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M.A./M.Sc

MATHEMATICS

MT - 501 : Real Analysis - I

(2008 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to right indicate full marks.***

Q1) a) Define norms $\|\cdot\|_1$, $\|\cdot\|_2$ and $\|\cdot\|_\infty$ on $\mathbb{R}^2$. Also find d ((1, 1), (2, 3)) in these norms. **[6]**

b) Let M be the set of all ordered n tuples of 0s and 1s. For $x, y \in$ M, define $d(x, y)$ = Number of places in which x and y differ. Show that d is a metric. **[5]**

c) State and prove Cauchy-Schwarz inequality. **[5]**

Q2) a) State and prove Arzela - Ascoli theorem. **[6]**

b) Define ring and $\sigma-$ ring. Give an example of a ring which is not a σ ring. **[5]**

c) What do you mean by a separable space. whethere R and $e[a, b]$ are separable? Jusitfy. **[5]**

Q3) a) With usual notations, prove that $\mathcal{M}_{\mathcal{J}}$ is a ring. **[6]**

b) Prove or disprove $\text{int}(A \cap B) = \text{int}(A) \cap \text{int}(B)$

$$\text{int}(A \cup B) = \text{int}(A) \cup \text{int}(B)$$ **[5]**

c) Show that an exterior measure is countably subadditive. **[5]**

Q4) a) Define a measurable function $f : \mathbb{R}^n \to \mathbb{R} \cup \{\pm \infty\}$. Show that following statements are equivalent. For $\forall a \in \mathbb{R}$, **[6]**

i) $\left\{x \in \mathbb{R}^n / f(x) > \mathrm{a}\right\}$ is a measurable set.

ii) $\left\{x \in \mathbb{R}^n / f(x) \geq a\right\}$ is a measurable set.

iii) $\left\{x \in \mathbb{R}^n / f(x) < a\right\}$ is a measurable set.

iv) $\left\{x \in \mathbb{R}^n / f(x) \leq a\right\}$ is a measurable set.

b) Give an example of a non-measurable function. **[5]**

c) State and prove monotone convergence theorem. **[5]**

Q5) a) Show that a constant function and a continuous function both are measurable. **[6]**

b) Show that characteristic function of Q is not Riemann integrable but it is Lebesgue integrable. **[5]**

c) Prove that $\mathrm{X}_{\mathrm{A} \cup \mathrm{B}} = \mathrm{X}_{\mathrm{A}} + \mathrm{X}_{\mathrm{B}} - \mathrm{X}_{\mathrm{A} \cap \mathrm{B}}$ **[5]**

Q6) a) State and prove Fatou's lemma. **[6]**

b) If $\int_{\mathrm{E}} f dm = 0$ for $\forall$ E $\in \mathcal{M}$ then show that **[5]**

$f = 0 = a.e.$ on E.

c) Find f_+ and f_- for a function

$f(x) = \sin x$ for $x \in [0, 2\pi]$ **[5]**

Q7) a) Apply Gram-Schmidt process to $f_n(x) = x^n$, $n \in \mathrm{N}$ to obtain formulas for first three Legendre polynomials. **[8]**

b) Show that the sequence $\frac{e^{inx}}{\sqrt{2\pi}}$ where $n \in \mathrm{Z}$ is completely orthonormal in $\mathrm{L}^2[-\Pi, \Pi]$. **[8]**

Q8) a) State and prove Bessel's inequality. **[8]**

b) State and prove Riesz - Fischer theorem. **[8]**

Total No. of Questions : 8]

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P1775

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M.A./M.Sc.

MATHEMATICS

MT-502: Advanced Calculus

(2008 Pattern) (Semester - I)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicates full marks.***

Q1) a) Define the following: **[6]**

i) Derivative of a scalar field.

ii) Directional derivative of a scalar field.

Explain and illustrate by an example the difference of the above two definitions.

b) Give an example to show a function of two variables that is continuous in each variable separately but it is discontinuous as a function of two variables together. **[5]**

c) Let S be an open subset of $\mathbb{R}^n$. If $f: S \to \mathbb{R}$ is linear prove that the derivative of f with respect to $\overline{y}$ is equal to the value of the function at $\overline{y}$. **[5]**

Q2) a) Let scalar field f is differentiable at $\overline{a}$ with total derivative $T_{\overline{a}}$. Prove that the derivative $f'(\overline{a};\overline{y})$ exists for every $\overline{y}$ in $\mathbb{R}^n$ and $T_{\overline{a}}(\overline{y}) = f'(\overline{a};\overline{y})$. Also show that $f'(\overline{a};\overline{y})$ is a linear combination of the components of $\overline{y}$. **[6]**

b) State and prove Mean Value theorem for derivatives of scalar field. **[5]**

c) Evaluate the directional derivative of $f(x, y, z) = 3x - 5y + 2z$ at (2, 2, 1) in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 9$. **[5]**

P.T.O.

Q3) a) Define the line integral of a vector field. State and prove the additive property with respect to the path of integration. **[6]**

b) Let $\overline{f}$ be a two dimensional vector field given by **[5]**

$\overline{f}(x, y) = \sqrt{y}\,\overline{i} + (x^3 + y)\overline{j}$ for all (x, y) with $y \geq 0$.

Calculate the line integral of $\overline{f}$ from (0, 0) to (1, 1) along each of the following paths.

i) the line with parametric equations $x = t,\ y = t,\ 0 \leq t \leq 1$.

ii) the path with parametric equations $x = t^2,\ y = t^3,\ 0 \leq t \leq 1$.

Show that the integral from one point to another may depend on the path joining the two points.

c) Compute the mass M of one coil of a spring having the shape of the helix whose vector equation is $\overline{\alpha}(t) = a\cos t\ \overline{i} + a\sin t\ \overline{j} + bt\ \overline{k}$ if the density at (x, y, z) is $(x^2 + y^2 + z^2)$. **[5]**

Q4) a) State and prove the first fundamental theorem for line integrals. **[6]**

b) Determine whether or not the vector field

$\overline{f}(x, y, z) = 3y^4z^2\overline{i} + 4x^3z^2\overline{j} - 3x^2y^2\overline{k}$ is a gradient on any open subset of $\mathbb{R}^3$. **[5]**

c) Find the amount of work done by the force $\overline{f}(x, y) = (x^2 - y^2)\overline{i} + 2xy\,\overline{j}$ in moving a particle (in a counter clockwise direction) once around the square bounded by the co-ordinate axes and the lines $x = a$ and $y = a$, $a > 0$. **[5]**

Q5) a) Define the double integral of a step function on a rectangle. Let s and t are step function defined on a non degenerate rectangle Q. If $s(x, y) \leq t(x, y)$ for every $(x, y) \in Q$ then prove that **[6]**

$$\iint_Q s(x, y)\,dxdy \leq \iint_Q t(x, y)\,dxdy$$

Also prove that if $t(x, y) \geq 0$ for every $(x, y) \in Q$ then $\iint_Q t(x, y)\,dxdy \geq 0$.

b) By Green's theorem express the area of the region as line integral. **[5]**

c) Evaluate $\oint_C y^2dx + xdy$ when C is the square with vertices (0, 0), (2, 0), (2, 2), (0, 2). **[5]**

Q6) a) State only the transformation formula for n-fold integral and explain the terms invalved in it. **[6]**

b) Evaluate $\iiint_S xy^2 z^3 dx\, dy\, dz$ where S is the solid bounded by the surface $z = xy$ and planes $y = x$, $x = 1$ and $z = 0$. **[5]**

c) Evaluate $\iint_S e^{\frac{y-x}{y+x}} dx\, dy$ where S is the triangle bounded by the line $x+y=2$ and the two co-ordinate axes. **[5]**

Q7) a) Define the fundamental vector product show that the fundamental vector product is normal to the surface. **[6]**

b) Compute the area of the region cut from the plane $x + y + z = a$ by the cylinder $x^2 + y^2 = a^2$. **[5]**

c) Let S be a parametric surface whose vector representation is

$\bar{r}(u,v) = (u+v)\bar{i} + (u-v)\bar{j} + (1-2u)\bar{k}$

Find the fundamental vector product and the unit normal to the surface. **[5]**

Q8) a) State and prove Stoke's theorem. **[8]**

b) Transform the surface integral $\iint_S (\text{curl}\, \bar{\text{F}}) \cdot \bar{n}\, ds$ to a line integral by the Stoke's theorem where $\bar{\text{F}}(x,y,z) = y^2\bar{i} + xy\,\bar{j} + xz\,\bar{k}$ where S is the hemisphere $x^2 + y^2 + z^2 = 1\ z \geq 0$ and $\bar{n}$ is the unit normal with nonnegative z-component. Evaluate the line integral. **[6]**

c) Determine the Jacobian matrix and also compute the divergence of

$\bar{F}(x,y,z) = (x^2 + yz)\bar{i} + (y^2 + xz)\bar{j} + (z^2 + xy)\bar{k}$. **[2]**

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Total No. of Questions :8]

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P 1776

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M.A/M.Sc.

MATHEMATICS

MT-503 :Linear Algebra

(2008 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Solve any five questions out of eight questions.***
2) ***Figures to he right indicate full marks.***

Q1) a) Let V and V' be finite dimensional vector spaces over K of dimensions n and m respectively. Prove that $dim\ L(V, V') = nm$. **[6]**

b) Let V be a finite dimensional vector space over K and let W be a subspace of V. Prove that $dim V = dim W + dim V/W$. **[6]**

c) Let $I = (-a, a)$, $a > 0$ be an open interval in R and let $V = R^I$, the space of all real valued functions defined on I. Show that $V = V_e \oplus V_0$, where V_e is the set of all even functions on I and V_0 is the set of all odd function on I. **[4]**

Q2) a) Let B be an ordered basis of an n-dimensional vector space V over K. If T is a linear operator on V, then prove that T is a bijecction if and only if $[T]_B$ is an invertible matrix. **[6]**

b) Let $V_1, V_2, ..., V_m$ be vector spaces over a field K. prove that $V = V_1 \oplus V_2 \oplus ... \oplus V_m$ is finite dimensional if and only if each V_i is finite dimentional. also prove that $dim\ V_1 \oplus V_2 \oplus ... \oplus V_m = dim V_1 + dim\ V_2 + ... + dim V_m$. **[6]**

c) Consider the vector space $R_3[x]$ of polynomials with real coefficients and of degree at most 3. The differential operator D is a linear operator on $R_3[x]$. Write the matrix representation of D with respect to $B_1 = \{1 + x, x + x^2, x^2 + x^3, x + x^3\}$. **[4]**

P.T.O.

Q3) a) Let A$\in$ $K^{n\times n}$. The left multiplication by A defines a linear operator $\lambda_A : K^{n\times m} \to K^{n\times m}$ such that $\lambda_A(B) = AB$. Prove that $\propto$ is an eigen value of λ_A if and only if $\propto$ is an eigen value of A. **[6]**

b) Prove that linearly independent subset of finite dimensional vector space can be extended to form a basis of vector space. **[6]**

c) Let V be a finite dimensional vector space over K and let X and Y be subspace of V. show that $(X + Y)^o = X^o + Y^o$, where S^o is annihilator of S. **[4]**

Q4) a) State and prove the primary decomposition theorem. **[10]**

b) Prove that the geometric multiplicity of an eigenvalue of a linear operator can not exceed its algebraic multiplicity. **[6]**

Q5) a) Prove that two triangulable $n \times n$ matrices are similar if and only if they have the same Jordon canonical form. **[6]**

b) Find the Jordan form for the matrix $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ **[6]**

c) State Cayley - Hamilton theorem and illustrate by an example. **[4]**

Q6) a) Give all possible rational canonical forms it the characteristic polynomial is :

i) $(x^2 + 2)\ (x - 3)^2;$

ii) $(x - 1)^2\ (x + 1)^2.$ **[6]**

b) Prove that a jordan chain consists of linearly independent vectors. **[6]**

c) The characteristic polynomial of matrix $(x - 1)^3\ (x - 2)^2$. Write its Jordon Canonical forms. **[4]**

Q7) a) Let V and W be finite dimensional inner product spaces over F and Let $T \in L(V, W)$. Show that

i) $\text{Ker}T^* = (im T)^{\perp}$ and $im T^* = (\text{Ker} T)^{\perp}$

ii) $V = \text{Ker} T \oplus im T^*$ and $W = im T \oplus \text{Ker} T^*$ **[6]**

b) Prove that the eigenvalues of a unitary operator have absolute value 1. **[5]**

c) Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Find a polar decomposition of A. **[5]**

Q8) a) Let V be a vector space over K and let f,g be Linear form on V. Prove that the mapping $\phi(x + y) = f(x)g(y)$ is a bilinear form on V. **[8]**

b) State and prove Schur's theorem. **[6]**

c) Find the eigenvalue of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P1777

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[5321]-14
M.A./M.Sc.
MATHEMATICS
MT-504: Number Theory
(2008 Pattern) (Semester-I) (Credit System)

Time : 3Hours] *[Max. Marks :80*

Instructions to the candidates

1) ***Attempt any five questions.***
2) ***Figures to the right indicates full marks.***

Q1) a) State and prove the chinese Remainder Theorem. **[6]**

b) Find values of x and y that satisfy 423 x + 198 y = g. **[6]**

c) Prove that if r is a quadratic residue modulo m > 2 then.

$$r^{\phi(m)/2} \equiv 1 (\text{mod m})$$ **[4]**

Q2) a) If Q is odd and Q > o then prove that

$$\left(\frac{-1}{Q}\right) = (-1)^{\frac{Q-1}{2}} \text{ and } \left(\frac{2}{Q}\right) = (-1)^{\frac{Q^2-1}{8}}$$ **[6]**

b) Decide whether $x^2 \equiv 150$ (mod 100g) is solvable or not. Justify. **[6]**

c) Find all quadratic residues of prime number 7. **[4]**

Q3) a) Let $f(n)$ be a multiplicative function and let $f(n) = \sum_{d/n} f(d)$. Then prove that $f(n)$ is multiplicative. **[6]**

b) Prove that for every positive integer n

$$\sum_{d/n} \mu(d)\, d(d) = (-1)^{w(n)}$$ **[6]**

c) Find the minimal polynomial of $\frac{1+\sqrt[3]{7}}{2}$ **[4]**

P.T.O.

Q4) a) If p is a prime number and $P \equiv 1 \pmod 4$ then prove that there exist positive integers a and b such that $a^2 + b^2 = P$. **[6]**

b) If 100! were written out in the ordinary decimal notation without the factorial sign, how many zeroes would there be in a row at the right end? **[5]**

c) If the norm of an integer α in $Q\left(\sqrt{m}\right)$ is $\pm$ p where p is a rational prime then prove that α is a prime. **[5]**

Q5) a) Prove that product of two primitive polynomials is primitive. **[6]**

b) Show that 1387 is a pseudoprime. **[5]**

c) What are the last two digits in the ordinary decimal representation of 3^{400}. **[5]**

Q6) a) If a monic polynomials $f(x)$ with integral coefficients factors into two monic polynomials with rational coefficients say $f(x) = g(x)\, h\,(x)$ then prove that $g(x)$ and $h(x)$ have integral coefficients. **[6]**

b) Find all solutions of equation $999\,x - 49\,y = 5000$. **[5]**

c) Is $\phi\left(\sqrt{m}\right)$ unique factorization domain for every integer m ? Justify your answer. **[5]**

Q7) a) Let m be a negative square-free rational integer. Prove that the field $\phi\left(\sqrt{m}\right)$ has units ± 1 and these are the only units except in the cases m = –1 and m = –3. The units for $\phi(i)$ are ± 1 and $\pm$ i. The units for $\phi\left(\sqrt{-3}\right)$ are $\pm 1, \dfrac{1 \pm \sqrt{-3}}{2}$, $\dfrac{-1 \pm \sqrt{-3}}{2}$ **[8]**

b) For which primes p do there exist integers x and y with $(x, p) = 1 (y, p) = 1$, such that $x^2 + y^2 \equiv 0 \pmod p$? **[5]**

c) If a is a quadratic nonresidue of each of the odd primes p and q , is $x^2 \equiv a \pmod{pq}$ solvable? **[3]**

Q8) a) State and prove wilson's Theorem. **[6]**

b) Prove that $\sum_{d/n} d = \sum_{d/n} \frac{n}{d}$ **[5]**

c) Find the value of $\left(\frac{2}{11}\right)$ where $\left(\frac{2}{11}\right)$ denote Legendre symbol. **[5]**

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Total No. of Questions :8]

SEAT No. :

P1778

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[5321]-15
M.A./M.Sc.
MATHEMATICS
MT-505 : Ordinary Differential Equations
(2008 Pattern) (Semester-I)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $y_1(x)$ and $y_2(x)$ are any two solutions of the equation $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$ then show that the Wronskian $W(y_1, y_2)$ is either identically zero or never zero on $[a, b]$. **[6]**

b) Form the differential equation of family of circles passing through origin and having centres on X-axis. **[4]**

c) Show that $y = c_1e^x + c_2e^{2x}$ is the general solution of $y'' - 3y' + 2y = 0$ on any interval, and find particular solution for which $y(0) = -1$, and $y'(0) = 1$. **[6]**

Q2) a) Explain the method of variation of parameters for determining a particular solution of the nonhomogeneous equation $y'' + P(x)y' + Q(x)y = R(x)$. Hence find particular solution of $y'' + y = \operatorname{cosec} x$. **[8]**

b) If k and b are positive constants, then find the general solution of $y'' + k^2y = \sin bx$ by using method of undetermined coefficients. **[8]**

Q3) a) Show that by applying the substitution $y = uv$ to the homogeneous equation $y'' + P(x)y' + Q(x)y = 0$ it is possible to obtain a homogeneous second order linear equation for V with no V' term present. Find u and the equation for V in terms of the coefficients $P(x)$ and $Q(x)$. **[8]**

b) If $\frac{dr}{d\theta} = F(r,\theta)$ is the differential equation of the given family of curves in polar coordinates then show that $\frac{dr}{d\theta} = -r^2 / F(r,\theta)$ is the differential equation of the orthogonal trajectories. Hence find orthogonal trajectories of the family of circles $r = 2\,C \sin\theta$. **[8]**

Q4) a) State and prove sturm comparison theorem. **[6]**

b) Find power series solution of the equation $y' = 2xy$ of the form $\Sigma a_n x^n$. **[6]**

c) Classify singular points of the differential equation

$$x^2\left(x^2-1\right)^2 y'' - x(1-x)y' + 2y = 0$$ **[4]**

Q5) a) Show that the Bessel's equation $x^2 y'' + xy' + \left(x^2 - 1\right)y = 0$ has only one frobenious series solution. Hence find the solution of it. **[8]**

b) Find general solution of the following differential equation near the singular point $x = 0$ in terms of hypergeometric functions.

$$x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0.$$ **[8]**

Q6) a) Find general solution of the system **[8]**

$$\begin{cases} \frac{dx}{dt} = x - 2y \\ \frac{dy}{dt} = 4x + 5y \end{cases}$$

b) Replace each of the following differential equations by an equivalent system of first order equatinos **[4]**

i) $y'' - x^2 y' - xy = 0$

ii) $y''' = y'' - x^2 (y')^2$

c) Describe the phase portrait of the following system : **[4]**

$$\begin{cases} \dot{x} = 0 \\ \dot{y} = 0 \end{cases}$$

Q7) a) Consider the following system of equation $\begin{cases} \dfrac{dx}{dt} = x \\ \dfrac{dy}{dt} = -x + 2y \end{cases}$. **[8]**

i) Find the critical points of the above system.

ii) Find general solution of the above system

iii) Find the differential equation of the paths.

b) Discuss nature and stability properties of the critical point (0, 0) for the following linear autonomous system :

$$\begin{cases} \dfrac{dx}{dt} = 4x - 2y \\ \dfrac{dy}{dt} = 5x + 2y \end{cases}$$ **[8]**

Q8) a) Find the exact solution of the initial value problem $y' - y = 0$ with $y(0)=1$. Starting with $y_0(x)=1$, apply Picard's method to calculate $y_1(x)$, $y_2(x)$, $y_3(x)$ and compare these results with exact solution. **[8]**

b) Show that $f(x, y) = xy$

i) satisfies a Lipschitz condition on any strip $a \le x \le b$ and $-\infty < y < \infty$.

ii) does not satisfy a Lipschitz condition on the entire plane.

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Total No. of Questions : 8]

SEAT No. :

P1779

[Total No. of Pages : 2

[5321]-21
M.A./M.Sc.
MATHEMATICS
MT - 601 : General Topology
(2008 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let $\mathfrak{B}$ be a collection of non-empty sets (not necessarily disjoint). Prove that there exists a function $C : \mathfrak{B} \to \bigcup_{B \in \mathfrak{B}} B$ such that C(B) is an element of B, for each $B \in \mathfrak{B}$. **[8]**

b) For a non-empty set X, let τ_C be a collection of all subsets U of X such that X\U is either countable or all of X. Then show that τ_C is a topology on X. **[6]**

c) Define order topology. Give an example of order topology. **[2]**

Q2) a) Let X be a topological space. Then prove that **[8]**

i) ϕ and X are closed.

ii) Arbitrary intersections of closed sets are closed.

iii) Finite unions of closed sets are closed.

b) Prove that the order topology on $\mathbb{Z}_+$ is the discrete topology on $\mathbb{Z}_+$. **[6]**

c) Let Y = [–1, 1], which of the following sets are open in Y? Justify. **[2]**

i) $A = \left\{ x \,\middle|\, \frac{1}{2} \le |x| \le 1 \right\}$

ii) $B = \left\{ x \,\middle|\, \frac{1}{2} < |x| \le 1 \right\}$

P.T.O.

Q3) a) Define homeomorphism between two topological spaces X and Y. Also give an example of a non-homeomorphic function between subset of real numbers and unit ball in $\mathbb{R}^2$. **[6]**

b) If $A \subseteq X$ and $B \subseteq Y$, show that in the topological space $X \times Y$, $\overline{A \times B} = \overline{A} \times \overline{B}$. **[6]**

c) Show that every order topology is Hausdorff. **[4]**

Q4) a) Show that if U is open in X and A is closed in X, then U\A is open in X and A\U is closed in X. **[6]**

b) Let (X, τ) be a topological space and $A \subset B \subset \overline{A} \subset X$. Show that, if A is connected then $\overline{A}$ is connected. **[6]**

c) Give an example of connected set which is not path connected. **[4]**

Q5) a) Show that a compact topological space is limit point compact. Is converse true? Justify. **[6]**

b) Define the box topology and the product topology. State the comparison of the box and product topologies. **[6]**

c) Let $f : X \to Y$. If the function f is continuous, then prove that for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ converges to $f(x)$. **[4]**

Q6) a) Prove that the product of finitely many compact spaces is compact. **[8]**

b) Show that the image of connected space under a continuous map is connected. **[5]**

c) Prove that every closed subspace of a compact space is compact. **[3]**

Q7) a) State and prove the tube lemma. **[8]**

b) Show that if X is compact Hausdorff under both τ and τ', then either τ and τ' are equal or they are not comparable. **[8]**

Q8) a) State and prove the Tychonoff theorem. **[10]**

b) State: **[6]**

i) The Urysohn lemma.

ii) The Tietze extension theorem.

iii) The Urysohn metrization theorem.

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Total No. of Questions :8]

SEAT No. :

P1780

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[5321] - 22
M.A./M.Sc.
MATHEMATICS
MT - 602 : Differential Geometry
(2008 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let S be a connected n-surface in R^{n+1}. Show that on S, there exists exactly two smooth unit normal vector fields N_1 and N_2. **[6]**

b) Find the velocity, acceleration and speed of the curve $\alpha(t) = (\cos t, \sin t)$. **[5]**

c) Show that the 1-form η on $R^2 - \{0\}$ defined by **[5]**

$$\eta = \frac{-x_2}{x_1^2 + x_2^2} dx_1 + \frac{x_1}{x_1^2 + x_2^2} dx_2 \text{ is not exact.}$$

Q2) a) Let S be an n-surface in R^{n+1}, let $\alpha : I \to S$ be a parameterized curve in S, let $t_0 \in I$ and $v \in S_{\alpha(t_0)}$. Prove that there exists a unique vector field V tangent to S along $\propto$, which is parallel and has $V(t_0) = v$. **[6]**

b) Show that the Weingarten map of the n-sphere of radius r oriented by inward normal is multiplication by $\frac{1}{r}$. **[4]**

c) Consider a vector field $X(x_1, x_2) = (x_1, x_2, 1, 0)$ on R^2. For $t \in R$ and $p \in R^2$, let $\phi_t(p) = \alpha_p(t)$ where $\propto$ is the maximal integral curve of X through p. Show that $F(t) = \phi_t$ is a homomorphism of additive group of real numbers into the invertible linear maps of the plane. **[6]**

P.T.O.

Q3) a) Show that the covariant differentiation has the following property : $(X.Y)' = X'.Y + X.Y'$. **[5]**

b) Let $a, b, c, d \in$ R be such that $ac - b^2 > 0$. Show that the maximum and minimum values of the function $g(x_1, x_2) = ax_1^2 + 2bx_1x_2 + cx_2^2$ on the unit circle $x_1^2 + x_2^2 = 1$ are eigen values of the matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$. **[6]**

c) Find the integral curve of the vector field X given by **[5]**

$X(x_1, x_2) = (x_1, x_2, -x_2, x_1)$ through the point (1, 0).

Q4) a) Let U be an open subset of R^{n+1} and $f : U \to R$ be a smooth function. Let $S = f^{-1}(c)$, $c \in$ R and $\nabla f(q) \neq 0$, $\forall q \in S$. If $g : U \to R$ is smooth function and $p \in$ S is an extreme point of g on S, then show that there exists a real number λ such that $\nabla g(p) = \lambda \nabla f(p)$. **[6]**

b) Let S denote the cylinder $x_1^2 + x_2^2 = r^2$ of radius r in R^3. Show that α is a geodesic of S if and only if α is of the form

$\alpha(t) = (r\cos(at + b), r\sin(at + b), ct + d)$ for some real numbers a, b, c, d. **[6]**

c) Sketch the following vector fields on R^2 : $X(p) = (p, X(p))$ where **[4]**

i) $X(p) = -p$.

ii) $X(x_1, x_2) = (-x_2, x_1)$.

Q5) a) Show that the Weingarten map L_p is self-adjoint.

(that is $L_p(v).w = v.L_p(w)$, for all $v, w \in S_p$). **[6]**

b) If an n-surface S contains a straight line segment, then show that it is geodesic in S. **[5]**

c) Show that the graph of any smooth function $f : R^n \to R$ is an n-surface in R^{n+1}. **[5]**

Q6) a) Prove that on each compact oriented n-surface S in R^{n+1} there exists a point p such that the second fundamental form at p is definite. **[6]**

b) Find the curvature of the circle with center (a, b) and radius r oriented by the outward normal. **[5]**

c) Let $\alpha(t) = (x(t), y(t))$ be a local parameterization of the oriented plane curve C. Show that $k \circ \alpha = \frac{x'y'' - y'x''}{\left(x'^2 + y'^2\right)^{\frac{3}{2}}}$. **[5]**

Q7) a) Let S be an n-surface in R^{n+1} and let $p \in$ S. Prove that there exists an open set V about p in R^{n+1} and a parameterized n-surface $\phi : U \to R^{n+1}$ such that ϕ is one to one map from U onto $V \cap S$. **[8]**

b) Show that the speed of geodesic is constant. **[8]**

Q8) a) Let S be the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$, a, b, c all non-zero, oriented by the outward normal. Show that the Gaussian curvature of S is

$$K(p) = \frac{1}{a^2 b^2 c^2 \left(\frac{x_1^2}{a^4} + \frac{x_2^2}{b^4} + \frac{x_3^2}{c^4}\right)^2}.$$ **[8]**

b) Let C be a connected oriented plane curve and let $\beta : I \to C$ be a unit speed global parameterization of C. Show that β is either one to one or periodic. **[8]**

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Total No. of Questions : 8]

SEAT No :

P 1781

[Total No. of Pages :3

[5321]-23

M.A./M.Sc.

MATHEMATICS

MT - 603: Groups and Rings

(2008 Pattern) (Semester-II) (New)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that in a group there exists unique identity element. Further prove that for every element there exists unique inverse in that group. **[5]**

b) Define the center of a group. Prove that the center of a group G is a subgroup of G. **[5]**

c) Prove that any cyclic group is isomorphic to either $\mathbb{Z}_n$ (for some $n \in \mathbb{N}$) or $\mathbb{Z}$. Also find all the generators of $\mathbb{Z}_{33}$. **[6]**

Q2) a) Prove that $(\mathbb{Z},+)$ and $(\mathbb{Q},+)$ are not isomorphic. Also, prove that group of quaternions Q_8 is not isomorphic to the dihedral group D_4 with 8 elements. **[5]**

b) Give examples of two non isomorphic groups of order 6 with justification. **[5]**

c) Find $\text{Aut}(\mathbb{Z}_6)$, the group of automorphisms of $\mathbb{Z}_6$. **[6]**

P.T.O.

Q3) a) Let p be a prime integer and let G be a group such $|G| = p^2$. Prove that G is abelian. **[5]**

b) Prove that every subgroup of a cyclic group is cyclic. Moreover prove that, if $|< a >| = n$, then the order of any subgroup of $< a >$ is a divisor of n; and, for each positive divisor k of n, the group $< a >$ has exactly one subgroup of order k. **[5]**

c) Let G be a finite group and p be a prime. If p^k divides $|G|$, then prove that G has atleast one subgroup of order p^k. **[6]**

Q4) a) Find the inverse and the order of each of the following permutation in S_{15}. **[5]**

i) (12 14 9) (1 3 10) (4 6 5 2)

ii) (2 1 3 15 14) (7 12 6) (5 8 9).

b) Prove that set of even permutations A_n forms a subgroup of S_n. Also prove that for $n > 1$, A_n has order $n!/2$. **[5]**

c) State and prove the Lagrange's theorem for finite groups. Is the converse of the theorem true? Justify. **[6]**

Q5) a) Let G be a finite group of permutations of a set S. Then prove that for any i from S, $|orb_G(i)| = |stab_G(i)|$. **[5]**

b) Find the inverse of the element $A = \begin{bmatrix} 1 & 6 \\ 3 & 2 \end{bmatrix}$ *in* $GL(2, \mathbb{Z}_{13})$. **[5]**

c) If $\tau = (5\ 7\ 9\ 4)\ (6\ 10)$, $\rho = (7\ 4\ 3\ 6\ 5\ 1)\ (8\ 2\ 9) \in S_{10}$. Then find $\tau^{-1}\rho\tau$ and $\rho^{-1}\tau\rho$. **[6]**

Q6) a) Find $Inn(D_4)$. **[5]**

b) Determine all the homomorphisms from $\mathbb{Z}_{30}$ to $\mathbb{Z}_{40}$. **[5]**

c) Find all the non isomorphic abelian groups of order 5400. **[6]**

Q7) a) Prove that every permutation of a finite set can be written as a cycle or a product of disjoint cycles. **[5]**

b) If |G| is a group of order pq where p and q are primes, $p < q$ and p does not divide $q - 1$, then prove that G is cyclic. **[5]**

c) Let G be a finite group and let p be a prime. Then prove that the number of Sylow p - subgroups of G is equal to 1 modulo p and divides |G|. Also prove that, any two Sylow p subgroups of G are conjugate. **[6]**

Q8) a) Define the index of a subgroup of a group G. Prove that a subgroup of G with index 2 is normal in G. **[5]**

b) Prove that every group of order 16 has non trivial center. Justify your answer. **[5]**

c) Prove that the group of order 200 is not simple. **[6]**

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Total No. of Questions : 8]

SEAT No. :

P1782

[Total No. of Pages : 3

[5321]-24
M.A./M.Sc.
MATHEMATICS
MT - 604 : Complex Analysis
(2008 Pattern) (Semester - II)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) *Answer any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) For a given power series $\sum_{n=0}^{\infty} a_n z^n$ define $R, 0 \le R \le \infty$, by $\frac{1}{R} = \limsup |a_n|^{1/n}$. **[6]**

Prove that :

i) If $|z| < R$, the series converges absolutely;

ii) If $|z| > R$, the series diverges.

b) Prove that the two series $\sum_{n=0}^{\infty} a_n z^n$ and $\sum_{n=1}^{\infty} n a_n z^{n-1}$ have same radius of convergence. **[5]**

c) i) Prove that $f(z) = |z|^2$ has derivative at only origin; **[5]**

ii) Show that $u(x, y) = e^x \cos y$ is a harmonic function on $\mathbb{C}$. Find a harmonic conjugate v of u.

Q2) a) Define a Mobius transformation. Prove that every Mobius transformation is the composition of translations, dilations, and the inversions. **[6]**

b) Let G be a region and let $f : G \to \mathbb{C}$ be analytic. Prove that if the image of f is contained in a circle, then f is constant. **[5]**

c) Find the image of the set $\left\{z : |\text{Im } z| < \pi/2\right\}$ under the exponential function e^z. **[5]**

Q3) a) Prove that an analytic function f on the disk B (a, R) has a power series expansion about a. **[6]**

b) Evaluate : $\int_{\gamma} \frac{\log z}{z^3} dz$, where $\gamma(t) = 1 + \frac{1}{2}e^{it}$ with $0 \le t \le 2\pi$. **[5]**

c) Suppose f and g are analytic on a region G such that $\overline{f}\,g$ is also analytic on G. Show that either f is constant or $g \equiv 0$. **[5]**

Q4) a) State and prove Liouville's theorem. Hence deduce that sin z function is unbounded. **[6]**

b) Show that every entire function with bounded real part is constant. **[5]**

c) Let $\gamma : [0,1] \to \mathbb{C}$ be a closed rectifiable curve and $a \notin \{\gamma\}$. Prove that $\int_{\gamma} \frac{dz}{z-a}$ is an integer. **[5]**

Q5) a) State and prove Morera's theorem. **[6]**

b) Let G be simply connected and $f : \mathrm{G} \to \mathbb{C}$ be an analytic function such that $f(z) \neq 0$ for any z in G. Prove that there is an analytic function $g : \mathrm{G} \to \mathbb{C}$ such that $f(z) = e^{g(z)}$. **[5]**

c) Suppose $f : \mathrm{G} \to \mathbb{C}$ is analytic and one-one. Show that $f'(z) \neq 0$ for any z in G. **[5]**

Q6) a) State and prove Rouche's Theorem. **[6]**

b) Let $\{f_n\}$ be a sequence of analytic functions on G which converges uniformly to a function f on G. Prove that f is analytic. **[5]**

c) Using residue calculus, show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$ **[5]**

Q7) a) State and prove Schwarz's lemma. **[6]**

b) State the first version of Maximum modulus theorem. Using it, prove that if f is a non-constant analytic function on a bounded open set G and is continuous on $\overline{G}$, then f has a zero in G or $|f|$ assumes its minimum value. **[5]**

c) Evaluate the intergral $\int_{\gamma} \frac{dz}{z^2+1}$ where $\gamma(\theta) = 2|\cos 2\theta| e^{i\theta}$ for $0 \le \theta \le 2\pi$. **[5]**

Q8) a) Let G be an open set and $f : G \to \mathbb{C}$ be a differentiable function. Prove that f is analytic. **[6]**

b) Suppose a function f has an essential singularity at $z = a$. Prove that for every $\delta > 0$, the image of the annulus ann $(a;\, 0, \delta)$ under f *is* dense in $\mathbb{C}$. **[5]**

c) Find the singularities of the following function and classify them. **[5]**

i) $\dfrac{1}{\sin\left(1/z\right)}$

ii) $\dfrac{\sin z}{z^3}$

iii) $\dfrac{z-1}{z\left(z^2-1\right)}$

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Total No. of Questions : 8]

SEAT No. :

P1783

[Total No. of Pages : 3

[5321] - 25

M/A./M.Sc.

MATHEMATICS

MT-605: Partial Differential Equations

(2008 Pattern) (Semester - II) (Old Course)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Eliminate the arbitrary function F from the equation $F(x+y+z, x^2+y^2-z^2)=0$ and find the corresponding partial differential equation. **[6]**

b) Find the general solution of $x(y-z)p+y(z-x)q=z(x-y)$. **[5]**

c) Explain the method of solving the following first order partial differential equation. **[5]**

i) $f(z,p,q)=0$

ii) $g(x,p)=h(y,q)$

Q2) a) Find the general solution of: **[6]**

$$(y^2+yz+z^2)dx+(z^2+xz+x^2)dy+(y^2+xy+x^2)dz=0.$$

b) Show that the equations $p^2+q^2-1=0$ and $(p^2+q^2)x-pz=0$ are compatible. Also find their common solution. **[6]**

c) Find the general integral of $y^2p-xyq=x(z-2y)$. **[4]**

P.T.O.

Q3) a) Prove that the pfaffian differential equaiton. **[6]**

$X.dr = P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0$ is integrable if and only if X. curlX=0.

b) Find the complete integral of $p^2x + q^2y = z$ by Charpit's method. **[6]**

c) Find the integral of pfaffian differential equation: **[4]**

$$yz(y+z)dx + xz(x+z)dy + xy(x+y)dz = 0.$$

Q4) a) Find the integral surface for the differential equation: **[6]**

$zx\,Z_x - yz\,Z_y = y^2 - x^2$ passing through intial curve (2s,s,s).

b) Verify that the pfaffian differential equation is integrable and find the corresponding integral of $(1+yz)dx + z(z-x)dy - (1+xy)dz = 0$. **[6]**

c) Find the complete integral of $xU_x + yU_y = U_z^2$ by Jacobi's method. **[4]**

Q5) a) Find by the method of characteristics, the integral Surface of $p\,q = x\,y$ which passes through curve $z = x$, $y = 0$. **[8]**

b) Find d' Alembert's solution of one dimensional wave equation which describes the vibration of infinite string. **[4]**

c) Reduce the equation $U_{xx} + x\,U_{yy} = 0$ in the region $x < 0$ to canonical form and solve. **[4]**

Q6) a) State and prove Harnack's theorem. **[8]**

b) Find the solution of the Heat - equation in an infinite rod which is defined as: **[8]**

$$U_t = kU_{xx}, -\infty < x < \infty, t > 0$$

$$U(x,0) = f(x), -\infty < x < \infty$$

Q7) a) If U(x, y) is harmonic in a bounded domain D and continuous in $\bar{D} = D \bigcup B$ then U attains it's maximum on the boundary B of D. **[8]**

b) State and prove Harnack's theorem. **[6]**

c) Classify the following equation into hyperbolic, parabolic or elliptic type **[2]**

$$u_{xx} + 2u_{yz} + \cos x u_z - e^{y^2} u = \cosh z .$$

Q8) a) Use Duhamel's principle and solve the non homogeneous wave **[8]**

equation $u_{tt} - c^2 u_{xx} = F(x,t), -\infty < x < \infty, t > 0$

with conditions: $u(x,0) = u_{t(x,0)} = 0 - \infty < x < \infty$.

b) Find the solution of Dirichlet's problem for the upper half plane which is defined as: $u_{xx} + u_{yy} = 0 ; -\infty < x < \infty, y > 0$ **[8]**

$u(x,0) = f(x), -\infty < x < \infty$ with the condition that u is bounded as $y \to \infty$, u and u_x vanish as $|x| \to \infty$.

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Total No. of Questions :4]

SEAT No. :

P1784

[Total No. of Pages :2

[5321] - 26

M. A/ M.Sc

MATHEMATICS

MT - 606 : C++ (Object Oriented Programming)

(2008 Pattern) (Semester - II)

Time : 2 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) ***Question 1 is compulsory***
2) ***Attempt any two from questions 2, 3, 4.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt the following questions. **[20]**

a) Explain the use of # define.

b) What does the following C statement define? Char * P [10];

c) Define a structure consisting of two floating point members called real and imaginary.

d) Explain the meaning of the following declaration int (* p [10]) (int);

e) What is data encapsulation?

f) Write a note on function overloading.

g) Give an example of union in C++.

h) What is use of scope resolution operation.

i) Write a short note an function prototype.

j) Write a function of find l.c.m. of two integers.

P.T.O.

Q2) a) Illustrate use of static member function. **[10]**

b) Write a short note on pointer arithmetic in C. **[5]**

Q3) a) Define a class rat having two data members num and den. Overload necessary construction. Overload operators + and –. Find addition and subtraction of two rational numbers. **[10]**

b) Write a short note an preprocessors in C. **[5]**

Q4) a) Write a note on compile time polymorphism and run time polymorphism. **[10]**

b) Describe the three bitwise operators. What is the purpose of each? **[5]**

Total No. of Questions :8]

SEAT No. :

P1785

[Total No. of Pages :3

[5321] - 31

M.Sc.

MATHEMATICS

MT - 701 : Functional Analysis

(2008 Pattern) (Semester - III)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to right indicate full marks.*

Q1) a) Let M be a closed linear subspace of a normed linear space N. If the norm of a coset x + M in the quotient space N/M is defined by

$$\|x+M\| = \inf\{\|x+m\| \mid m \in M\},$$

then prove that N/M is a normed linear space. Further, if N is a Banach space, then prove that N/M is also Banach space. **[8]**

b) If M is a closed linear subspace of a normed linear space N, and if T is the natural mapping of N onto N/M defined by T(x) = x + M, show that T is a continuous linear transformation for which $\|T\| \le 1$. **[6]**

c) Give an example of reflexive Banach space. **[2]**

Q2) a) State and prove Riesz Representation theorem. **[8]**

b) Let Y be a closed subspace of a normed linear space X. Show that a sequence $\{x_n + Y\}$ converges to x + Y in X/Y if and only if there is a sequence $\{y_n\}$ in Y such that $\{x_n + y_n\}$ converges to x in X. **[6]**

c) Give an example of a normal operator. **[2]**

Q3) a) Prove that an infinite dimensional Banach space cannot have a denumerable Hamel basis. **[6]**

b) If a Banach space B is reflexive, then prove that B* is reflexive. **[5]**

c) Give examples of two non-equivalent norms. Justify. **[5]**

Q4) a) If T is an operator on a Hilbert space H, then prove that T is normal if and only if its real and imaginary parts commute. **[6]**

b) If a Hilbert space H is separable, then prove that every orthonormal set in H is countable. **[6]**

c) Find $M^{\perp}$ if $M = \{(x, y) | x - y = 0\} \subset \mathbb{R}$. **[4]**

Q5) a) If T is an operator on a Hilbert space H for which $(Tx, x) = 0$ for all $x \in H$ then prove that $T = 0$. **[5]**

b) With usual notations prove that $l_1^* = l_\infty$. **[5]**

c) Prove that the adjoint operation $T \to T^*$ on B(H) has the following properties: **[4]**

i) $(\alpha T)^* = \alpha T^*$

ii) $(T_1 T_2)^* = T_2^* T_1^*$

Q6) a) If M is a closed linear subspace of a Hilbert space, then prove that

$H = M \oplus M^{\perp}$. **[6]**

b) Show that a projection P on a Hilbert space H satisfies $0 \le P \le I$. Under what conditions.

i) $P = 0$,

ii) $P = 1$? **[5]**

c) Show that $\left\{ \frac{e^{inx}}{\sqrt{2\pi}} \right\}$ is an orthonormal set in $L_2[0, 2\pi]$. **[5]**

Q7) a) Let T be a normal operator on H with spectrum $\{\lambda_1, \lambda_2, ..., \lambda_m\}$. Show that T is self-adjoint if and only if each λ_i is real. **[6]**

b) Let S and T be normal operators on a Hilbert space H. If S commutes with T*, then prove that S + T and ST are normal. **[6]**

c) Let the dimension *n* of *a* Hilbert space H be 2, let B = $\{e_1, e_2\}$ be a basis for H, and assume that the determinant of a 2 ×2 matrix $[a_{ij}]$ is given by $a_{11}a_{22} - a_{12}a_{21}$. If T is an arbitrary operator on H whose matrix relative to B is $[a_{ij}]$, show that $T^2 - (a_{11} + a_{22})T + (a_{11}a_{22} - a_{12}a_{21})I = 0$. **[4]**

Q8) a) State and prove the Closed Graph Theorem. **[8]**

b) Prove that every finite dimensional subspace of a normed linear space X is closed. Give an example to show that an infinite dimensional subspace of a normed linear space may not be closed. **[8]**

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Total No. of Questions :8]

SEAT No. :

P1786

[Total No. of Pages :3

[5321] - 32

M.A./M.Sc. (Mathematics)

MT - 702 : RING THEORY

(2008 Pattern) (Semester - III)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Each question carries 16 marks.*

Q1) a) State true or false and justify your answers: **[10]**

i) Every UFD (Unique Factorzation Domain) is a PID (Principal Ideal Domain).

ii) Every prime ideal is a maximal ideal in a ring.

iii) $\mathbb{Z}_6[X]$ is an integral domain.

iv) In $\mathbb{Z}_{13}$ every non-zero element has a multiplicative inverse.

v) Quotient of an integral domain by an ideal is always integral domain.

b) If R is a Boolean ring with 1 (unity) then show that $x=-x$ for all $x \in R$ and that R is commutative. Is there a Boolean ring without unity? **[6]**

Q2) a) Suppose R is a PID and $I_1 \subseteq I_2 \subseteq \ldots \subseteq I_n \subseteq \ldots$ is an ascending chain of ideals in R then show that there is a positive integer n_0 such that $I_{n_0} = I_{n_0+1} = \ldots.$ **[6]**

b) Show that if a commutative ring R with 1 has no ideals other than $\{0\}$ and R then R is a field. Give an example of a commutative ring with unity which has exactly one non-zero proper ideal. **[6]**

c) Show that in $\mathbb{Z}_n$ every elements is either a zero divisor or unit (i.e. has a multiplicative inverse). **[4]**

P.T.O.

Q3) a) Define the term PID. Show that every non-zero prime ideal in a PID is a maximal ideal. **[6]**

b) Show that in a commutative integral domain with 1, a prime element is irreducible but not conversely. **[6]**

c) Show that in a commutative ring with 1 (unity) every maximal ideal is a prime ideal. **[4]**

Q4) a) Show that in a ring with identity every proper ideal is contained in a maximal ideal. **[8]**

b) Prove that any subfield of $\mathbb{C}$ must contain $\mathbb{Q}$. **[4]**

c) Is $\mathbb{Q}[X, Y]$ a Euclidean domain? Justify your answer. **[4]**

Q5) a) Let R be a commutative ring with $1 \neq 0$ and G be a finite group. Define group ring RG of G clearly defining the addition and multiplication on RG. Show that if G is Abelian group then RG is commutative ring. **[6]**

b) Show that in a UFD a non-zero element is prime if and only if it is irreducible. **[6]**

c) Show that every Euclidean domain is a PID. **[4]**

Q6) a) Let K be a field. Show that $K[x]/(f(x))$ is field if and only if $f(x)$ is irreducible polynomial in $K[x]$. **[6]**

b) Define the term affine algebraic set and give two examples of affine algebraic sets. Identify all the affine algebraic sets in A^1 over $\mathbb{R}$. **[6]**

c) Define the term Artinian ring and give two examples. **[4]**

Q7) a) Let V be a finite dimensional vector space over a field F and given a linear transformation $T:V \to V$ give a F[x]-module structure to V. Also show that if V is a F[x]-module then there is a linear transformation associated to it. **[8]**

b) Give example of a free $\mathbb{Z}$-module and non-free $\mathbb{Z}$-module. Is every finite abelian group afree $\mathbb{Z}$-module? Justify. **[8]**

Q8) a) Prove that a quotient of a PID by a nonzero prime ideal is a field. Illustrate this by an example. **[8]**

b) Define the term ring homomorphism. Give two examples of ring homomorphisms with non-zero kernels. Also determine all ring homomorphisms from $\mathbb{Q}$ to $\mathbb{R}$. **[8]**

Total No. of Questions :8]

SEAT No :

P 1787

[Total No. of Pages :3

[5321]-33

M.A/M.Sc.

MATHEMATICS

MT-703 :Mechanics

(2008 Pattern) (Semester - III) (Credit System)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Explain the following terms: **[6]**

i) Linear momentum

ii) Generalized momentum

iii) Angular momentum

b) A particle of mass m moves in one dimension such that it has the Langrangian $L = \frac{m^2 x^4}{12} + mx^2 V(x) - V^2(x)$, where V is some differentiable function of *x*. Find the equation of motion for *x(t)* and describe the physical nature of the system on the basis of this equation. **[5]**

c) A particle is constrained to move on the surface of a cylinder of fixed radius. Find the Lagrange's equation of motion. **[5]**

Q2) a) Use D' Alembert's principle to determine the equation of motion of a simple pendulum. **[6]**

b) Explain the D' Alembert's principle and derive Lagrange's equations of motion using the same. **[10]**

P.T.O.

Q3) a) Explain Atwood machine and discuss its motion. **[7]**

b) Find E-L differential equation satisfied by twice differentiable function y(x) which extremizes the functional $I(y(x)) = \int_{x_1}^{x_2} f(x,y,y')\,dx$ where y is prescribed at the end points. **[7]**

c) Explain the Basic lemma. **[2]**

Q4) a) Describe the Routh's Procedure to solve the problem involving cyclic and non-cyclic co-ordinates. **[6]**

b) Prove that angular momentum of a particle in a central force field remains constant. **[5]**

c) Show that the Hamilton's Principle $\delta\int_{t_0}^{t} Ldt = 0$ also holds for the non-conservative system. **[5]**

Q5) a) If the Lagrangian function does not contain time t explicity, show that the total energy of the conservative system is conserved. **[7]**

b) State Modified Hamilton's Principle. **[2]**

c) Explain Legendre transformations and the Hamiltonian equations of motion. **[7]**

Q6) a) Explain the Principle of least action. **[3]**

b) Prove that field force motion is always motion in plane. **[5]**

c) Prove the Kepler's third law of planetary motion. **[8]**

Q7) a) Prove that Poisson brackets are invariant under canonical transformation. **[8]**

b) Infinitesimal rotation of a right body with one point fixed is commutative. Also find the inverse matrix of infinitesimal rotation. **[8]**

Q8) a) Find extremal of the functional $I = \int_{0}^{\frac{\pi}{2}} (y'^2 - y^2 + 2xy)\, dx$, subject to the conditions that $y(0) = 0, y\left(\frac{\pi}{2}\right) = 0$ **[8]**

b) Prove that a co-ordinate which is cyclic in the Lagrangian is also cyclic in the Hamiltonian. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P1788

[Total No. of Pages : 3

[5321]-34
M.A./M.Sc.
MATHEMATICS
MT-704 : Measure and Integration
(2008 Pattern) (Semester - III)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***
3) ***All symbols have their usual meanings.***

Q1) a) Define following terms with suitable example. **[6]**

i) σ - algebra.

ii) Outer Measure.

iii) Signed measure.

b) Let $\{E_i\}$ be a sequence of measurable sets. If $E_1 \subseteq E_2 \subseteq .....$, then show that $\mu(\lim E_i) = \lim \mu(E_i)$. **[5]**

c) Let f be a bounded measurable function defined on the finite interval (a, b) then show that $\lim_{\beta \to \infty} \int_a^b f(x) \sin(\beta x)\, dx = 0$. **[5]**

Q2) a) Let $\{(X_\alpha, \mathfrak{B}_\alpha . \mu_\alpha)\}$ be a collection of measurable spaces, and suppose that the sets $\{X_\alpha\}$ are disjoint and define $X = \bigcup X_\alpha$, $\mathfrak{B} = \{B : (\alpha)[B \cap X_\alpha \in \mathfrak{B}_\alpha]\}$ and $\mu(B) = \Sigma \mu_\alpha (B \cap X_\alpha)$. Show that $\mathfrak{B}$ is a σ - algebra. **[6]**

b) Let $(X, \mathfrak{B}, \mu)$ be a measure space and $Y \in \mathfrak{B}$. Let $\mathfrak{B}_Y$ consist of those sets of $\mathfrak{B}$ that are contained in Y and $\mu_Y(E) = \mu(E)$ if $E \in \mathfrak{B}_Y$. Then show that $(Y, \mathfrak{B}_Y, \mu_Y)$ is a measure space. **[5]**

c) Show that $\mu(E_1 \Delta E_2) = 0$ implies $\mu E_1 = \mu E_2$ provided that $E_1, E_2 \in \mathfrak{B}$. **[5]**

P.T.O.

Q3) a) Let c be any real number and let f and g be real valued measurable functions defined on the same measurable set E. Show that $f + c$, cf, $f + g$, $f - g$ and fg are measurable. **[6]**

b) Let $\{A_n\}$ be Borel sets and let α_n be the Hausdorff dimension of A_n. Find the Hausdorff dimension of $A = \bigcup_{n=1}^{\infty} A_n$. **[5]**

c) Show that each nonempty open set G in $\mathbb{R}$ is a union of disjoint open intervals at most countable in number. **[5]**

Q4) a) Suppose that to each α in a dense set D of real numbers there is assigned a set $B_\alpha \in \mathfrak{B}$ such that $B_\alpha \subset B_\beta$ for $\alpha < \beta$. Then show that there exist a unique measurable extended real valued function f on X such that $f \leq \alpha$ on B_α and $f \geq \alpha$ on $X \sim B_\alpha$. **[6]**

b) Let F be a bounded linear functional on $L^p(\mu)$ with $1 \leq p < \infty$ and μ a σ - finite measure. Then show that there is a unique element g in L^q where $1/p + 1/q = 1$, such that $F(f) = \int fg \, d\mu$ with $\|F\| = \|g\|_q$. **[6]**

c) Give an example of a function such that $|f|$ is measurable but f is not. **[4]**

Q5) a) Let μ^* be a topologically regular outer measure on X then prove that each Borel set is μ^* - measurable. **[6]**

b) If $\{f_n\}$ be a sequence of nonnegative measurable functions then show that $\int \sum_{n=1}^{\infty} f_n = \sum_{n=1}^{\infty} \int f_n$. **[6]**

c) Show that if f is a non-negative measurable function, then $f = 0$ a.e. if and only if $\int f \, dx = 0$. **[4]**

Q6) a) Let $(X, \mathfrak{B}, \mu)$ be a σ - finite measure space and let υ be a measure defined on $\mathfrak{B}$ which is absolutely continuous with respect to μ. Then prove that there is a nonnegative measurable function f such that for each set E in $\mathfrak{B}$ we have $\upsilon E = \int_E f \, d\mu$. **[6]**

b) Let $<f_n>$ be a sequence of measurable functions that converge to a function f except at the points of set E of measure zero. Then prove that f is a measurable function if μ is complete. **[6]**

c) Let B be a μ^*-measurable set with $\mu^*B < \infty$ then prove that $\mu_*B = \mu^*B$. **[4]**

Q7) a) Let μ be a measure on an algebra $\mathscr{G}$ and μ^* the outer measure induced by μ. Then prove that the restriction $\bar{\mu}$ of μ^* to the μ^* - measurable sets is an extension of μ to σ-algebra containing $\mathscr{G}$. **[8]**

b) If μ^* is a Caratheodory outer measure with respect to Γ then prove that every function in Γ is μ^*-measurable. **[8]**

Q8) a) State and prove Fatou's Lemma. **[6]**

b) Let μ be a finite measure defined on σ-algebra which contains all the Baire sets of a locally compact space X. If μ is inner regular then show that it is regular. **[6]**

c) Give an example of a non measurable set. **[4]**

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Total No. of Questions :8]

SEAT No. :

P1789

[Total No. of Pages : 2

[5321]-35
M.A./M.Sc.
MATHEMATICS
MT-705: Graph Theory
(2008 Pattern) (Semester-III)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Find the number of labeled trees with n vertices. Draw all labeled trees with 4 distinct vertices. **[5]**

b) Prove that an edge is a cut edge if and only if it belongs to no cycle. **[5]**

c) Prove that a graph G is Eulerian if it has at most one non trivial component and its vertices all have even degree. **[6]**

Q2) a) Show that the connectivity of the hypercube Q_k is k. **[6]**

b) Determine the number of isomorphism classes of simple 7-vertex graphs in which every vertex has degree 4. **[5]**

c) Show that the Petersen graph has girth 5. **[5]**

Q3) a) Prove that for a connected nontrivial graph with exactly $2k$ odd vertices, the minimum number of trails that decompose it is max $\{k, 1\}$. **[8]**

b) Show that if G is a simple n-vertex graph with $\delta(G) \geq \frac{(n-1)}{2}$, then G is connected. **[4]**

c) Decompose K_7 into copies of C_n, for $n = 5, 7, 9$. **[4]**

Q4) a) Prove that if T is a tree with k edges and G is a simple graph with $\delta(G) \geq k$, then T is a subgraph of G. **[7]**

b) Show that a simple graph is a forest if and only if every induced subgraph has a vertex of degree at most 1. **[3]**

c) Show that there exists a simple graph with 12 vertices and 28 edges such that the degree of each vertex is either 3 or 5. Draw this graph. **[6]**

P.T.O.

Q5) a) Let $_T$(G) denote the number of spanning trees of a graph G. Prove that if $e \in E(G)$ is not a loop, then ${}_T(G) = {}_T(G-e) + {}_T(G.e)$. **[7]**

b) Let T be a tree with average degree a. Determine $n(T)$ in terms of a. **[3]**

c) Prove that P_n is the only tree that maximizes D(T), where $D(T) = \sum_{u,v} d(u,v)$ is the Winer index of G. **[6]**

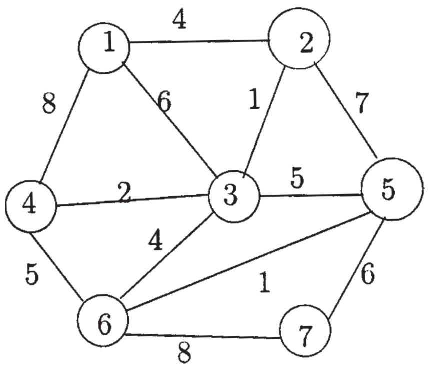
Q6) a) Let $\alpha'(G), \beta'(G)$ and $n(G)$ denotes maximum size of matching, minimum size of edge cover and number of vertices in G respectively. Prove that if G is a graph without isolated vertices, then $\alpha'(G) + \beta'(G) = n(G)$. **[10]**

b) Prove that every component of the symmetric difference of two matchings is a path or an even cycle. **[6]**

Q7) a) Prove that if G is a bipartite graph, then the maximum size of a matching in G equals the minimum size of a vertex cover of G. **[7]**

b) Find (with proof) the smallest 3-regular simple graph having connectivity 1. **[4]**

c) Use Dijkstra's algorithm to find shortest distance from a vertex 1 to any other vertex in the following graph. **[5]**



Q8) a) Prove that in a connected weighted graph G, Kruskal's Algorithm constructs a minimum-weight spanning tree. **[6]**

b) Explain the Ford-Fulkerson labeling algorithm to find an f-augmenting path. **[5]**

c) Prove that if G is a connected graph, then an edge cut F is a bond if and only if $G - F$ has exactly two components. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P1790

[Total No. of Pages : 3

[5321]-41
M.A./M.Sc.
MATHEMATICS
MT - 801 : Field Theory
(2008 Pattern) (Semester - IV) (Old)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *All questions carry equal marks.*
3) *Figures to the right indicate full marks.*

Q1) a) Let F be a field and let $f(x) \in F[x]$ be a polynomial of degree 2 or 3. Then show that $f(x)$ is reducible if and only it has a root in F. **[6]**

b) Let E be an extension of a field F. If K is the subset of E consisting of all the elements that are algebraic over F, then show that K is a subfield of E and also that K is an algebraic extension of F. **[6]**

c) Determine the minimal polynomials of $\sqrt{2}+5$ and $\sqrt{-1+\sqrt{2}}$ over $\mathbb{Q}$. **[4]**

Q2) a) Let E be a finite extension of a field F. Then prove that E = F (α) for some $\alpha \in E$ if and only if there are only finite number of intermediate fields between F and E. **[8]**

b) Show that $x^3 - x - 1 \in \mathbb{Q}[x]$ is irreducible over $\mathbb{Q}$. **[4]**

c) Let E be the splitting field of a polynomial of degree n over a field F. Show that $[E : F] \le n!$. **[4]**

Q3) a) Let $F \subseteq E \subseteq K$ be fields. If $[K : E] < \infty$ and $[E : F] < \infty$ then show that **[6]**

i) $[K:F] < \infty$ and

ii) $[K:F] = [K:E][E:F]$.

b) Show that the splitting field of $f(x) = x^4 - 2 \in \mathbb{Q}[x]$ over $\mathbb{Q}$ is $\mathbb{Q}\left(2^{1/4}, i\right)$, and its degree of extension is 8. **[6]**

c) Show that every finite extension of a finite field is normal. **[4]**

P.T.O.

Q4) a) Let F be a finite field. Prove that **[8]**

i) The characteristic of F is a prime number p,

ii) F contains a subfield $F_p \simeq \mathbb{Z}/(p)$ and

iii) The number of elements of F is p^n for some positive integer n.

b) Let E be a finite extension of F. Then show that E is a normal extension of F if and only if E is a splitting field of a polynomial $f(x) \in F[x]$. **[4]**

c) Let $f(x)$ be a polynomial over a field F with no multiple roots. Show that $f(x)$ is irreducible over F and only if the Galois group G of $f(x)$ is isomorphic to a transitive permutation group. **[4]**

Q5) a) Show that it is impossible to construct a cube with a volume equal to twice the volume of a given cube by using ruler and compass only. **[6]**

b) Let E be the splitting field of $x^4 - 2 \in \mathbb{Q}[x]$ over $\mathbb{Q}$. Show that Galois group $G(E/\mathbb{Q})$ is isomorphic to the group of symmetries of a square. **[6]**

c) Let K = F (x) be the field of rational functions in one variable x over a field F of characteristic 3. Show that the polynomial $y^3 - x$ in the polynomial ring K[y] is irreducible over K and has multiple roots. **[4]**

Q6) a) Let K and K$'$ be algebraic closures of a field F. Show that $K \simeq K'$ under an isomorphism that is an identity on F. **[6]**

b) Prove that the multiplicative group of nonzero elements of a finite field is cyclic and deduce that finite extension of a finite field is simple. **[6]**

c) Let F be field of characteristic not equal to 2. If $x^2 - a \in F[x]$ is an irreducible polynomial over F then show that its Galois group is of order 2. **[4]**

Q7) a) Let E be a finite separable extension of a field F. Prove that E is a normal extension of F if and only if F is the fixed field of Galois group G (E/F). **[8]**

b) Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radicals over $\mathbb{Q}$. **[4]**

c) Show that the Galois group of $x^4 + 1 \in \mathbb{Q}[x]$ is the Klein four-group. **[4]**

Q8) a) Let H be a finite subgroup of the group of automorphisms of a field E. Prove that $[E : E_H] = |H|$, where $E_H = \{x \in E : \sigma(x) = x \text{ for all } \sigma \in H\}$. **[8]**

b) Show that the angle $2\pi/5$ can be trisected using ruler and compass. **[4]**

c) If a field F contains a primitive n^{th} root of unity, then show that the characteristic of F is 0 or a prime p that does not divide n. **[4]**

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Total No. of Questions :8]

SEAT No. :

P1791

[Total No. of Pages : 3

[5321]-42
M.A./M.Sc.
MATHEMATICS
MT-802 : Combinatorics
(2008 Pattern) (Semester-IV)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) How many 8-digit sequences are there involving exactly six different digits? **[6]**

b) How many permutations of the 26 letters are there that contain none of the sequences MATH, RUNS, FROM or JOE? **[6]**

c) Find all derangements of 1, 2, 3, 4, 5 with the help of associated chessboard of darkened squares. **[4]**

Q2) a) i) How many even five digit numbers (leading zeros not allowed) are there? **[6]**

ii) How many five-digit numbers are there with exactly one 3?

iii) How many five digit numbers are there that are the same when the order of their digits is inverted (e.g. 15251)?

b) Find ordinary generating function whose coefficient a_r equals r. Hence evaluate the sum 0+1+2+......+n. **[6]**

c) Find a generating function for the number of integers between 0 and 9,99,999 whose sum of digits is r. **[4]**

Q3) a) How many ways are there to collect \$24 from 4 children and 6 adults if each person gives atleast \$1, but each child can give at most \$4 and each adult at most \$7? **[6]**

b) Prove by combinatorial argument $\binom{r}{r}+\binom{r+1}{r}+.....+\binom{n}{r}=\binom{n+1}{r+1}$.

Hence, evaluate the sum 1×2×3+2×3×4+......+(n–2) (n–1)n. **[6]**

c) How many nonnegative integer solutions are there to the in equalities

$x_1 + x_2 + ... + x_6 \leq 20$ and $x_1 + x_2 + x_3 \leq 7$? **[4]**

P.T.O.

Q4) a) How many ways are there to make an r-arrangement of pennies, nickels, dimes and quarters with at least one penny and an odd number of quarters? **[6]**

b) Solve the recurrence relation. **[6]**

$a_n = a_{n-1} + 3n^2$ with $a_0 = 10$

c) Show that in any set of n integers, n ≥ 3 there always exists a pair of integers whose difference is divisible by n–1. **[4]**

Q5) a) How many ways are there to assign 20 different people to three different rooms with atleast one person in each room? **[6]**

b) How many sequences of length 5 can be formed using the digits 0, 1, 2,.... 9 with the property that exactly two of the 10 digits appear. **[6]**

c) How many words can be formed by rearranging INQUISITIVE so that U does not immediately followed Q? **[4]**

Q6) a) Using generating function, solve the recurrence relation **[6]**

$a_n = 2a_{n-1} + 2^n$ with $a_0 = 1$

b) How many 10 letter words are there in which each of the letters e, n, r, s occur **[6]**

i) at most once?

ii) at least once?

c) Solve the recurrence relation $a_n = 4a_{n/2} + 3n$

(assuming that n is a power of 2). **[4]**

Q7) a) Suppose a bookcase has 200 books, 70 in French and 100 about mathematics. How many non-French books not about mathematics are there if. **[6]**

i) There are 30 french mathematics books?

ii) There are 60 French nonmathematics books?

b) Solve the recurrence relation $a_n^2 = 2a_{n-1}^2 + 1$ with $G_0=1$ **[6]**

c) Show that any subset of n+1 distinct integers between 2 and 2n (n ≥ 2) always contains a-pair of integers with no common divisor. **[4]**

Q8) a) How many ways are there to assign 6 city cars, denoted C_1, C_2, C_3, C_4, C_5, C_6, to 6 persons denoted P_1, P_2, P_3, P_4, P_5, P_6,; if person P_1 will not drive cars C_2 and C_4; P_2 will not drive cars C_1 or C_5; Person P_3 drives all cars; Person P_4 will not drive cars 'C_2 or C_5; P_5 will not drive C_4 and person P_6 will not drive C_6? **[8]**

b) An elf has a staircase of n stairs to climb. Each step it takes can cover either one stair or two stairs. Find and solve the recurrence relation for an, the number of different ways for the elf to ascend the n-stair staircase. **[8]**

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Total No. of Questions : 8]

SEAT No :

P 1792

[Total No. of Pages :3

[5321]-43

M.A./M.Sc.

MATHEMATICS

MT - 803: Differentiable Manifolds

(2008 Pattern) (Semester-IV)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Define orientation of a manifold M and induced orientation on ∂M. **[4]**

b) State Green's theorem for compact, oriented 2-manifold. **[4]**

c) Let $\alpha:(0,1)^2 \to \mathbb{R}^3$ be given by $\alpha(u,v)=(u,v,u^2+v^2+1)$. Let Y be the image set of α. **[8]**

Evaluate $\int_Y x_2 dx_2 \wedge dx_3 + x_1 x_3 dx_1 \wedge dx_3$.

Q2) a) What is the dimension of $A^k(V)$, the space of alternating k - tensors on an n dimensional vector space V? Justify. **[8]**

b) State Stoke's theorem. **[4]**

c) Define a closed form and give an example. **[4]**

Q3) a) Let U be an open set in $\mathbb{R}^n$ and $f:U\to\mathbb{R}^n$ be a class C^r. Let $M=\{x: f(x)=0$ and $N=\{x: f(x)\geq 0$. If M is nonempty and D$f(x)$ has rank one at each point of M, then prove that N is an n - manifold in $\mathbb{R}^n$ and $\partial N = M$. **[8]**

b) Define an exact form and give an example. **[4]**

c) Give an example of a manifold without boundary. Justify. **[4]**

P.T.O.

Q4) a) Define the differential operator d and for any k - form ω, show that $d(d\omega)=0$. **[7]**

b) Show that $g(X, Y, Z) = \det\begin{pmatrix} x_i & y_i & z_i \\ x_j & y_j & z_j \\ x_k & y_k & z_k \end{pmatrix}$ is an alternating 3 - tensor on $\mathbb{R}^n$. Further, express g as a combination of elementary tensors. **[6]**

c) Define volume of parametrized surface in $\mathbb{R}^n$. **[3]**

Q5) a) Let F be a k - tensor. With usual notation, if $AF = \sum_{\sigma\in S_k}(\operatorname{sign}\sigma)F^\sigma$, then prove that AF is an alternating tensor. Find AF if F is already alternating. **[7]**

b) If $\omega = x^2 yzdx + xyzdy + ze^x ydz$ and $\eta = yz\sin xdx + xyzdy + 2xyzdz$, then find $(\omega\wedge\eta)$. **[5]**

c) Define an alternating tensor and give an example. **[4]**

Q6) a) Let M be a k - manifold in $\mathbb{R}^n$. If ∂M is nonempty, then prove that ∂M is a k - 1 manifold without boundary. **[7]**

b) If $\omega = x^2yz^2dx + 2xz\cos ydy + xye^z dz$ find $d\omega$. **[5]**

c) Find the tangent plane $T_p(S^2)$ where $p \equiv \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$. **[4]**

Q7) a) With usual notation, show that $\alpha^*(d\omega) = d(\alpha^*\omega)$. **[8]**

b) Let $A = \mathbb{R}^2 - \{0\}$. If $\omega = \dfrac{xdx + ydy}{x^2 + y^2}$, then show that ω is closed and exact on A. **[8]**

Q8) a) It $T: V \to W$ is a linear transformation, and if f and g are alternating tensors on W, then prove that $T^*(f \wedge g) = T^* f \wedge T^* g$. **[8]**

b) Let $\omega = y^2 z dx + x^2 z dy + x^2 y dz$, and $\alpha(u,v) = (u - v, uv, u^2)$. Find $\alpha^*(d\omega)$. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P1793

[Total No. of Pages : 2

[5321]-44
M.A./M.Sc.
MATHEMATICS
MT - 804 : Algebraic Topology
(2008 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Solve any five questions.*
2) *Figures to the right indicate marks.*

Q1) a) Prove that the homotopy relation between continuous functions is an equivalence relation. **[6]**

b) Let $f, g : X \to S^n$ be continuous mappings such that $f(x) \neq -g(x)$ for all $x \in X$. Show that f is homotopic to g. **[5]**

c) Let f and g be homotopic mappings of X into Y and let h be a continuous mapping of Y into Z. Prove that $h\,f$ and $h\,g$ are homotopic mappings of X into Z. **[5]**

Q2) a) Define a contractible space. Prove that if Y is contractible, then every continuous function $f : X \to Y$ is homotopic to a constant. **[6]**

b) Define: **[5]**

i) A deformation retract and

ii) A strong deformation retract

prove that S^n is a strong deformation retract of $\mathbb{R}^{n+1} - 0$.

c) Let $A \subset B \subset X$. Suppose that B is a retract of A and A is a retract of X. Show that B is a retract of X. **[5]**

Q3) a) Prove that every non-empty open connected subset of $\mathbb{R}^n$ is path connected. **[6]**

b) Prove that a path connected space is connected but not conversely. **[5]**

c) Let $A \subset X$ and $\{A_i : i = 1, 2, ...\}$ is a collection of connected subsets of X each of which intersects A. Show that $A \cup \{\cup_{i=1}^{\infty} A_i\}$ is connected. **[5]**

P.T.O.

Q4) a) With usual notations, prove that $\pi_1(X, x_0)$ is a group. **[6]**

b) Let $x_0, x_1 \in X$. Prove that if there is a path in X from x_0 to x_1, then the fundamental groups $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are isomorphic. **[5]**

c) Prove that the fundamental group of the real projective plane is isomorphic to a cyclic group of order two. **[5]**

Q5) a) Prove that the fundamental group $\pi_1(S^1)$ of the circle S^1 is isomorphic to the additive group $\mathbb{Z}$ of integers. **[6]**

b) Prove that the circle S^1 is not a retract of the disc B^2. **[5]**

c) Determine the fundamental groups of torus and $\mathbb{R}^n$. **[5]**

Q6) a) Using the techniques of Algebraic Topology, prove that every non-constant polynomial has a root. **[6]**

b) Determine the fundamental groups of Sphere, $\mathbb{R}^2 - 0$ and $S^1 \times \mathbb{R}$. **[5]**

c) Define a fibration. Prove that the composition of fibrations with unique path lifting is a fibration with unique path lifting. **[5]**

Q7) a) Define a covering map. Prove that a covering map is a local homomorphism but the converse is not true. **[6]**

b) Prove that a covering map $p : \tilde{X} \to X$ is open. **[5]**

c) Let $p : \tilde{X} \to X$ and $q : \tilde{Y} \to Y$ are covering maps. Show that $p \times q : \tilde{X} \times \tilde{Y} \to X \times Y$ is also a covering map. **[5]**

Q8) a) Prove that a fibration has unique path lifting if and only if every fiber has no non-null path. **[6]**

b) Prove that the closed ball $B^n \, (n \geq 1)$ has the fixed point property. **[6]**

c) Prove that $\mathbb{R}^n$ is homeomorphic to $\mathbb{R}^m$ if and only if $n = m$ **[4]**

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Total No. of Questions : 8]

SEAT No. :

P1794

[Total No. of Pages : 2

[5321] - 45

M/A./M.Sc.

MATHEMATICS

MT-805: Lattice Theory

(2008 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicates full marks.*

Q1) a) Prove that L is distributive so is $I(L)$. **[5]**

b) Prove that a poset $(L, \leq)$ is a lattice if and only if sup(H) and inf(H) exist for any nonempty finite subset of H of L. **[6]**

c) If a poset satisfies Ascending Chain Condition then prove that it has maximal element. **[5]**

Q2) a) Draw all non isomorphic lattice with six element. **[5]**

b) Prove that homomorphic image of distributive lattice is distributive. **[6]**

c) Let Θ be a congruence relation on L then prove that for $a \in L, [a]\Theta$ is convex sublattice of L. **[5]**

Q3) a) Prove that I is a prime ideal of L if and only if there is a homomorphism ϕ of L onto C_2 with $I = \phi^{-1}(0)$. **[5]**

b) Let I be an ideal and let D be dual ideal if $I \cap D \neq \phi$, then prove that $I \cap D$ is convex sublattice and every convex sublattice can be expressed in this form in one and only one way. **[7]**

c) Draw Hasse diagram of divisors of 30 with $a \leq b$ if and only if a divides b. **[4]**

P.T.O.

Q4) a) Prove that in a bounded distributive lattice, an element can have only one complement. **[6]**

b) If P be a poset, $\wedge H$ exists for all $H \subset P$ then prove that P is complete lattice. **[6]**

c) Prove that collection of all complemented elements in a distributive lattice forms a sublattice. **[4]**

Q5) a) Define: Distributive Element, Normal Element, Neutral Element. **[5]**

b) Prove that a collection of subgroup of any group G is lattice $L(G)$. **[6]**

c) Prove that every ideal I of a distributive lattice is the intersection of all prime ideals containing it. **[5]**

Q6) a) Prove that every homomorphic image of a lattice L is isomorphic to a suitable quotient lattice of L. **[5]**

b) Let L be distributive lattice, $a, b \in L$ and $a \neq b$, then prove that there exist a prime ideal P of L containing exactly one of a and b. **[5]**

c) Prove that a lattice is Boolean if and only if it is isomorphic to ring of sets. **[6]**

Q7) a) Prove that a lattice L is modular if and only if it does not contain a sublattice isomorphic to N_5. **[9]**

b) Prove that every modular lattice is semimodular. Is the converse true? Justify your answer. **[7]**

Q8) a) Prove that every distributive lattice satisfies median condition. **[3]**

b) State and prove Jordan-Holder Theorem for semimodular lattices. **[7]**

c) Let L be a bounded distributive lattice with $0 \neq 1$. Then prove that L is a Boolean lattice if and only if $P(L)$ is unordered. **[6]**

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