61214

ROLL No. | | | | | |

TEST BOOKLET No. **591**

TEST FOR POST GRADUATE PROGRAMMES

MATHEMATICS

Time: 2 Hours

Maximum Marks: 450

INSTRUCTIONS TO CANDIDATES

1. You are provided with a Test Booklet and an Optical Mark Reader (OMR) Answer Sheet to mark your responses. Do not soil the Answer Sheet. Read carefully all the instructions given on the Answer Sheet.

2. Write your Roll Number in the space provided on the top of **this page**.

3. Also write your Roll Number, Test Code, and Test Subject in the columns provided for the same on the **Answer Sheet**. Darken the appropriate bubbles with a **Ball Point Pen.**

4. The paper consists of 150 objective type questions. All questions carry equal marks.

5. Each question has four alternative responses marked **A, B, C** and **D** and you have to **darken** the bubble fully by a **Ball Point Pen** corresponding to the correct response as indicated in the example shown on the Answer Sheet.

6. Each correct answer carries **3** marks and each wrong answer carries **1** minus mark.

7. Space for rough work is provided at the end of this Test Booklet.

8. You should return the Answer Sheet to the Invigilator before you leave the examination hall. However, you can retain the Test Booklet.

9. Every precaution has been taken to avoid errors in the Test Booklet. In the event of any such unforeseen happenings, the same may be brought to the notice of the Observer/Chief Superintendent in writing. Suitable remedial measures will be taken at the time of evaluation, if necessary.

SEAL

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MATHEMATICS

1. The term independent of x in the expansion of $\left(\sqrt[6]{x}-\frac{1}{\sqrt[3]{x}}\right)^9$ is

   (A) ${}^9C_2$ (B) ${}^9C_3$

   (C) $-{}^9C_3$ (D) $-{}^9C_2$

2. Which one of the following is a subspace of $\mathbb{R}^n$?

   (A) $\{(x_1, x_2, \ldots, x_n) | \text{either } x_1 = 0 \text{ or } x_2 = 0\}$

   (B) $\{(x_1, x_2, \ldots, x_n) | x_1 = x_2 = 0\}$

   (C) $\{(x_1, x_2, \ldots, x_n) | x_1 \neq 0\}$

   (D) $\{(x_1, x_2, \ldots, x_n) | 5x_1 - 9x_2 = 6\}$

3. $\log(-ei) =$

   (A) $1+\frac{\pi}{2}i$ (B) $1-\frac{\pi}{2}i$

   (C) $1+\pi i$ (D) $1-\pi i$

4. The order of the alternating group A_n is

   (A) $\frac{n}{2}$ (B) n

   (C) $\frac{n!}{2}$ (D) $n!$

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5. The sequence $\{x_n\}$ defined by $x_1 = \sqrt{7}$, $x_{n+1} = \sqrt{7 + x_n}$, for $n \geq 1$, converges to

(A) $\sqrt{7}$

(B) 0

(C) $\frac{1-\sqrt{29}}{2}$

(D) $\frac{1+\sqrt{29}}{2}$

6. The sum of the infinite series $\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \ldots$ is

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

7. The fixed points of the transformation $w = \frac{z-1}{z+1}$ are

(A) $\pm 2i$

(B) $1 \pm i$

(C) $\pm i$

(D) $1 \pm 2i$

8. Which one of the following is a unit in the integral domain $\mathbb{Z}\left[\sqrt{5}\right]$?

(A) $1-\sqrt{5}$

(B) $3+\sqrt{5}$

(C) $9+4\sqrt{5}$

(D) $3-2\sqrt{5}$

9. $\lim_{x\to 0} \frac{3^x - 2^x}{x} =$

(A) $e^{3/2}$

(B) $\log\frac{2}{3}$

(C) $\log\frac{3}{2}$

(D) $e^{2/3}$

10. Which one of the following statements need not be true?

(A) A convergent sequence is bounded
(B) A bounded sequence is convergent
(C) A monotonic bounded sequence is convergent
(D) A sequence is convergent if and only if it is a Cauchy sequence

11. Which one of the following subsets of $\mathbb{R}^2$ is not a basis of $\mathbb{R}^2$ over $\mathbb{R}$?

(A) $\{(1,1),(-1,-1)\}$

(B) $\{(1,-1),(-1,0)\}$

(C) $\{(0,1),(-1,0)\}$

(D) $\{(1,1),(1,-1)\}$

12. $\int_0^{\pi/4} \log(1+\tan\theta)\,d\theta =$

(A) $\frac{\pi}{2}\log 2$

(B) $\frac{\pi}{4}\log 2$

(C) $\frac{\pi}{8}\log 2$

(D) $\pi\log 2$

13. If C_n is the n^{th} binomial coefficient in the expansion of $(1+x)^n$, then $C_0+\frac{C_1}{2}+\frac{C_2}{3}+....+\frac{C_n}{n+1}=$

(A) $\frac{2^n-1}{n+1}$
(B) $\frac{2^{n+1}-1}{n+1}$
(C) $\frac{2^n-1}{n}$
(D) $\frac{2^n-1}{n+2}$

14. If S and T are finite dimensional subspaces of a vector space V over a field F, then

(A) $d[S]+d[T]=d[S\cap T]+d[S+T]$
(B) $d[S]+d[T]=d[S\cap T]+d[S\cup T]$
(C) $d[S\cap T]+d[S\cup T]=d[S]+d[S+T]$
(D) $d[S\cup T]+d[S\cap T]=d[S\cup T]+d[T]$

15. The order of the permutation $\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8\\ 2 & 5 & 4 & 7 & 1 & 8 & 3 & 6\end{pmatrix}$ is

(A) 8
(B) 4
(C) 5
(D) 6

16. The polynomial x^2+10 is reducible over the domain

(A) $\mathbb{Z}_3$
(B) $\mathbb{R}$
(C) $\mathbb{Z}_{11}$
(D) $\mathbb{Q}$

17. The function $f(x)=\frac{1+x}{1+x^2}$ decreases for

(A) $-1-\sqrt{2}<x<-1+\sqrt{2}$
(B) $x<-1-\sqrt{2}$ and $x>-1+\sqrt{2}$
(C) $-\sqrt{2}<x<\sqrt{2}$
(D) $x<-\sqrt{2}$ and $x>\sqrt{2}$

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18. Which one of the following is correct?

(A) $\pi^3 > 3^\pi$
(B) $\pi^3 < 3^\pi$
(C) $\pi^3 = 3^\pi$
(D) $\pi^3 \geq 3^\pi$

19. If S and T are linear transformations on $\mathbb{R}^2$ defined by $S(x,y) = (y,x)$ and $T(x,y) = (0,x)$, then

(A) $S^2 = S,\ T^2 = I$
(B) $S^2 = S,\ T^2 = 0$
(C) $S^2 = I,\ T^2 = 0$
(D) $S^2 = I,\ T^2 = I$

20. The set of all generators of the cyclic group $G = \langle a \rangle$ of order 8 is

(A) $\{a^2, a^4, a^6\}$
(B) $\{a, a^3, a^5, a^7\}$
(C) $\{a^4, a^8\}$
(D) $\{a^3, a^5, a^7\}$

21. The function $f(x) = \begin{cases} \cos x, & \text{for } 0 \leq x < \pi \\ \sin x - 1, & \text{for } \pi \leq x \leq 3\pi/2 \\ -3, & \text{for } x \geq 3\pi/2 \end{cases}$ *is*

(A) continuous at $x = \pi$ and $x = 3\pi/2$
(B) continuous at $x = \pi$ and discontinuous at $x = 3\pi/2$
(C) continuous at $x = 3\pi/2$ and discontinuous at $x = \pi$
(D) discontinuous at $x = \pi$ and $x = 3\pi/2$

22. $\int_0^\pi e^x \sin x \, dx =$

(A) $\frac{1}{2}(e^\pi + 1)$
(B) $\frac{1}{2}(e^\pi - 1)$
(C) $(e^\pi + 1)$
(D) $(e^\pi - 1)$

23. If $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are roots of the equation $x^2+px+q=0$, then the equation whose roots are α and β is

(A) $x^2+qx+p=0$
(B) $x^2+px+1=0$
(C) $px^2+qx+1=0$
(D) $qx^2+px+1=0$

24. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$ are respectively

(A) 2, 3
(B) 2, 2
(C) 2, 6
(D) 6, 2

25. The inverse of an element 'a' in the group $G=\{a\in\mathbb{R}|a>0, a\neq 1\}$ under the operation $*$ defined by $a*b=a^{\log b}$ is

(A) $e^{\frac{1}{\log a}}$
(B) $\frac{1}{\log a}$
(C) $\frac{1}{e^{\log a}}$
(D) 1

26. If $f(x)=(x^2+3x+1)g(x)$, $g(0)=2$ and $\lim_{x\to 0}\frac{g(x)-2}{x}=3$, then $f'(0)=$

(A) 9
(B) 8
(C) 6
(D) 2

27. If a, b and c are positive, then $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} =$

(A) $a^2+b^2+c^2$
(B) $a+b+c$
(C) $1+a^2+b^2+c^2$
(D) 0

28. A solution of the simultaneous congruences $2x \equiv 1 \pmod{3}$, $3x \equiv 2 \pmod{5}$ is

(A) 2
(B) 4
(C) 14
(D) 13

29. Which one of the following statements is not correct?

(A) The Klein four group is abelian
(B) The Klein four group is not cyclic
(C) S_3 is abelian
(D) $\mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$ are nonisomorphic groups

30. If $f(x) = \sqrt{5}x^3 + x^2$, then the numbers $c \in (-1,1)$ which satisfy the equation $[f(1) - f(-1)] = 2f'(c)$ are

(A) $\frac{\sqrt{5}}{3}, -\frac{1}{\sqrt{5}}$
(B) $-\frac{\sqrt{5}}{3}, -\frac{1}{\sqrt{5}}$
(C) $-\frac{\sqrt{5}}{3}, \frac{1}{\sqrt{5}}$
(D) $\frac{\sqrt{5}}{3}, \frac{1}{\sqrt{5}}$

31. If $A=\begin{pmatrix}3 & 4\\ 5 & 7\end{pmatrix}$, then $A^2-10A+I=$

(A) $\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$
(B) $\begin{pmatrix}0 & 16\\ 23 & 49\end{pmatrix}$
(C) $\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$
(D) $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

32. Which one of the following is a subgroup of the group of all nonzero real numbers under usual multiplication?

(A) The set of all nonzero integers
(B) The set of all irrational numbers
(C) The set of all rational numbers
(D) The set of all nonzero rational numbers

33. The area (in square units) enclosed between the lines $|x|+|y|=1$ is

(A) 4
(B) 3
(C) 2
(D) 1

34. The derivative of $f(x)=(x+1)\tan^{-1}\left(e^{-2x}\right)$ at $x=0$ is

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{4}+1$
(C) $\frac{\pi}{4}-1$
(D) $\frac{\pi}{2}-1$

35. Let T be the linear transformation on $\mathbb{R}^2$, defined by $T(3,1)=(2,-4)$ and $T(1,1)=(0,2)$. Then $T(-1,1)=$

(A) $(0,2)$
(B) $(-2,2)$
(C) $(2,8)$
(D) $(-2,8)$

36. Which one of the following sets is not a subgroup of S_3 under the composition of maps?

(A) $\left\{\begin{pmatrix}1 & 2 & 3\\1 & 2 & 3\end{pmatrix}, \begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix}\right\}$

(B) $\left\{\begin{pmatrix}1 & 2 & 3\\1 & 2 & 3\end{pmatrix}, \begin{pmatrix}1 & 2 & 3\\2 & 3 & 1\end{pmatrix}\right\}$

(C) $\left\{\begin{pmatrix}1 & 2 & 3\\1 & 2 & 3\end{pmatrix}, \begin{pmatrix}1 & 2 & 3\\3 & 2 & 1\end{pmatrix}\right\}$

(D) $\left\{\begin{pmatrix}1 & 2 & 3\\1 & 2 & 3\end{pmatrix}, \begin{pmatrix}1 & 2 & 3\\2 & 1 & 3\end{pmatrix}\right\}$

37. The number of non units in the ring $\mathbb{Z}_{15}$ of integers modulo 15 is

(A) 7
(B) 6
(C) 5
(D) 9

38. The sum of the infinite series $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+\ldots$ is

(A) $e+e^{-1}$
(B) e
(C) $e-1$
(D) $\frac{1}{e}$

39. If $f(x)=\begin{cases}4(3^x), & \text{for } x<0\\ 2a+x, & \text{for } x\geq 0\end{cases}$ is continuous at $x=0$, then $a=$

(A) -1
(B) 2
(C) -2
(D) 1

40. The maximum value of $f(x)=\left(\frac{1}{x}\right)^x$ is

(A) e^{2e}
(B) e
(C) $e^{1/e}$
(D) e^{-e}

41. Which one of the following statements is not true?

(A) If $x^2 = x$ for all x in a ring $\mathbb{R}$, then $\mathbb{R}$ is commutative

(B) A finite integral domain is a field

(C) If U is an ideal of a ring $\mathbb{R}$ and $1 \in U$, then $U = \mathbb{R}$

(D) $U = \{10n \mid n \in \mathbb{Z}\}$ is a prime ideal of the ring $\mathbb{Z}$ of all integers

42. Let f be a real valued continuous function defined on $[a,b]$. Which one of the following statements need not be true?

(A) f is bounded on $[a,b]$

(B) f is uniformly continuous on $[a,b]$

(C) f attains its bounds on $[a,b]$

(D) f is differentiable on (a,b)

43. If $\{x_n\}$ is a sequence of real numbers such that $\lim_{n\to\infty}(2x_{n+1} - x_n) = x$. Then $\lim_{n\to\infty} x_n =$

(A) x

(B) 0

(C) $2x$

(D) 2

44. If $[x]$ is the greatest integer less than or equal to x, then $\int_1^n [x]\,dx =$

(A) $n(n-1)$

(B) $\frac{n(n-1)}{2}$

(C) $\frac{n(n+1)}{2}$

(D) $n(n+1)$

45. If $D = \frac{d}{dx}$, then the general solution of $(D^2+1)y = e^{2x}$ is

(A) $y = c_1 \cos x + c_2 \sin x + \frac{1}{5}e^{2x}$

(B) $y = c_1 e^{x} + c_2 e^{-x} + \frac{1}{5}e^{2x}$

(C) $y = c_1 \cos x + c_2 \sin x + e^{2x}$

(D) $y = c_1 \cos x + c_2 \sin x + \frac{1}{2}e^{2x}$

46. A point of intersection of the line $\frac{x+3}{4} = \frac{y+4}{3} = \frac{z-8}{-5}$ and the sphere $x^2 + y^2 + z^2 + 2x - 10y - 23 = 0$ is

(A) $(-1,1,-3)$
(B) $(1,1,-3)$
(C) $(1,1,3)$
(D) $(1,-1,3)$

47. Which one of the following series is divergent?

(A) $\sum_{n=1}^{\infty} \frac{1.3.5...(2n-1)}{1.4.7...(3n-2)}$
(B) $\sum_{n=1}^{\infty} \frac{(2n)!(3n)!}{n!(4n)!}$
(C) $\sum_{n=1}^{\infty} \frac{\log n}{2^{n-1}}$
(D) $\sum_{n=1}^{\infty} \frac{2n+3}{3n+5}$

48. If $z = \log(x^2 + y^2)$, then

(A) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$
(B) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$
(C) $x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial y^2} = 0$
(D) $x\frac{\partial^2 z}{\partial x^2} - y\frac{\partial^2 z}{\partial y^2} = 0$

49. If C is the curve $x = e^t$, $y = e^{-t}$, $z = t^2$, $0 \le t \le 1$, then $\int_c xydx + x^2zdy + xyzdz =$

(A) $\frac{5}{2} - e$

(B) $e - \frac{5}{2}$

(C) $\frac{3}{2}$

(D) $-\frac{5}{2}$

50. The equation $\left|\frac{z-2}{z+2}\right| = 3$ represents a

(A) parabola

(B) circle

(C) hyperbola

(D) pair of lines

51. If C is the circle $|z| = 1$, then $\oint_c \frac{z^2 - 4}{z(z^2+9)} dz =$

(A) $\frac{4\pi i}{9}$

(B) $\frac{8\pi i}{9}$

(C) $-\frac{8\pi i}{9}$

(D) $-\frac{4\pi i}{9}$

52. All values of i^i, where $i = \sqrt{-1}$, are

(A) $e^{-\left(\frac{\pi}{2}+2k\pi\right)}$, $k \in \mathbb{Z}$

(B) $e^{i\left(\frac{\pi}{2}+2k\pi\right)}$, $k \in \mathbb{Z}$

(C) $e^{\left(\frac{\pi}{2}+2k\pi\right)}$, $k \in \mathbb{Z}$

(D) $e^{(\pi+2k\pi)}$, $k \in \mathbb{Z}$

53. If $x_n = \frac{n}{2^n}$, then $\lim_{n\to\infty} x_n =$

(A) $\frac{1}{2}$
(B) 0
(C) 1
(D) $+\infty$

54. In the group $((Q-\{-1\},*)$; where $*$ is defined by $a*b=a+b+ab$, for all $a,b \in Q-\{-1\}$, the inverse of 15 is

(A) -15
(B) $\frac{15}{16}$
(C) $-\frac{15}{16}$
(D) $\frac{1}{15}$

55. The angle between the line $x-1=2-y=z+1$ and the plane $2x-y+z=4$ is

(A) $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$
(B) $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$
(C) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$
(D) $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$

56. Let H be a subgroup of G and $N(H)=\{g \in G \mid g^{-1}Hg=H\}$. Then which one of the following statements is not true?

(A) $N(H)$ is not a subgroup of G
(B) $N(H)$ is a subgroup of G
(C) H is normal in $N(H)$
(D) H is normal in G if $N(H)=G$

57. If A and B are symmetric matrices of same order and $AB \neq BA$, then

(A) AB is symmetric
(B) $AB+BA$ is symmetric
(C) $AB-BA$ is symmetric
(D) $BA-A$ is symmetric

58. If $f(x,y)$ is a homogeneous function of degree n in x and y, then

(A) $x\frac{\partial^2 f}{\partial x^2}+y\frac{\partial^2 f}{\partial y^2}=nf$
(B) $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf$
(C) $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$
(D) $y\frac{\partial f}{\partial x}+x\frac{\partial f}{\partial y}=nf$

59. $\left[\vec{a}+\vec{b}\quad \vec{b}+\vec{c}\quad \vec{c}+\vec{a}\right]=$

(A) $2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$
(B) 0
(C) $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$
(D) $\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$

60. The differential equation of the family of circles with center on the x-axis is

(A) $xy''-(y')^3-(y')^3-y'=0$
(B) $xy''+(y')^2+1=0$
(C) $yy''+(y')^2+1=0$
(D) $yy''+(y')^2=0$

61. The unit vector normal to the surface $x^2+2y-3z=5$ at the point $(1,2,0)$ is

(A) $\frac{2\hat{i}+2\hat{j}-3\hat{k}}{\sqrt{17}}$ (B) $\frac{2\hat{i}-2\hat{j}-3\hat{k}}{\sqrt{17}}$

(C) $\frac{2\hat{i}-2\hat{j}+3\hat{k}}{\sqrt{17}}$ (D) $\frac{-2\hat{i}+2\hat{j}-3\hat{k}}{\sqrt{17}}$

62. The function $f(x)=\begin{cases} x^3, & \text{for } x<1 \\ 2-x, & \text{for } x\geq 1 \end{cases}$ is

(A) differentiable at $x=1$
(B) not continuous at $x=1$
(C) not differentiable at $x=1$
(D) both continuous and differentiable at $x=1$

63. If the vector $\vec{F}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is irrotational, then the constants a,b and c are respectively equal to

(A) $-2,4$ and 1 (B) $2,4$ and 1
(C) $4,2$ and -1 (D) $4,2$ and 1

64. $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n=$

(A) $\sin n\left(\frac{\pi}{2}-\theta\right)+i\cos n\left(\frac{\pi}{2}-\theta\right)$

(B) $\cos n\left(\frac{\pi}{2}-\theta\right)+i\sin n\left(\frac{\pi}{2}-\theta\right)$

(C) $\cos n(\pi-\theta)+i\sin n(\pi-\theta)$

(D) $\sin n(\pi-\theta)+i\cos n(\pi-\theta)$

65. Which of the following is a field under usual operations?

(A) The ring of integers
(B) The ring of Gaussian integers
(C) The ring of $\mathbb{Z}_p$, where p is a prime
(D) The ring of quaternions

66. The angle of intersection of the curves $x^2 - y^2 = 1$ and $xy = \sqrt{2}$ at $\left(\sqrt{2},1\right)$ is

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{6}$

67. The derivative of $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ is

(A) -1
(B) 1
(C) -2
(D) 2

68. $\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{\frac{3n^2}{n+1}} =$

(A) e^{-3}
(B) e^3
(C) e
(D) e^2

69. The matrix $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix}$ is

(A) skew-symmetric
(B) symmetric
(C) non-singular
(D) singular

70. The area (in square units) of the triangle formed by the x-axis, the tangent and the normal to the curve $y(2a-x)=x^2$ at (a,a) is

(A) $\frac{2a^2}{3}$ (B) $\frac{4a^2}{3}$

(C) $\frac{5a^2}{3}$ (D) $\frac{10a^2}{3}$

71. Equation of the normal line to the curve $x^3+y^3=6xy$ at (3,3) is

(A) $x-y=0$ (B) $x+y=0$

(C) $x-y=1$ (D) $x+y-6=0$

72. The sum of the infinite series $1+\frac{1}{3}-\frac{1}{2}+\frac{1}{5}+\frac{1}{7}-\frac{1}{4}+\cdots$ is

(A) $\frac{2}{3}\log 2$ (B) $\frac{1}{2}\log 2$

(C) $\frac{3}{2}\log 2$ (D) $\frac{3}{4}\log 2$

73. The kernel of the homomorphism $\phi:(\mathbb{Z},+)\to(\mathbb{C},\cdot)$ defined by $\phi(n)=e^{\pi in}$ is

(A) $\{0\}$ (B) $4\mathbb{Z}$

(C) $2\mathbb{Z}$ (D) $\mathbb{Z}$

83. The set of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4}+\frac{d^2y}{dx^2}=0$ is

(A) $\{1, x, \cos x, \sin x\}$
(B) $\{1, x, \cos x, x\sin x\}$
(C) $\{1, x, x\cos x, \sin x\}$
(D) $\{1, x, x\cos x, x\sin x\}$

84. The value of $\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{1}{\sqrt{4n^2-r^2}}$ is

(A) 0
(B) $\frac{\pi}{2}$
(C) π
(D) 2π

85. The point of intersection of the lines given by $x^2-xy-2y^2-x+5y-2=0$ is

(A) (1, 1)
(B) (−1, 1)
(C) (1, 2)
(D) (0, 0)

86. A printer numbers the pages of a book starting with 1 and uses 3189 digits in all. How many pages does the book have?

(A) 1000
(B) 1074
(C) 1075
(D) 1080

87. How many iron rods each of length 7m and diameter 2cm can be made out of 0.88 cubic m of iron?

(A) 400
(B) 500
(C) 800
(D) 1200

70. The area (in square units) of the triangle formed by the x-axis, the tangent and the normal to the curve $y(2a-x)=x^2$ at (a,a) is

(A) $\frac{2a^2}{3}$ (B) $\frac{4a^2}{3}$

(C) $\frac{5a^2}{3}$ (D) $\frac{10a^2}{3}$

71. Equation of the normal line to the curve $x^3+y^3=6xy$ at (3,3) is

(A) $x-y=0$ (B) $x+y=0$

(C) $x-y=1$ (D) $x+y-6=0$

72. The sum of the infinite series $1+\frac{1}{3}-\frac{1}{2}+\frac{1}{5}+\frac{1}{7}-\frac{1}{4}+\cdots$ is

(A) $\frac{2}{3}\log 2$ (B) $\frac{1}{2}\log 2$

(C) $\frac{3}{2}\log 2$ (D) $\frac{3}{4}\log 2$

73. The kernel of the homomorphism $\phi:(\mathbb{Z},+)\to(\mathbb{C},\cdot)$ defined by $\phi(n)=e^{\pi i n}$ is

(A) $\{0\}$ (B) $4\mathbb{Z}$

(C) $2\mathbb{Z}$ (D) $\mathbb{Z}$

74. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$, then $A^{-1} =$

(A) $\frac{1}{4}(A-3I)$

(B) $\frac{1}{4}(3I-A)$

(C) $\frac{1}{3}(4I-A)$

(D) $\frac{1}{3}(A-4I)$

75. Let $f(x)$ be a real valued differentiable function defined for all $x \geq 1$ satisfying $f(1)=1$ and $f'(x) = \frac{1}{x^2+(f(x))^2}$. Then $\lim_{x\to\infty} f(x)$ exists and is less than

(A) $1+\frac{\pi}{4}$

(B) $1+\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

76. The series $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n^x}$ converges for

(A) $x > \frac{1}{2}$

(B) $x > 0$

(C) $x = \frac{1}{2}$

(D) $x = \frac{1}{3}$

77. The length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 sq.m. assuming $\pi = \frac{22}{7}$, is

(A) 3136

(B) 996

(C) 856

(D) 56

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78. Let $u(x, y) = 2x(1+y)$ for all x and y. Then a function $v(x, y)$, so that $f(z) = f(x, y) = u(x, y) + iv(x, y)$ is analytic, is

(A) $x^2 - (y+1)^2$
(B) $(x+1)^2 - y^2$
(C) $(x+1)^2 + y^2$
(D) $-x^2 + (y+1)^2$

79. Running at $\frac{5}{6}$ of its usual speed a train is 10 minutes late. The usual time to cover the journey is

(A) 85 minutes
(B) 60 minutes
(C) 50 minutes
(D) 20 minutes

80. The coefficient of $\frac{1}{z}$ in the expansion of $\log\left(\frac{z}{z+1}\right)$ valid in $|z|>1$ is

(A) -1
(B) 1
(C) $-\frac{1}{2}$
(D) $\frac{1}{2}$

81. Suppose f and g are maps from R^2 to R^2 defined by $f(x, y) = (x+y, |x|)$ and $g(x, y) = (|x-y|, y)$. Then

(A) both f and g are linear
(B) f is linear, but not g
(C) g is linear, but not f
(D) neither f nor g is linear

82. A bag contains 50p, 25p, 10p coins in the ratio 5:9:4, amounting to Rs.206. Find the number of 10p coins.

(A) 40
(B) 90
(C) 160
(D) 360

83. The set of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4}+\frac{d^2y}{dx^2}=0$ is

(A) $\{1, x, \cos x, \sin x\}$
(B) $\{1, x, \cos x, x \sin x\}$
(C) $\{1, x, x\cos x, \sin x\}$
(D) $\{1, x, x\cos x, x \sin x\}$

84. The value of $\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{1}{\sqrt{4n^2-r^2}}$ is

(A) 0
(B) $\frac{\pi}{2}$
(C) π
(D) 2π

85. The point of intersection of the lines given by $x^2-xy-2y^2-x+5y-2=0$ is

(A) (1, 1)
(B) (−1, 1)
(C) (1, 2)
(D) (0, 0)

86. A printer numbers the pages of a book starting with 1 and uses 3189 digits in all. How many pages does the book have?

(A) 1000
(B) 1074
(C) 1075
(D) 1080

87. How many iron rods each of length 7m and diameter 2cm can be made out of 0.88 cubic m of iron?

(A) 400
(B) 500
(C) 800
(D) 1200

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88. $\int_0^\infty x^{n-1}e^{-x}\,dx$ is

(A) divergent
(B) convergent for all the values for n
(C) converges for $n > 0$
(D) converges for $n < 0$

89. Let the characteristic equation of the matrix M be $\lambda^2 + \lambda - 1 = 0$. Then

(A) M^{-1} does not exist
(B) $M^{-1} = M - I$
(C) $M^{-1} = M + I$
(D) M^{-1} exists but cannot determine from the data

90. The arithmetic mean of the scores of a group of students in a test was 52.The brightest 20% of them secured a mean score of 80 and the dullest 25% a mean of 31. The mean score of the remaining is

(A) 52.50
(B) 51.41
(C) 50.50
(D) 50

91. The orthogonal trajectory of the family of circles $x^2 + y^2 + c = 2\mu x$ (μ is the parameter) is described by the equation

(A) $(x^2 + y^2 - c)y' = 2xy$
(B) $(x^2 - y^2 - c)y' = 2xy$
(C) $(-x^2 + y^2 - c)y' = 2xy$
(D) $(-x^2 + y^2 + c)y' = 2xy$

92. The initial value problem corresponding to the integral equation $y(x) = 2 + \int_0^x y(t)dt$ is

(A) $y' - y = 0,\ y(0) = 2$
(B) $y' + y = 0,\ y(0) = -2$
(C) $y' - y = 0,\ y(0) = 0$
(D) $y' + y = 0,\ y(0) = 1$

93. The set of complex numbers $z = x + iy$ such that $|e^{z^2}| \leq 1$ is given by

(A) $-x \leq y \leq x$
(B) $-y \leq x \leq y$
(C) $-x^2 \leq y \leq x^2$
(D) $-y^2 \leq x \leq y^2$

94. The average of 5 consecutive numbers is n. If the next two numbers are also included, then the average will

(A) remain the same
(B) increase by 1
(C) increase by 1.4
(D) increase by 2

95. The equation $x^2 + y^2 + gx + fy + 1 = 0$ represents a pair of lines if

(A) $f^2 + g^2 = 4$
(B) $f^2 - g^2 = 4$
(C) $f^2 - g^2 = 1$
(D) $f + g = 1$

96. The circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch if

(A) $\frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{c}$
(B) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$
(C) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{-1}{c}$
(D) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

97. Let z be complex number satisfying $z^2 + z + 1 = 0$. If n is not a multiple of 3, then the value of $z^n + z^{2n}$ is

(A) -1
(B) 0
(C) -2
(D) 1

98. The area enclosed by the curves $y = 4x^3$ and $y = 16x$ is

(A) 32
(B) 16
(C) 64
(D) 2π

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99. Two perpendicular tangents to $y^2 = 4ax$ always intersect on the line

(A) $x - a = 0$
(B) $x + a = 0$
(C) $x + 2a = 0$
(D) $x + 4a = 0$

100. The point which is equidistant from the points (0,0,0) , (2,0,0) , (0,2,0) and (2,2,2) is

(A) (1,0,1)
(B) (0,1,0)
(C) (1,1,–1)
(D) (1,1,1)

101. The volume of the parallelepiped whose edges are represented by the vectors $i+j,\ j+k,\ k+i$ is

(A) 2
(B) 0
(C) 1
(D) 6

102. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors, then $|\vec{a} + \vec{b} + \vec{c}|$ is

(A) $\sqrt{3}$
(B) 3
(C) 2
(D) 0

103. If the roots of the equation $x^n - 1 = 0$ are $1, \alpha_{1,}, \alpha_2, \ldots, \alpha_{n-1}$, then $(1-\alpha_{1,})(1-\alpha_2)\ldots(1-\alpha_{n-1})$ is

(A) 0
(B) n
(C) 1
(D) $-n$

104. If for the equation $x^3 - 3x^2 - kx + 3 = 0$ one root is the negative of the other, then the value of k is

(A) 3
(B) –3
(C) 1
(D) –1

105. The domain of the function $f(x) = \frac{x+3}{\sqrt{x^2-5x+4}}$ is

(A) $(-\infty, 1) \cup (4, \infty)$ (B) $[-\infty,1] \cup [4,\infty]$

(C) $[1,4]$ (D) $(1,4)$

106. The equation $\sum_{i=0}^{n} a_i x^{n-i} = 0$ has at least one root between 0 and 1 if

(A) $\sum_{i=0}^{n-1} \frac{a_i}{n-i} = 0$ (B) $\sum_{i=0}^{n} \frac{a_i}{n+1-i} = 0$

(C) $\sum_{i=0}^{n} \frac{a_i}{n} = 0$ (D) $\sum_{i=0}^{n-1} \frac{a_i}{n+1+i} = 0$

107. If $\frac{e^x}{1-x} = \sum_{i=0}^{\infty} B_i x^i$, then $B_n - B_{n-1}$ is given by

(A) $n!$ (B) $\frac{1}{n!}$

(C) n (D) $2n$

108. Value of the $\iint_A e^{-x^2-y^2}\ dx\,dy$ where $A = \{(x,y) \in R^2 | x^2 + y^2 \le 4\}$ is

(A) $\pi\,(e^{-2} - 1)$ (B) $\pi\,(1 - e^{-2})$

(C) $e^{\pi} - 1$ (D) $\pi\,(e^2 - 1)$

109. The distance moved by a particle at time t is $s = t^3 - 6t^2 - 18t + 12$. Then the velocity of the particle when acceleration is zero is

(A) -30 (B) 20

(C) 0 (D) -40

110. A random variable X has a uniform distribution over $(-3, 3)$. The value of k for which $P(X > k) = \frac{1}{3}$ is

(A) 1
(B) $\frac{1}{2}$
(C) $\frac{1}{3}$
(D) $\frac{1}{6}$

111. Let $a \in R^+$. Define a sequence $\{x_n\}$ as $x_0 = 0$ and $x_{n+1} = a + x_n^2$, $n \geq 0$. Then $\{x_n\}$ converges for

(A) $a \geq 1$
(B) $a \geq 0$
(C) $a \leq \frac{1}{4}$
(D) $a \geq \frac{1}{4}$

112. The number of linearly independent eigenvectors of the matrix $\begin{bmatrix} 2 & 2 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 5 & 4 \end{bmatrix}$ is

(A) 1
(B) 2
(C) 3
(D) 4

113. The solution of $xu_x + yu_y = 0$ is of the form

(A) $f(y/x)$
(B) $f(xy)$
(C) $f(x + y)$
(D) $f(x - y)$

114. Let $1+x$ and e^x be two solutions of $y''(x) + P(x)\, y'(x) + Q(x) y(x) = 0$. Then $P(x)$ is

(A) $1 + x$
(B) $1 - x$
(C) $\frac{1+x}{x}$
(D) $\frac{-1-x}{x}$

115. Consider the following statements
I. Every cyclic group is abelian
II. Every abelian group is cyclic
III. Every group of order < 4 is cyclic. Then

(A) I alone is correct
(B) I and II are correct
(C) I and III are correct
(D) II and III are correct

116. If $f(x)=10^x$ and $g(x)=\ln(x)$, then $\frac{d}{dx}((g \circ f)(x))$ is

(A) $\log_e 10$
(B) $\log_{10} e$
(C) 0
(D) $\frac{1}{x}$

117. The number of real roots of the equation $x^2+5|x|+6=0$ is

(A) 1
(B) 2
(C) 3
(D) 4

118. If $\prod_{i=1}^{n}(a_i - i\, b_i) = A + i\,B$, then $\sum_{j=1}^{n}\tan^{-1}\frac{b_j}{a_j}$ is

(A) $\tan^{-1}\frac{A}{B}$
(B) $\tan^{-1}\frac{B}{A}$
(C) $\cot^{-1}\frac{A}{B}$
(D) $\cot^{-1}\frac{B}{A}$

119. If $f(x)=\int_0^x t\sin t\, dt$, then $f'(x)$ is

(A) $x\sin x$
(B) $x+\sin x$
(C) $x-\cos x$
(D) $x\cos x$

120. The value of the natural number a for which $\sum_{k=1}^{n} f(a+k) = 32(2^n - 1)$ where the function satisfies $f(x+y) = f(x)$ $f(y)$ and $f(1) = 2$, is

(A) 1
(B) 2
(C) 4
(D) 8

121. If $y = \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$, then x is

(A) $\frac{y}{1} - \frac{y^2}{2} + \frac{y^3}{3} - \ldots\ldots$
(B) $\frac{y}{1} + \frac{y^2}{2} + \frac{y^3}{3} + \ldots\ldots$
(C) $1 + \frac{y}{1} - \frac{y^2}{2} + \frac{y^3}{3} - \ldots\ldots$
(D) $-1 + \frac{y}{1} - \frac{y^2}{2} + \frac{y^3}{3} - \ldots\ldots$

122. The eccentricity of the hyperbola $\frac{\sqrt{2013}}{13}(x^2 - y^2) = 1$ is

(A) $\sqrt{2}$
(B) 1
(C) $\sqrt{3}$.
(D) 2

123. If $(x_i, \frac{1}{x_i})$, for $i = 1, 2, 3, 4$ are four points on circle, then $x_1 x_2 x_3 x_4$ is

(A) 0
(B) -1
(C) 1
(D) π

124. The distance between the parallel lines given by the equation $x^2 + 2xy + y^2 - 7x - 7y + 6 = 0$ is

(A) $\frac{1}{\sqrt{3}}$
(B) $\frac{5}{\sqrt{3}}$
(C) $\frac{1}{\sqrt{2}}$
(D) $\frac{5}{\sqrt{2}}$

125. If $\sqrt{1-y^2} + \sqrt{1-x^2} = a(x-y)$, then $\frac{dy}{dx}$ is

(A) $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ (B) $-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

(C) $\frac{\sqrt{1+y^2}}{\sqrt{1+x^2}}$ (D) $\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$

126. If $f(x) = \dfrac{2x-3}{x-1}$, then inverse of $f(x)$ is

(A) $\frac{2x+3}{1+x}$ (B) $\frac{x-3}{x-2}$

(C) $\frac{x-3}{2-x}$ (D) $\frac{x-1}{3-2x}$

127. $\lim\limits_{x\to 0} \dfrac{\tan x - \cos x}{x}$ is

(A) 1 (B) 2

(C) 3 (D) 6

128. The value of the series $\sum_{n=1}^{\infty} \frac{1}{2n(2n-1)}$ is

(A) 1 (B) $1 - \log 2$

(C) $\log 2 - 1$ (D) $\log 2$

129. The function $f(x,y) = xy + 2x - \log x^2y$ has the point $\left(\frac{1}{2}, 2\right)$ which corresponds to, (for $x > 0$ and $y > 0$).

(A) local minimum (B) local maximum

(C) global minimum (D) global maximum

130. If $x = r\cos\theta$ and $= r\sin\theta$, then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is given by

(A) r (B) $-r$

(C) $\frac{1}{r}$ (D) 1

131. $\frac{1}{2}\int_C (x\,dy - y\,dx)$ gives

(A) the volume enclosed by the curve
(B) area enclosed by the curve
(C) length of the curve
(D) surface area of the curve

132. Let $f(z) = \cos z - \frac{\sin z}{z}$ for non zero $z \in \mathbb{C}$ and $f(0) = 0$. Then $f(z)$ has a zero

(A) at $z = 0$ of order 1 (B) at $z = 0$ of order 2
(C) at $z = 1$ of order 3 (D) at $z = 1$ of order 2

133. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then the value of A^n

(A) $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ (B) $\begin{bmatrix} 1-2n & 4n \\ n & 1+2n \end{bmatrix}$

(C) $\begin{bmatrix} n & 2n \\ -n & 1 \end{bmatrix}$ (D) $\begin{bmatrix} n & 1-2n \\ 1+2n & 1-n \end{bmatrix}$

134. The value of $\int_{|z|=1} \frac{|dz|}{|z-a|^2}$ where a is a complex number such that $|a|<1$ is

(A) $\frac{2\pi}{1-|a|^2}$ (B) $\frac{2\pi}{1+|a|^2}$

(C) 0 (D) 1

135. The value of $\int_{-\frac{1}{2}}^{\frac{1}{2}} sin^2 x \log\frac{1-x}{1+x}\, dx$ is

(A) 1 (B) 0
(C) −1 (D) 2

136. If $f(x) = \sin^{-1}(\frac{2(\log x)}{1+(\log x)^2})$, then $f'(e)$ is

(A) e (B) $-e$

(C) $\frac{1}{e}$ (D) $\frac{-1}{e}$

137. $w = \frac{iz+2}{4z+i}$ will transform the real axis into

(A) real axis (B) imaginary axis
(C) straight line (D) a circle

138. The value of the $\int_1^3 |x-2|\, dx$ is

(A) 0 (B) 1
(C) 2 (D) 4

139. The function $f(x) = |x-1|$ is

(A) differentiable at $x = 1$ (B) continuous at $x = 1$
(C) nowhere differentiable (D) nowhere continuous

140. The maximum value of the function $f(x) = \left(\frac{1}{x}\right)^{\frac{1}{x}}$ $(x > 0)$ is

(A) 1
(B) e^{-1}
(C) $(e^{-1})^{-e}$
(D) $(e^{-1})^{e^{-1}}$

141. The equation $(\alpha xy^3 + y\cos x)dx + (x^2y^2 + \beta \sin x)dy = 0$ is exact for

(A) $\alpha = \frac{3}{2}, \beta = 1$
(B) $\alpha = 1, \beta = \frac{3}{2}$
(C) $\alpha = \frac{2}{3}, \beta = 1$
(D) $\alpha = 1, \beta = \frac{2}{3}$

142. For vector spaces M and N such that $M \subseteq N$, consider the following statements.
(I) $\dim M \leq \dim N$;
(II) If $\dim M = \dim N$, then $M = N$.
Then

(A) (I) and (II) are true
(B) only (I) is true
(C) only (II) is true
(D) (I) and (II) are false

143. Consider the following statements
(I) If X and Y are subspaces of a vector space V, then $\dim (X + Y) = \dim X + \dim Y - \dim (X \cap Y)$;
(II) $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$.
Then

(A) (I) and (II) are true
(B) only (I) is true
(C) only (II) is true
(D) (I) and (II) are false

144. If A is $m \times n$ and B is $n \times p$, consider the statements
(I) $\text{rank}(AB) \leq \min\{\text{rank}(A), \text{rank}(B)\}$
(II) $\text{rank}(A) + \text{rank}(B) + n \leq \text{rank}(AB)$. Then

(A) (I) and (II) are true
(B) only (I) is true
(C) only (II) is true
(D) (I) and (II) are false

145. Let $a_n \geq 0$ and suppose $\sum_{n=0}^{\infty} a_n$ converges. Then $\sum_{n=0}^{\infty} \frac{a_n}{1+a_n^2}$ is

(A) convergent
(B) divergent
(C) oscillatory
(D) doubtful

146. Which of the following is the imaginary part of a possible value of $\log(\sqrt{i})$?

(A) π
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{8}$

147. Let $f: \mathbb{C} \to \mathbb{C}$ be analytic except for a simple pole at $z=0$ and let $g: \mathbb{C} \to \mathbb{C}$ be analytic. Then value of $\frac{\text{Res}_{z=0}\{f(z)g(z)\}}{\text{Res}_{z=0}\{f(z)\}}$ is

(A) $g(0)$
(B) $g'(0)$
(C) $\lim_{z\to 0} zf(z)$
(D) $\lim_{z\to 0} zf(z)g(z)$

148. The sum of n terms of the series 4 + 44 + 444 + ... is

(A) $\frac{4}{81}[10^{n+1} - 9n - 1]$
(B) $\frac{4}{81}[10^{n-1} - 9n - 1]$
(C) $\frac{4}{81}[10^{n+1} - 9n - 10]$
(D) $\frac{4}{81}[10^{n} - 9n - 10]$

149. Given that $f(y) = \frac{|y|}{y}$ and q is any non zero real number, then the value of $|f(q) - f(-q)|$ is

(A) 0
(B) −1
(C) 1
(D) 2

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150. Let f be bilinear transformation that maps 1 to 0, -1 to ∞ and i to 1. Then $f(i)$ is equal to

(A) 1
(B) -1
(C) i
(D) $-i$

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SPACE FOR ROUGH WORK

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