}{4096}$

147. Suppose 60% of a herd of cattle is infected with a particular disease. Let Y = the number of non-diseased cattle in a sample of size 5. The distribution of Y is

(A) Binomial with $n=5$ and $p=0.6$
(B) Binomial with $n=5$ and $p=0.4$
(C) Binomial with $n=5$ and $p=0.5$
(D) Poisson with $\lambda=.6$

148. Suppose the probability that a cross between two varieties will express a particular gene is 0.20. What is the probability that in 8 progeny plants, two or fewer plants will express the gene?

(A) .2936
(B) .3355
(C) .1678
(D) None of the above

149. The value of $\int_0^\infty e^{-3t}t^3dt$ is

(A) $\frac{1}{27}$

(B) $\frac{2}{27}$

(C) $\frac{3}{29}$

(D) $\frac{5}{29}$

150. The bilinear transformation which transform 2, 10, 29 to 0, ∞, 1 respectively is

(A) $\frac{z-2}{z-10}$

(B) $\frac{19}{27}\left(\frac{z-2}{z-10}\right)$

(C) $\frac{27}{19}\left(\frac{z-2}{z-10}\right)$

(D) $\frac{29}{19}\left(\frac{z-2}{z-10}\right)$

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