Total No. of Questions :8]

SEAT No. :

P1247

[Total No. of Pages : 2

[5121]-101
M.A./M.Sc. (Semester - I)
MATHEMATICS
MT- 501: Real Analysis
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define rectangles and almost disjoint rectangles in $\mathbb{R}^d$. If a rectangle R is the almost disjoint union of finitely many other rectangles, say $R = \bigcup_{k=1}^{N} R_k$ then with usual notations prove that $|R| = \sum_{K=1}^{N} |R_k)$ **[5]**

b) Find i) interior of $A = (1,4) \cup \{5\} \subset R$

ii) Bd (B) = $\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \geq 1\}$ = Boundary of B. **[3]**

iii) Limit points of Q

c) Define a compact set in $\mathbb{R}^d$. Whether (–10,10) is compact? Justify. **[2]**

Q2) a) Define exterior measure of $E \subset \mathbb{R}^d$ and show that exterior measure of a closed cube is its volume. **[5]**

b) Show that an exterior measure is countably subadditive. **[3]**

c) Calculate exterior measure of a finite set **[2]**

$A = \{x_1, x_2, ...x_n / x_i \in R, \text{ for } i = 1,..,n\}$

Q3) a) Define a Lebesgue measurable set $E \subset \mathbb{R}^d$. Show that countable union of Lebesgue measurable sets is also a measurable set. **[5]**

b) If $\{E_1, E_2, ..., E_n, ...\}$ is a collection of measurable sets in $\mathbb{R}^d$ and $E_k \searrow E$ with $m(E_k) < \infty$ for some K then show that m (E) $= \lim_{n\to\infty}$ M (E_n).
Check validity of above statement. for $E_n = (n, \infty)$
where n = 1,2,3... **[3]**

c) Show that if *f* and *g* are two measurable functions then their product is also measurable. **[2]**

P.T.O.

Q4) a) Show that a Riemann Integrable function is also a Lebesgue integrable function; but the converse is not true. Justify with an example. **[5]**

b) If E and F are subsets of $\mathbb{R}^d$ then show that $\chi_{E \cup F} = \chi_E + \chi_F - \chi_{E \cap F}$ **[3]**

c) State any two properties of Lebesgue integral of non-negative measurable functions. **[2]**

Q5) a) Define Lebesgue integral of a measurable function f on $\mathbb{R}^d$. Show that f is Lebesgue integrable iff $|f|$ is Lebesgue integrable. **[5]**

b) Verify Fatou's lemma for $\{f_n\}$ where f_n is given by **[3]**

$$f_n(x) = n \ \text{if} \ \frac{1}{n} \le x \le \frac{2}{n}$$

$$= 0 \ \text{otherwise.}$$

c) State monotone convergence theorem for a sequence of non-negative measurable function. **[2]**

Q6) a) When two functions f and g are said to be equivalent in $L^1(\mathbb{R}^d)$? Hence prove.

i) $\| \alpha f \|_{L^1(R^d)} = |\alpha| \, \| f \|_{L^1(R^d)}$ **[5]**

ii) $\| f + g \|_{L^1(R^d)} \le \| f \|_{L^1(R^d)} + \| g \|_{L^1(R^d)}$

b) State Lebesgue differentiation theorem. **[3]**

c) Give statement of fubini's theorem. **[2]**

Q7) a) Define Dini derivatives and find them for $f(x) = |x|$, $x \in \mathbb{R}$. **[5]**

b) Define an absolutely continuous function and show that an absolutely continuous function is continuous and uniformely continuous. **[5]**

Q8) a) State and prove rising sun lemma. **[5]**

b) State what do you mean by boundeded variation. Show that a function of bounded variation is always bounded but converse may not be true. **[5]**

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Total No. of Questions :8]

SEAT No. :

P1248

[Total No. of Pages : 3

[5121]-102
M.A./M.Sc (Semester - I)
MATHEMATICS
MT- 502:Advanced Calculus
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let f: $S \to \mathbb{R}$, S$\subset\mathbb{R}^n$ be a scalar field defined on S. Assume that the partial derivative $D_1 f, D_2 f, \ldots, D_n f$ exists in some n – ball $B(\bar{a}) \subset S$ and are continuous at $\bar{a}$. Then prove that f is differentiable at $\bar{a}$. **[5]**

b) A differentiable scalar field f has, at the point (1,2) directional derivative +2 in the direction towards (2,2) and –2 in the direction towards (1,1). Determine the gradient vector at (1,2) and compute the directional derivative in the direction towards (4,6) **[3]**

c) Let $\bar{f} : \mathbb{R}^n \to \mathbb{R}^m$ be a vector field and $f_1, \ldots, f_m$ are components of the vector field $\bar{f}$.Prove that $\bar{f}$ is continuous at a point if, and only if, each component f_K is continuous at that point. **[2]**

Q2) a) Let $\bar{f} : \mathbb{R}^n \to \mathbb{R}^m$ is a vector field differentiable at $\bar{a}$ with total derivative $\bar{T}_{\bar{a}}$, then prove that $\bar{T}_{\bar{a}}(\bar{y}) = (\nabla f_1(\bar{a}), \bar{y}\nabla f_2(\bar{a})\bar{y}, \ldots, \nabla f_m(\bar{a})\bar{y})$ where $\bar{f} = (f_1, \ldots, f_m)$ *and* $\bar{y} = (y_1, \ldots y_n)$. **[5]**

b) Let f be a non constant scalar field, differentiable everywhere in the plane, and Let C be a constant.Assume the cartesian equation $f(x,y) = C$ describes a curve C having tangent at each of its points. Prove that f has the following properties at each point of C: **[3]**

i) The gradient vector ∇f is normal to C.
ii) The directional derivative of f is zero along C.
iii) The directional derivative of f has its largest value in a direction normal to C.

P.T.O.

c) Let the two equations $x = e^u$ cas v and $y = e^u \sin v$ defines $u = U(x,y)$ and $v = V(x,y)$. Find explicit formula for $U(x,y)$ and $V(x,y)$ when $x > 0$. **[2]**

Q3) a) Define line integral. Show that a line integral remains unchanaged under a change of parameter that preserves orientation, it reverses its sign if the change of parameter reverses orientation. **[4]**

b) A particle of mass m moves along a curve under the action of a force field $\overline{f}$. If the speed of the particle at time t is V (t), its kinetic energy is defined to be $\frac{1}{2}MV^2(t)$. Prove that the change in kinetic energy in any time interval is equal to the work done by $\overline{f}$ during this time interval. **[4]**

c) Calculate the line integral of $\overline{f} = (x, y, z) = (y^2 - z^2)\overline{i} + 2yz\,\overline{j} - x^2\overline{k}j$ along the path described by $\overline{\alpha}(t) = t\overline{i} + t^2\overline{j} + t^3\overline{k}$ **[2]**

Q4) a) Prove that the line integral of a gradient is independent of the path in any open connected set in which the gradient is continuous. **[5]**

b) A force field $\overline{f}$ in 3-space is given by $\overline{f}(x, y, z) = x\overline{i} + y\overline{j} + (xz - y)\overline{k}$. compute the work done by this force in moving a particle from (0,0,0) to (1,2,4) along the line segment joining these two points. **[3]**

c) Determine whether or not the vector field. **[2]**

$\overline{f}(x, y, z) = 2xy^3 i + x^2 z^2 \overline{j} + 3x^2 yz^2 \overline{k}$. is a gradient on any open subset of $\mathbb{R}^2$.

Q5) a) Let f be bounded function on a rectangle Q in $\mathbb{R}^2$. Show that upper integral $\overline{I}(f)$ and lower integral $\underline{I}(f)$ exist. Also prove that f is integrable over Q if and only if $\overline{I}(f) = \underline{I}(f)$. **[4]**

b) Evaluate $\iint\limits_Q \left(\sqrt{y} + x - 3xy^2\right) dx\, dy$ where $Q = [0,1] \times [1,3]$ **[3]**

c) Use Green's theorem to compute the work done by the force field $\overline{f}(x, y) = (y + 3x)\overline{i} + (2y - x)\overline{j}$ in moving a particle once around the ellipse $4x^2 + y^2 = 4$ in the counterclockwise direction. **[3]**

Q6) a) Define fundamental vector product of a parametric surface.find the fundamental vector product where.

$\bar{r}(u,v) = a \cos u \cos v\bar{i} + a \sin u \cos v\bar{j} + a \sin v\bar{k}$

$(u,v) \in T = [o, 2\pi] \times \left[o, \frac{\pi}{2}\right]$. What are singular points of this surface? **[4]**

b) A parametric surface is described by the vector equation.

$\bar{r}(u,v) = u\cos v\bar{i} + u\sin v\bar{j} + u^2\bar{k}$ where $0 \le u \le 4$ *and* $0 \le V \le 2\pi$.

Compute the area of this surface. **[4]**

c) Define surface integral and explain the terms involved in it. **[2]**

Q7) a) State the formula for change of variables in double integrals. Prove this formula for particular case when the region of integration is rectangle and the function with constant value1. **[5]**

b) Evaluate $\iiint_S dx\,dy\,dz$ where S is a solid sphere of radius a and center at origin by transforming to spherical co-ordinates. **[5]**

Q8) a) State and prove stoke's theorem. **[5]**

b) Determine the Jacobian matrix and compute divergence and curl of $\bar{F}$ where $\bar{F}(x,y,z) = x^2 \sin y\bar{i} + y^2 \sin(xz)\bar{j} + xy\sin(\cos z)\bar{k}$ **[5]**

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Total No. of Questions :8]

SEAT No. :

P1249

[Total No. of Pages : 2

[5121]-103
M.A./M.Sc (Semester - I)
MATHEMATICS
MT- 503 Group Theory
(2013 Pattern) (Credit System)

Time : 3 Hour] *[Max. Marks :50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Show that $\mathbb{R}^2$ forms a group under the map $T_{(a,b)}$, where a,b are fixed real numbers and $T_{(a,b)}$ is defined as $T_{(a,b)}(x,y) = (x+a, y+b)$. **[4]**

b) Let G be a group of rigid motions of tetrahedron in $\mathbb{R}^3$. Show that $|G| = 12$. **[3]**

c) Find order of $A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ in $SL(2, Zp)$ where P is a prime. **[3]**

Q2) a) State and prove lagrauye's theorem. **[5]**

b) Is $Z_2 \times Z_2$ cyclic? Justify. **[2]**

c) Show that any cyclic group of order in is isomorphic To Z_n. **[3]**

Q3) a) Show that order of A_n is $\frac{n!}{2}$ **[4]**

b) Show that $Z(S_n)$ is trivial. **[3]**

c) Show that a permutation with odd order must be an ever premutation. **[3]**

Q4) a) Prove that for every positive integer n, Aut (Z_n) is isomorphic to U (n), **[4]**

b) Show that Inn $(G) \cong \frac{G}{Z(G)}$ also find Inn (S_n). **[4]**

c) Is $(\mathbb{R}^*, \bullet) \cong (Q, +)$? Justify. **[2]**

Q5) a) Let G be a non-abelian group of order P^3 (P is a prime) and Z (G) $\neq \{e\}$. Prove that $|Z(G)|=P$. **[5]**

b) If H and K are subgroups of a group G. then show that HK is a subgroup of G if and only if HK=KH. **[5]**

Q6) a) State and prove fundamental theorem of finite abeliam groups. **[5]**

b) Show that $G \oplus H$ is abelian group if and only if G and H are abelian groups. **[3]**

c) How many abeliam groups upto isomorphism are there having order 4Z? **[2]**

Q7) a) State and prove sylow's third theorem. **[5]**

b) Let $|G| = 2P$, where P is an odd prime prove that G is either isomorphic to Z_{2p} or Dp. **[5]**

Q8) a) Show that group of order 175 is abelian. **[4]**

b) Prove that there is no simple group of order 396. **[3]**

c) How many sylow 5-subgroups of S_5 are there? Justify. **[3]**

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Total No. of Questions :8]

SEAT No. :

P1250

[Total No. of Pages : 4

[5121]-104
M.A./M.Sc (Semester - I)
MATHEMATICS
MT- 504: Numerical Analysis
(2013 Pattern) (Credit System)

Time : 3 Hours] ***[Max. Marks :50***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *Use of non-programmable, scientific calculator is allowed.*

Q1) a) Let f be a twice continuously differential function on the interval [a,b] with P ∈ (a,b) and f (p) = 0. Further, suppose that f '(P)≠0. Then show that there exist a δ > 0 such that for $P_o \in I=[p-\delta, P+\delta]$. the sequence $\{P_n\}$ generated by Newton's method converges to P. **[5]**

b) Determine the rate of convergence of the function $f(x)=\dfrac{\cos x-1+\frac{1}{2}x^2}{x^4}$ **[3]**

c) Show that when Newton's method is applied to the equation $\dfrac{1}{x}-a=0,$ the resulting iteration function is g (*x*) = *x* (2–*ax*) **[2]**

Q2) a) Show that the function $g(x) = e^{-x}$ has a unique fixed point near x = 0.6 by using fixed point Iteration method and starting Value $P_o = 0$ (Do at least 5 iterations) **[5]**

b) Verify that the equation $x^4-18x^2+45=0$ has a root on the interval (1,2), perform five iterations of the secant method, using $P_o = 1$ and $P_2 = 2$. **[3]**

c) Given $x=\sqrt{a}$ is a fixed point of the function $g(x)=\dfrac{x^3+3xa}{3x^2+a}$ **[2]**

Determine the order of convergence and the asymptotic error constant of the sequence $P_n = g(P_{n-1})$ towards $x=\sqrt{a}$

P.T.O.

Q3) a) Solve the following system of equation by using Gaussian elimination with partial pivoting. **[5]**

$$x_2 + x_3 + x_4 = 0$$
$$3x_1 + 3x_3 - 4x_4 = 7$$
$$x_1 + x_2 + x_3 + 2x_4 = 6$$
$$2x_1 + 3x_2 + x_3 + 3x_4 = 6$$

b) Explain the QR algorithm for finding eigenvalues of symmetric tridigonal matrix. **[3]**

c) Compute the condition number K_∞ for the matrix A=$\begin{bmatrix} 1 & -2 \\ -0.99 & 1.99 \end{bmatrix}$ **[2]**

Q4) a) Solve the following system of linear equations by Gauss - seidel method. start with $x^{(0)} = [0 \quad 0 \quad 0]^T$ (Perform 3 iterations)

$4x_1 - x_2 = 2$; $-x_1 + 4x_2 - x_3 = 4$; $-x_2 + 4x_3 = 10$ **[5]**

b) Solve the following system of linear equations by SOR method, start with $x^{(0)} = [0 \quad 0 \quad 0]^T$ and W = 0.9 (Perform 2– iterations). **[3]**

c) Show that the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ has no LU decomposition. **[2]**

Q5) a) Solve the following non-linear system of equations by Newtons method. (Starting Vector $x^{(0)} = [1 \quad 1 \quad 1]^T$) **[5]**

$$x_1^3 - 2x_2 - 2 = 0$$
$$x_1^3 - 5x_3^2 + 7 = 0$$
$$x_2 x_3^2 - 1 = 0$$

(Perform 2-iterations)

b) Approximate the smallest eigenvalue and its associated eigenvector for the matrix. **[3]**

$$A = \begin{bmatrix} 5 & -2 & 2 \\ 4 & -3 & 4 \\ 3 & -6 & 7 \end{bmatrix}$$

Use a convergence tolerance of 5×10^{-5}

c) For the non-linear system. **[2]**

$$x_1 + x_2 - x_1^3 = 0$$
$$x_1 + x_2 + x_2^3 = 0$$

Compute the Jacobian of F.

Q6) a) Derive the forward difference approximations for the second derivative. **[5]**

$$f''(x_o) \approx \frac{f(x_o) - 2f(x_o + h) + f(x_o + 2h)}{h^2}$$

What is the error term associated with this formula? Numerically verify the order of approximations using $f(x) = e^x$ and $x_o = 0$

b) Approximate the value of the integral $\int_0^1 \frac{1}{1+x^2} dx$ using the trapezoidal rule. Verify that the theoretical error bound holds. **[2]**

c) Determine the values for the coefficients A_o, A_1 and A_2 so that the quadrature formula.

$$I(f) = \int_{-1}^{1} f(x)dx = A_o f\left(\frac{-1}{3}\right) + A_1 f\left(\frac{1}{3}\right) + A_2 f(1)$$

has degree of precision atleast 2. **[3]**

Q7) a) Verify that the composite midpoint rule has rate of convergence $O(h^2)$ by approximating the value of $\int_o^1 \sqrt{1+x^3}\,dx$ **[5]**

b) Apply Euler's method to approximate solution of the initial value problem.

$$\frac{dx}{dt} = 1 + \frac{x}{t}, \quad 1 \le t \le 3, \; x(1) = 1$$

Using 4 steps. Find the corresponding error in each step. **[5]**

Q8) a) Find solution of the initial value problem

$$\frac{dx}{dt} = \frac{t}{x}, \quad 0 \le t \le 5, \ x(0) = 1$$

Using fourth order Runge Kutta method with a step size h = 1 **[5]**

b) Define: **[5]**

i) Rate of convergence.

ii) House holder matrix.

iii) The degree of precision.

iv) Triangular matrix.

v) Orthogonal matrix.

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Total No. of Questions :8]

SEAT No. :

P1251

[Total No. of Pages : 3

[5121]-105
M.A./M.Sc (Semester - I)
MATHEMATICS
MT- 505:Ordinary Differential Equations
(2013 Pattern) (Credit System)

Time : 3 Hours] ***[Max. Marks :50***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $y_1(x)$ and $y_2(x)$ are two solutions of equation $y'' + P(x)y' + Q(x)\ y = 0$ on [a,b], then prove that their wronskian W (y_1, y_2) is identically equal to zero or never zero on [a,b]. **[5]**

b) Find the general solution of $x^2 y''+3xy' + 10\ y = 0$ **[3]**

c) Show that $y = c_1e^{2x} + c_2xe^{2x}$ is the general solution of $y'' - 4y' + 4y = 0$ on any interval. **[2]**

Q2) a) Discuss the method of variation of parameters to find the solution of second order differential equation with constant coefficients. **[5]**

b) Find the general solution of $y'' + 4y = 3\sin x$ by using method of undetermined coefficients. **[3]**

c) Verify that $y_1 = x$ is one solution of differential equation $x^2y'' + xy' - y = 0$ and find another solution y_2 and the general solution. **[2]**

Q3) a) State and prove Sturm's separation theorem. **[5]**

b) Let $u(x)$ be any non-trivial solution of $u'' + q(x)u = 0$ where $q(x) > 0$ for all $x > 0$. If $\int_1^\infty q(x)dx = \infty$, then prove that $u(x)$ has infinitely many zeros on the positive x-axis. **[3]**

c) Find the normal form of Bessel's equation $x^2 y'' + xy' + (x^2 - p^2)\ y = 0$. **[2]**

P.T.O.

Q4) a) Find the general solution of the system. **[5]**

$$\frac{dx}{dt} = 7x + 6y$$
$$\frac{dy}{dt} = 2x + 6y$$

b) Find the indicial equation and its root of the differential equation $x^3 y'' + (\cos 2x - 1)\, y' + 2xy = 0$. **[3]**

c) Locate and classify the singular points on the x–axis of $x^3 (x-1)\, y'' - 2(x-1)\, y' + 3xy = 0$. **[2]**

Q5) a) Find the two independent Frobenius series solution of the differential equation $2x^2 y'' + x(2x+1)\, y' - y = 0$. **[5]**

b) Prove that the function $E(x,y) = ax^2 + bxy + cy^2$ is positive definite if and only if $a > o$ and $b^2 - 4ac < 0$. **[3]**

c) Find the critical points of **[2]**

$$\frac{dx}{dt} = y^2 - 5x + 6$$
$$\frac{dy}{dt} = x - y$$

Q6) a) Find the general solution near $x = 0$ of the hypergeometric function.

$x(1-x)\, y'' + [c - (a + b + 1)\, x]\, y' - aby = 0$ where a , b and c are constants. **[5]**

b) Prove that $\lim_{a \to \infty} F\left(a, b, b, \frac{x}{a}\right) = e^x$. **[3]**

c) Replace the differential equation $y'' - x^2 y' + xy = 0$ by an equivalent system of first order equation. **[2]**

Q7) a) If m_1 and m_2 are roots of the auxiliary equation of the system.

$$\frac{dx}{dt} = a_1 x + b_1 y$$

$$\frac{dy}{dt} = a_2 x + b_2 y$$

which are real, distict, and of opposite sign, then prove that the critical point (0,0) is a saddle point. **[5]**

b) Find the exact solution of initial value problem $y' = y^2, y(0) = 1$, starting with $y_0(x) = 1$. Apply Picard's method to calculate $y_1(x), y_2(x), y_3(x)$ and compare it with exact solution. **[5]**

Q8) a) Show that the function $f(x,y) = xy$ satisfies Lipschitz condition on any rectangle $a \leq x \leq b$ and $c \leq y \leq b$; but it does not satisfy a Lipschitz condition on any strip $a \leq x \leq b$ and $-\infty < y < \infty$. **[5]**

b) Find the general solution of $y'' + (1 + x)\, y' - y = 0$ about $x = 0$ by power series. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P1253

[Total No. of Pages : 3

[5121]-202

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT-602 : General Topology

(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let $\mathcal{B}$ be a collection of non-empty sets (not necessarily disjoint). Prove that there exists a function.

$$c:\mathcal{B}\to\bigcup_{B\in\mathcal{B}}B$$

such that $c(B)$ is an element of B for each $B\in\mathcal{B}$. **[5]**

b) Let $X=\{a,b,c\}$. Write all distinct topologies on X. **[3]**

c) If X is any set, then prove that the collection of all one - point subsets of X is a basis for the discrete topology on X. **[2]**

Q2) a) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies τ and τ' respectively on X. Prove that τ' is finer than τ if and only if for each $x\in X$ and each basis element $B\in\mathcal{B}$ containing x, there is a basis element $B'\in B'$ such that $x\in B'\subseteq B$. **[5]**

b) Show that the countable collection $\mathcal{B}=\{(a,b)\,|\,a<b;a \text{ and } b \text{ are rational}\}$ is a basis that generates the standard topology on $\mathbb{R}$. **[3]**

P.T.O.

c) Let Y = [–1, 1], which of the following sets are open in Y? Open in $\mathbb{R}$? Justify. **[2]**

i) $A=\left\{x \mid \frac{1}{2} \leq |x| \leq 1\right\}$

ii) $B=\left\{x \mid \frac{1}{2} < |x| \leq 1\right\}$

Q3) a) Let A be a subset of the topological space X. Prove that $x \in \overline{A}$ if and only if every open set U containing x intersects A. **[4]**

b) Let X be a space satisfying T_1 axiom and A be a subset of X. Prove that the point x is a limit point of A if and only if every neighborhood of x contains infinitely many points of A. **[4]**

c) Let X be an ordered set in the ordered topology and let Y be a subset of X. Whether the order topology on Y is same as the topology that Y inherits as a subspace of X? Justify. **[2]**

Q4) a) Show that the product of two Hausdorff spaces is Hausdorff. **[5]**

b) State and prove pasting lemma. **[3]**

c) In the finite complement topology on $\mathbb{R}$, to what point or points does the sequence $x_n = 1/n$ converge? **[2]**

Q5) a) Let A be any set, X and Y be two topological spaces with $f: A \to X \times Y$ defined by $f(a) = (f_1(a), f_2(a))$. Then show that f is continuous if and only if f_1 and f_2 are continuous, where $f_1 : A \to X$ and $f_2 : A \to Y$. **[5]**

b) Define box topology and product topology. What is the relation between them? **[3]**

c) Give an example of two discountinuous functions whose composite function is a continuous function, with proper justification. **[2]**

Q6) a) State and prove the sequence lemma. **[5]**

b) Show that the image of connected space under a continuous map is connected. **[3]**

c) Define quotient topology and given an example of a quotient map which is not an open map? **[2]**

Q7) a) State and prove the tube lemma. **[5]**

b) Let X be a Hausdorff space. Prove that X is locally compact if and only if for given $x \in X$ and neighborhood U of x, there is neighborhood V of x such that $\bar{V}$ is compact and $\bar{V} \subseteq U$. **[5]**

Q8) a) Show that : **[8]**

i) A subspace of a regular space is regular.

ii) A closed subspace of a normal space is normal.

b) State : **[2]**

i) The Urysohn lemma.

ii) The Tychonoff theorem.

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Total No. of Questions : 8]

SEAT No. :

P1254

[Total No. of Pages : 3

[5121]-203

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT-603 : Rings and Modules

(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let P be a positive prime number. Define $Q^p = \left\{\frac{a}{b} \in Q | b \neq 0, a, b \in Z, p | b\right\}$

Show that Q^p is a subring of rational numbers Q. Also find units of Q^p. **[4]**

b) Let R be a ring with 1. Prove that R is a division ring if and only if (0) and R are the only left ideals in R. **[4]**

c) Suppose R denotes the matrix ring M_2 (Z). Find a matrix $A \in R$ such that A is a zero divisor but not nilpotent. **[2]**

Q2) a) Let R be a ring with 1. Show that R is a local ring if and only if it has a unique maximal ideal. **[4]**

b) Let X be a non-empty set. Let P(X) denote the ring of power set of X under addition is the symmetric difference of sets, and multiplication is the intersection of sets. Find units and idempotents of P(X). **[3]**

c) Show that the characteristic of a simple ring is either zero or a prime number. **[3]**

P.T.O.

Q3) a) Show that the set of all nilpotent elements in a commutative ring R with 1 is the intersection of all prime ideals of R. **[5]**

b) Let S be a subring of the ring R. Show by example that an element $a \in S$ may be zero-divisor in R but not in S. **[2]**

c) Let X be a discrete toplogical space with at least two elements, and let R denote the ring of continuous maps from X to R under pointwise addition and multiplication of maps. Show that R is not an integral domain. **[3]**

Q4) a) Let $f: R \to S$ be a surjective ring homorphism. Show that the inverse image of a maximal ideal in S is a maximal ideal in R. **[4]**

b) Prove or disprove. **[3]**

An element $\overline{5} + \overline{4}x + \overline{6}x^2$ is a unit in $Z_8[x]$.

c) Show that there are infinitely many irreducible polynomials of degree four in Z [x]. **[3]**

Q5) a) Let I be an ideal in a ring R. Prove that I is 2-sided ideal in R if and only if I is the Kernel of some homomorphism $f: R \to S$ for a suitable ring S. **[5]**

b) Let R be a commutative ring with 1. If R [x] is a principal ideal domain, then prove that R is a field. **[3]**

c) Give examples of ring homomorphisms $f: R \to S$ and $g: S \to T$ such that *gof* is a monomorphism but *g* is not. **[2]**

Q6) a) Prove that Euclidean domain has the unity. **[4]**

b) Show that in the ring Z [i], the elements $3 + 4i$ and $4 - 3i$ are associates of each other. **[2]**

c) If I is a 2-sided ideal of R, then prove that the quotient ring R[x]/ I[x] is naturally isomorphic to (R/I) [x]. **[4]**

Q7) a) Show that a gcd of 6 and $2+2i\sqrt{5}$ does not exist in $Z\left[i\sqrt{5}\right]$. **[4]**

b) Prove or disprove.

A subring of UFD is UFD. **[3]**

c) Suppose M and N are submodules of a module P over ring R such that $M \cap N = (0)$. Prove that every element $z \in M + N$ can be uniquely written as $z = x + y$ with $x \in M$ and $y \in N$. **[3]**

Q8) a) Let M and N be simple R-modules. Prove that any R-Linear map $f : M \to N$ is either O or an isomorphism. **[4]**

b) Prove that a vector space is a free module. **[4]**

c) Prove or disprove.

Any minimal submodule is a simple module. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P1255

[Total No. of Pages : 4

[5121]-204

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT-604 : Linear Algebra

(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Answer any five questions.*
2) *Figures to the right indicate full marks.*
3) *Use of non-programmable, scientific calculator is allowed.*

Q1) a) Let $(W_i) i \in \Delta$ be a family of subspaces of a vector space V. Then show that, the following are equivalent. **[5]**

i) $\sum_{i \in \Delta} W_i$ is a direct sum.

ii) If $\sum_{i \in \Delta} X_i = 0$ where $x_i \in W_i$ then xi = 0 for all $i \in \Delta$.

iii) $W_i \cap \sum_{\substack{j \in \Delta \\ j \neq i}} W_j = 0$ for all $i \in \Delta$

b) Show that, in a vector space F^n, the set of n-tuples $(x_1, ..., x_i, ..., x_n)$ with $x_i = 0$ is a subspace. **[3]**

c) Complete the set {(2, 1, 4, 3), (2, 1, 2, 0} to form a basis of R^4. **[2]**

Q2) a) Let U, V be vector spaces over F. Let $(e_1, e_2, ... e_n)$ be an ordered basis of U. Given a list $f_1, f_2, ..., f_n$ of elements of V. Prove that, there is a unique linear mapping $f : U \rightarrow V$ such that

$f(e_i) = f_i, \quad i = 1, 2, ..., n.$

Further, f is an isomorphism if and only if $(f_1, f_2, ..., f_n)$ is a basis of V. **[5]**

b) If W is the subspace of V = $\mathbb{R}^4$. generated by e_1 = (1, 2, 3, 4) and e_2 = (0, 0, 0, 1) then find a basis of V/W. **[3]**

c) If $f : R^2 \to R^2$ is a linear mapping with $f(1,0) = (2,3)$ and $f(0,1) = (-1,1)$ then find $f(a,b)$. **[2]**

Q3) a) Let U, V be vector spaces over F. Then prove that, Hom (U,V) is a vector space over F. Moreover, If dim U = m and dim V = n then dim Hom (U, V) = mn **[5]**

b) Let V be a finite dimensional vector space over F, and let f, g, $\in$ Hom (v, v) such that fg = 1 then show that gf = 1. **[3]**

c) Find the matrix of the linear mapping $\phi : v \to v$, Where $v = R^{2\times 2}$, defined by $\phi(v) = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} v$ with respect to the standard basis

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}.$$ **[2]**

Q4) a) Let U, V be vector spaces over F with dimensions m, n respectively. Let A be the matrix of a linear mapping $\phi ; V \to U$ with respect to a given pair of ordered bases B, C of U, V respectively. Then prove that, the matrix of ϕ with respect to a new pair of bases B', C' is $A^1 = P^{-1}AQ$.

Where - P, Q are the matrices of transformations from B^1 to B and C^1 to C respectively. **[5]**

b) Show that matrices $A = \begin{pmatrix} -1 & 3 & 0 \\ 0 & 2 & 0 \\ 2 & 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ are similar matrices over C. **[3]**

c) Find the matrix associated with the linear map $f : R^4 \to R^4$ defined by f(a, b, c, d) = (2a, o, b, c+d) with respective to standard bases. **[2]**

Q5) a) Let $\phi \in$ Hom (v, v) and let $f(t)$ be a polynomial over F such that $f(\phi) = 0$. If $f(t) = g(t)h(t)$ is a factorization of $f(t)$ into relatively prime polynomials $g(t)$, $h(t)$ then show that. **[5]**

$$V = \ker g(\Phi) \oplus \ker h(\phi)$$

b) Show that the roots of the characteristic polynomial of the matrix. **[3]**

$$A = \begin{pmatrix} 1 & -2 & -2 & -2 \\ -2 & 1 & -2 & -2 \\ -2 & -2 & 1 & -2 \\ -2 & -2 & -2 & 1 \end{pmatrix}$$

are 3, 3, 3 and –5. Also show that the eigenspaces associated with the eigenvalues 3 and –5 are of dimensions 3 and 1 respectively. **[3]**

c) Determine the eigen values of the matrix. **[2]**

$$A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}, \text{ if exist.}$$

Q6) a) Reduce the following matrix into triangular form. **[5]**

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 1 \end{pmatrix}.$$

b) Find the Jordan canonical form of the matrix, **[5]**

$$A = \begin{pmatrix} 5 & 1 & -2 & 4 \\ 0 & 5 & 2 & 2 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

Q7) a) Let V be a vector space over f of characteristic $\neq$ 2 then show that, the mapping $f: B \rightarrow Q$ where $Q(x) = B(x, x), x \in V$ from the set of symmetric bilinear forms on V into the set of quadratic forms on V is a 1-1 correspondence. **[5]**

b) Prove that, every finite-dimensional Euclidean vector space has an orthonormal basis. **[5]**

Q8) a) Prove that, quadratic form. **[5]**

$Q(x) = \sum_{i,j=1}^{n} a_{ij} x_i x_j, x = t(x_1, x_2, ..., x_n) \in R^n$ on R^n can be reduced to a diagonal form $Q(x) = \lambda_1 x_1^{1\ 2} + \lambda_2 x_1^{1\ 2} + ...\lambda n x_n^{1\ 2}$ by an orthogonal transformation of co-ordinates where $\lambda_1, \lambda_2 ...., \lambda_n$ are the eigenvalues of A.

b) State and prove sylvester's theorem. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P1256

[Total No. of Pages : 3

[5121]-205

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT-605 : Partial Differential Equations

(2013 Pattern) (Credit System)

Time : 3 Hours] ***[Max. Marks : 50***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Eliminate the arbitrary function 'F' from the equation **[4]**

$$x + y + z = F(x^2 + y^2 + z^2)$$

b) Find the general integral of equation. **[4]**

$$z(xp - yq) = y^2 - x^2$$

c) Define the following terms and example of each **[2]**

i) Linear equation.

ii) Quasi-linear equation

Q2) a) Verify that the equation is integrable and find it's solution. **[4]**

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$$

b) Show that the equations : $p^2 + q^2 = 1$ and $(p^2 + q^2)x = pz$. **[4]**

are compatible and solve them.

c) Find the complete integral of : the partial differential equation. **[2]**

$$pqz = p^2(xq + p^2) + q^2(yp + q^2)$$

P.T.O.

Q3) a) If $h_1 = 0$ and $h_2 = 0$ are compatible with $f = 0$, then prove that h_1 and h_2 satisfy : **[4]**

$$\frac{\partial(f,h)}{\partial(x,u_x)} + \frac{\partial(f,h)}{\partial(y,u_y)} + \frac{\partial(f,h)}{\partial(z,u_z)} = 0$$

b) Find the complete integral of : **[4]**

$$(1+yz)dx + x(z-x)dy - (1+xy)dz = 0$$

c) Solve the equation : **[2]**

$$(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$$

Q4) a) Find the complete integral of first order partial differential equation : $z^2(p^2z^2 + q^2) = 1$ by Charpit's method. **[4]**

b) Find the general integral of the partial differential equation : $(x-y)y^2p + (y-x)x^2q = (x^2+y^2)$ and particular solution through, $xz = a^2, y = 0$. **[4]**

c) Derive the analytic expression for the monge cone at (x_0, y_0, z_0), **[2]**

Q5) a) Find the integral surface of the equation, $pq = z$; passing through curve C : $x_0 = 0,\ y_0 = s, z_0 = s^2$ **[4]**

b) Reduce the equation to canonical form the solve it $u_{xx} + 2u_{xy} + 17u_{yy} = 0$. **[4]**

c) Find the initial strip for the equation : $pq = xy$ which passes through the curve C : $z = x$, $y = 0$. **[2]**

Q6) a) If $u(x,y)$ is harmonic in a bounded domain D and continuous in $\bar{D} = D \cup B$. Then u attains it's maximum on the boundary B of D. **[4]**

b) State and prove Harnack's theorem. **[4]**

c) Classify the following equation into hyperbolic, parabolic or elliptic type : $e^z u_{xy} - u_{xx} = \log(x^2 + y^2 + z^2 + 1)$. **[2]**

Q7) a) Using D-Alembert's solution of infinite string find the solution of:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}, 0 < x < \infty, t > 0$$

$$y(x,0) = u(x), yt(x,0) = V(x), x \geq, 0$$

$$y(0,t) = 0,\ t \geq, 0$$ **[5]**

b) State and prove Kelvin's Inversion Theorem. **[5]**

Q8) a) Using Duhamel's principle find the solution of non-homogenous Heat equation : $u_t + ku_{xx} = f(x,t), -\infty < x < \infty, t > 0$.

$$u(x,0) = 0, \quad -\infty < x < \infty.$$ **[5]**

b) Find the solution of : $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta Q} = 0,\ r < a$ subject to the boundary

$$\frac{\partial u}{\partial r} = f(\theta) \text{ on } r = a, \int_0^{2\pi} f(\theta) d\theta = 0.$$ **[5]**

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Total No. of Questions : 8]

SEAT No. :

P1257

[Total No. of Pages : 3

[5121]-301
M.A./M.Sc. (Semester - III)
MATHEMATICS
MT-701 : Combinatorics
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Find ordinary generating function whose coefficient is $a_r = (r+1)r(r-1)$. Hence, evaluate the sum : 3 × 2 × 1 + 4 × 3 × 2 + ... + (n + 1) n (n–1). **[5]**

b) How many non-negative integer solutions are there to the inequalities $x_1 + x_2 + x_3 + ... + x_6 \leq 20$ and $x_1 + x_2 + x_3 \leq 7$? **[3]**

c) Solve the recurrence relation : **[2]**

$a_n = 2a_{n/2} + 5$ with $a_2 = 1$

[Assume that, n is a power of 2]

Q2) a) Find a recurrence relation for the number of n-digit quaternary (0, 1, 2, 3) sequences, with at least one 1 and the first 1 occuring before the first 0. **[5]**

b) How many arrangements of letters in REPETITION are there with the first E occuring before the first T? **[3]**

c) Find the rook polynomial for a full 4 × 4 board. **[2]**

Q3) a) Prove by combinatorial argument, that **[5]**

$$\binom{r}{r}+\binom{r+1}{r}+\binom{r+2}{r}+.........+\binom{n}{r}=\binom{n+1}{r+1}.$$

b) Suppose that among 40 toy robots, 28 have a broken wheel or rusted, but not both, 6 are not defective, and the number with a broken wheel equals the number with rust. How many robots are rusted? **[3]**

c) Find a generating function for the number of ways to write the integer r as a sum of positive integers in which no integer appears. More than three times. **[2]**

Q4) a) How many arrangement are there of MATHEMATICS with both T's before both A's or both A's before both M's or both M's before the E? **[5]**

[By before, we mean, any where before, not just immediately before]

b) How many sequences of length 5 can be formed using the digits 0, 1, 2, 3 ....9 with the property that exactly two of the ten digits appear (e.g.05550). **[3]**

c) How many words can be formed by rearranging INQUISITIVE so that U does not immediately follow Q? **[2]**

Q5) a) How many ways are there to divide five pears, five apples, five doughnuts five lollipops, five chocolate cats and five candy rocks into two (unordered) piles of 15 objects each? **[5]**

b) How many arrangements of 1, 1, 1, 1, 2, 3, 3 are there with the 2 not beside either 3? **[3]**

c) Using a generating function, find the number of distributions of 18 chocolate bunny rabbits into four easter baskets with at least 3 rabbits in each basket. **[2]**

Q6) a) How many ways are there to make an arrangement of pennies & nickels, dimes and quarters with at least one penny and an odd number of quarters. **[5]**

[coins of the same denomination are identical]

b) Solve the recurrence relation, **[3]**

$an = 3a_{n-1} + n^2 - 3$ with $a_0 = 1$

c) Find a generating function for the number of integer solutions of $2x + 3y + 7z = r$, with $x, y, z \geq 0$? **[2]**

Q7) a) Find recurrence relation for the number of n-digit ternary sequences with an even number of zero's and an even number of one's? **[5]**

b) How many ways are there to split 6 copies of one book, 7 copies of a second book and 11 copies of a third book between two teachers if each teacher gets 12 books and each teacher gets at least 2 copies of each book? **[5]**

Q8) a) How many ways are there to assign 6 cars, denoted C_1, C_2, C_3, C_4, C_5, C_6 to six men M_1, M_2, M_3, M_4, M_5, M_6, if man M_1 will not drive cars C_2 and C_4; Man M_2 will not drive cars C_1 or C_5; If man M_3 drives all cars.; Man M_4 will not drive C_2 or C_5 man M_5 will not drive C_4 and man M_6 will not drive C_6? **[5]**

b) Using generating functions, solve the recurrence relation : **[5]**

$a_n = 3a_{n-1} - 2a_{n-2} + 2$ with $a_0 = a_1 = 1$.

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Total No. of Questions : 8]

SEAT No. :

P1258

[Total No. of Pages : 3

[5121]-302

M.A./M.Sc. (Semester - III)

MATHEMATICS

MT-702 : Field Theory

(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let α be an algebraic over a field F and F (α) be the field generated by α over F then prove that, $F(\alpha) \cong F[x]/\langle m_\alpha(x)\rangle$ hence show that $[F(\alpha); F] = \deg(m_\alpha(x))$ where $M_\alpha(x)$ is minimal polynomial for α over F. **[5]**

b) Show that $P(x) = x^2 + 1$ is an irreducible polynomial over the field Z_3. Find an extension K of Z_3 in which P(x) has a root. **[3]**

c) Show that the characteristic of a field F is either zero or a prime. **[2]**

Q2) a) Prove that the extension K/F is finite if and only if K is generated by a finite number of algebraic elements over F. **[5]**

b) Show that $[Q(\sqrt[6]{2}):Q] = 6$ and hence show that $x^3 - \sqrt{3}$ is irreducible polynomial over $Q(\sqrt{2})$. **[3]**

c) Determine the degree of $\alpha = 2 + \sqrt{3}$ over Q. **[2]**

Q3) a) Find the splitting field of $f(x) = x^4 - 2 \in Q[x]$ over Q and it's degree of extension. **[5]**

b) Suppose α is a rational root of a monic polynomial in $Z[x]$ then prove that α is an integer. **[3]**

c) Determine wether the polynomial $p(x) = (x-2)^2 \in Q[x]$ is separable over Q. **[2]**

Q4) a) Let $\phi : F \to G$ be an isomorphism of fields. Let $f(x) \in F[x]$ and $g(x) \in G[x]$ be the polynomial obtained by applying ϕ to the coefficients of $f(x)$. Let E_1 be a splitting field for f (x) over f and E_2 be splitting Hing Field F or $g(x)$ over G, then prove that the isomorphism ϕ extends to an ismorphism. $\sigma : E_1 \to E_2$ **[5]**

b) Define algebraic closure of a field. If K is an algebraically closed field and F is a subfield of K then prove that the collection of elements of K that are algebraic over F is an algebraic closure of F. **[3]**

c) Show that doubling the cube is impossible by using straightfedge and compass. **[2]**

Q5) a) Let E be the splitting field over F of the polynomial $f(x) \in F[x]$ then prove that$|\text{Aut (K/F)}| \leq [\text{E : F}]$. **[5]**

b) Find all automorphisms of $Q\left(\sqrt{2}\right)$ over Q. Is the extension $Q\left(\sqrt{2}\right)$ of Q Galois? **[3]**

c) Prove that $Q\left(\sqrt{2}\right)$ and $Q\left(\sqrt{3}\right)$ are not isomorphic. **[2]**

Q6) a) Show that the Galois group of $x^3 - 2 \in Q[x]$ is the symmetric group on three lefters. **[5]**

b) Show that any quadratic extension K of any field F of characterisitic not equal to two is Galois. **[3]**

c) Find the discriminant D of a polynomial $f(x) = x^3 - x + 1$ in Q $[x]$. **[2]**

Q7) a) State the fundamental theorem of Galois theory. **[5]**

b) Prove that any cyclic extension of degree n over a field F of characteristic not dividing n which contains the nth root of unity is of the form $F\left(\sqrt[n]{a}\right)$ for some $a \in F$. **[5]**

Q8) a) Show that the field F_{p^n} is the splitting field of $x^{p^n} - x$ over F_p with cyclic Galois group of order 'n' generated by the Frobenius automorphims σ_p. Hence show that the subfield of F_{p^n} are all Galois over F_p. **[5]**

b) Show that the field generated over F by α and β is the field generated by β over the field $F(\alpha)$ generated by α. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P1259

[Total No. of Pages : 3

[5121]-303

M.A./M.Sc. (Semester - III)

MATHEMATICS

MT-703 : Functional Analysis

(Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and prove : Riesz Representation Theorem for a Hilbert Space. **[5]**

b) If $\{x_n\}_{n\in N}$ is a sequence in a Banach space x such that $\sum_{n=1}^{\infty} \| x_n \| < \infty$, then prove that the series $\sum_{n=1}^{\infty} x_n$ converges in X. **[3]**

c) Define a reflexive space and give an example. **[2]**

Q2) a) If A is normal operator and λ,μ are distinct eigenvalues of A, then prove that $\ker(A-\lambda) \perp \ker(A-\mu)$. **[4]**

b) Prove that if p $\geq$ q ≥ 1, then $l_q \subset l_p$. **[4]**

c) State Hahn-Banach theorem. **[2]**

Q3) a) State and prove the principle of uniform Boundedness. **[5]**

b) Give an example of an orthonormal basis of $L^2[0, 2\pi]$. Justify. **[5]**

P.T.O.

Q4) a) For an operator A on a Hilbert space H; if $A = A^*$, then prove that $\|A\| = \sup \{|\langle Ah, h\rangle| : \|h\|=1\}$. **[4]**

b) Prove that the identity operator on an infinite dimensional Hilbert space is not compact. **[3]**

c) Give an example of an operator A on a Hilbert space and a subspace M that is invariant under A but not reducing for A. **[3]**

Q5) a) Let H be a Hilbert space and M be a closed subspace of H.

Prove that $\left(M^{\perp}\right)^{\perp} = M$. **[4]**

b) Give an example of an isometry on a Hilbert space that is not surjective. Justify. **[4]**

c) Give an example of a (non-identity) unitary operator on an infinite dimensional Hilbert space. **[2]**

Q6) a) Let X be a normed space and f be a linear functional on X. If ker f is closed, then prove that f is continuous. **[4]**

b) Give an example of a convex set in a Hilbert space. **[2]**

c) For any operator A, prove that Ker A = $(\text{ran } A^*)^{\perp}$. **[4]**

Q7) a) State and prove open Mapping Theorem. **[5]**

b) Let H be a separable Hilbert Space with basis $\{e_n\}_{n\in N}$.

If A is an operator defined by $Ae_n = \frac{1}{n}e_n$ for $n \in N$, then show that A is a compact operator. **[3]**

c) Give an example of a Banach space which is not a Hilbert space. **[2]**

Q8) a) Prove that the operator T is normal if and only if real and imaginary parts of T commute. **[4]**

b) If T is compact, self-adjoint operator on a Hilbert space, then prove that either $\|T\|$ or $-\|T\|$ is an eigenvalue of T. **[4]**

c) A linear operator $T : l^2 \rightarrow l^2$ is defined by $T(x_1, x_2, x_3, \ldots\ldots\ldots) = (0, x_1, x_2, x_3, \ldots\ldots\ldots)$ Find T*. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P1260

[Total No. of Pages : 3

[5121]-401

M.A./M.Sc. (Semester - IV)

MATHEMATICS

MT-801 : Number Theory

(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) If (a, m) = 1, then prove that $a^{\phi(m)} \equiv 1 (\mod m)$. **[5]**

b) Determine the value of 999^{179} (mod 1763). **[3]**

c) Find the minimal polynomial of $\sqrt{3}+\sqrt{5}$. **[2]**

Q2) a) State and prove the Gauss lemma. **[5]**

b) If (a, m) =1, then prove that there is an x such that $ax \equiv 1 (\mod m)$. Further prove that any two such x are congruent (mod m). Also prove that if (a, m) > 1, there is no x such that $ax \equiv 1 (\mod m)$. **[3]**

c) Find all primes q, such that $\left(\frac{5}{q}\right) = -1$. **[2]**

Q3) a) Let x and y be any two real numbers. Prove that **[5]**

i) $[x]+[y] \le [x+y] \le [x]+[y]+1$

ii) $\left[\frac{[x]}{m}\right] = \left[\frac{x}{m}\right]$, if m is an integer.

b) Let μ (n) denotes the Möbius μ function. Prove that $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$, if n is a positive integer. **[3]**

c) Prove that 1+i is prime in Q(i). **[2]**

P.T.O.

Q4) a) If $f(n)=\sum_{d/n} f(d)$ for every positive integer n, then prove that $f(n)=\sum_{d/n}\mu(d)F(n/d)$. **[5]**

b) For every positive integer n, prove that $\sigma(n)=\prod_{P^{\alpha}\|n}\left(\frac{p^{\alpha+1}-1}{p-1}\right)$. **[3]**

c) Prove that 19 is not a divisor of $4n^2+4$ for any integer n. **[2]**

Q5) a) Let a, b and c be integer with not both a and b equal to O, and let g = gcd (a, b) and $ax + by = c$. **[5]**

i) If $g \nmid c$, then show that the equation $ax + by = c$ has no solution in integer.

ii) If g/c, then prove that the equation $ax + by = c$ has infinitely many solutions.

iii) If the pair (x_1, y_1) is one integral solution, then prove that all other are of the form $x = x_1 + \frac{kb}{g}, y = y_1 - \frac{ka}{g}$, where k is an integer.

b) Prove that the number $\beta = \sum_{j=0}^{\infty} 10^{-j!}$ is transcendental number. **[3]**

c) Find smallest integer x such that $d(x) = 6$ **[2]**

Q6) a) Show that **[5]**

i) The norm of product equals the product of norms;

$N(\alpha\beta) = N(\alpha)\cdot N(\beta)$

ii) $N(\alpha) = 0$ if and only if $\alpha = 0$.

iii) The norm of an integer in $Q(\sqrt{m})$ is a rational integer.

b) Show that there is no x for which both $x \equiv 29 \pmod{52}$ and $x \equiv 19 \pmod{72}$. **[5]**

Q7) a) Show that every Euclidean quadratic field has the unique factorization property. **[5]**

b) Find all solutions of the congruence : **[5]**

$57x \equiv 87 (\text{mod} 105)$

Q8) a) Let $Q(\sqrt{m})$ have the unique factorization property. Then prove that any rational prime p is either a prime π of the field or a product π_1, π_2 of two primes, not necessarily distinct of $Q(\sqrt{m})$. **[5]**

b) Find all solutions in the positive integer $15x + 7y = 111$. **[3]**

c) Evaluate $\left(\frac{51}{71}\right)$. **[2]**

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Total No. of Questions :8]

SEAT No. :

P1261

[Total No. of Pages : 2

[5121]-402

M.A. / MSc. (Semester IV)

MATHEMATICS

MT-802 Differential Geometry

(2013 Pattern) (Credit System)

Time : 3 Hour] *[Max. Marks :50*

Instructions to the candidates:

1) *Attempt any five questions of the following.*
2) *Figures at the right indicate full marks.*

Q1) a) State and prove lagranges multiplier theorem for smooth surfaces. **[5]**

b) Show that the gradient of f at PEf ^{y}(c) is orthogonal to all vectors tangent to f^{y} (C) at p. **[3]**

c) Find and sketch the gradient field for F $(x_1, x_2) = (x_1^2 + x_2^2) / g$ **[2]**

Q2) a) Show that for an smooth real valued function f on an open set U, the set of all vectors tangent to level set at p is equal to $[\nabla f(p)]1$. **[5]**

b) Show that graph of a smooth function is n - Surface in $\mathbb{R}^{n+1}$ **[3]**

c) show that the cylinder $x_1^2 + x_2^2 = 1$ in $\mathbb{R}^3$ can be represented as level set of the function $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + \text{Sin}(x_1^2 + x_2^2)$. **[2]**

Q3) a) Show that there exists a maximal integral curve for the smooth surface in $\mathbb{R}^{n+1}$. **[5]**

b) Show that a mobius band is unorientable 2 - Surface. **[3]**

c) Sketch the vector field on $\mathbb{R}^2$, $\overline{X}$ (P) = P, X (P)) (2) where X (P) = (0,1). **[2]**

P.T.O

Q4) a) Sketch and describe the spherical image when n = 1 and n = 2 of given n-surface oriented by outward normal for the cone $-x_1^2 + x_2^2 + ... + x_{n+1}^2 = 0.\ x_1 > 0.$ **[5]**

b) Find velocity, accelaration and the speed of parametrized curve $\alpha(t) =$ (cost, sint) **[3]**

c) Show that geodesics have constant speed **[2]**

Q5) a) State five properties of levi-Civita parallelism and prove the same. **[5]**

b) Show that the vector field $\bar{X}$ tangent to 2 - surface S in $\mathbb{R}^3$ along a geodisic $\alpha : I \to S$ is parallel along α iff both $\| \bar{X} \|$. and the angle between $\bar{X}$ and $\dot{\alpha}$ are constant along α. **[3]**

c) Show that the parallel transport from Sp onto Sq with all usual notations is a linear map. **[2]**

Q6) a) Show that weingarten Map is a self adjoint operator . **[5]**

b) Compute $\nabla_{\upsilon}\bar{X}$ where $\bar{\upsilon} \in \mathbb{R}_P^{n+1}$, $P \in \mathbb{R}^{n+1}$ and $\bar{X}$ is given by $\bar{X}$ $(x_1, x_2) = (x_1\ x_2, x_1 x_2, x_2^2)$, $\bar{\upsilon} = (1, 0, 0, 1)$. **[3]**

c) Show that covariant differentiation is destribitive over addition for the smooth vector field on n - surface. **[2]**

Q7) a) Find the curvature k of the oriented plane curve $x_2 - ax_1^2 = c$, $a \neq o$ also. Find its global parametrization. **[5]**

b) Show that for connected oriented plane curve there exists a global parametrization. **[3]**

c) Find the length of the connected oriented plane curve f^{-1} (C) oriented by $\nabla f / \| \nabla f \|$ where $f : U \to \mathbb{R}$ and c are given as $f(x_1, x_2)$

$$= \frac{1}{2} x_1^2 + \frac{1}{2}(x_2 - 1)^2,\ U = \mathbb{R}^2, C = 2$$ **[2]**

Q8) a) Find the gaussian curvature for the ellipsoid in $\mathbb{R}^3$ **[5]**

b) Show that for a compact connected oriented n-surface S , the Gauss map maps S onto unit Sphere S^n. **[5]**

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Total No. of Questions :8]

SEAT No. :

P1262

[Total No. of Pages : 3

[5121]-403

M.A. / MSc. (Semester IV)

MATHEMATICS

MT-803 : Fourier Analysis and Boundary Value Problems

(2013 Pattern) (Credite System)

Time : 3 Hour] *[Max. Marks :50*

Instruction to the candidates:

1) *Attempt any five questions.*
2) *Figures at the right indicate full marks.*

Q1) a) If F $\in$ Cp (o,π), then prove that the fourier sine series Coefficient b_n tend to zero as n tends to infinity. **[5]**

b) Find the fourier cosine series for the function F (x) = x^2 $(o < x < \pi)$ **[3]**

c) Find the fourier sine series for the function f (x) = x $(o < x < 1)$. **[2]**

Q2) a) Let f denote a function such that (i) f is continuous on the interval $-\pi \leq x \leq \pi$ (ii) $f(-\pi) = f(\pi)$ (iii) its derivative $f^1$is piecewise continuous on the interval $-\pi < x < \pi$. Prove that the fourier series $\frac{a_0}{2} + \sum_{n=1}^{\infty}$ (a_n cos nx + b_n Sin nx) for f, with coefficients $a_n \frac{1}{\pi} \int_{-\pi}^{\pi}$ f (x) cosnx dx and $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi}$ f (x) sin nx dx converges absolutely and uniformly to f (x) on the interval $-\pi \leq x \leq \pi$. **[5]**

b) Find the fourier series for the function $f(x) = \begin{cases} \frac{-\pi}{2} & when\ -\pi < x < o \\ \frac{\pi}{2} & when\ o < x < \pi \end{cases}$ **[3]**

P.T.O

c) If $f(x) = \frac{e^x - 1}{x}$ $(x \neq o)$, then find f (o+) and $f_R^1(o)$. **[2]**

Q3) a) If f is piecewise continuous on the interval $-\pi < x < \pi$, then prove that

$$\int_{-\pi}^{x} f(s)\ ds = \frac{a_0}{2}(x+\pi) + \sum_{n=1}^{\infty} \left\{a_n Sin\ nx - b_n[\cos nx + (-1)^{n+1}]\right\}.$$ **[5]**

b) Solve the following linear boundary value problem $u_t(x, t) = k\ Uxx(x,t)$ $(0 < x < c, t > 0)$ $u_x(0,t) = 0$, $u_x(c,t) = 0$ $(t > 0)$ $u(x,0) = f(x)$ $(0 < x < c)$**[5]**

Q4) a) Solve the following boundary value problem. $u_{xx}(x,y) + u_{yy}(x,y) = o$ $(0 < x < a, 0 < y < b)$ $u(0,y) = 0$, $u(a,y) = 0$ $(0 < y < b)$ $u(x,0) = f(x)$, $u(x,b) = 0$ $(0 < x < a)$. **[5]**

b) Solve the following boundary value problem :

$\rho^2 u_{\rho\rho}(\rho,\phi) + \rho u_{\rho}(\rho,\phi) + u\,\phi\,\phi(p,\phi) = o(k\,p < b, o < \phi < \pi)$

$u(\rho, 0) = 0$, $u(\rho,\pi) = 0$ $(1 < \rho < b)$ $u(1,\phi) = 0$ $u(b,\phi) = u_0$ $(o < \phi < \pi)$.

[5]

Q5) a) Let C_n (n = 1, 2, 3, ...) be the fourier constants for a function f in C_p (a,b) with respect to an orthogonal set $\{\phi_n(x)\}$ (n = 1, 2, 3, ...) in that space. Then prove that all possible linear combination of the function ϕ_1 (x), ϕ_2 (x), ... ϕ_N (x) the combination $C_1\ \phi_1(x) + C_2\ \phi_2(x) + ... + C_N\ \phi_N(x)$ is the best approximation in the mean to f (x) on the fundamental interval a < x < b. **[5]**

b) Show that the function $\psi_1(x) = 1$ and $\psi_2(x) = x$ are orthogonal on the interval $-1 < x < 1$ and determine constants A and B such that the function $\psi_3(x) = 1 + Ax + Bx^2$ is orthogonal to both ψ_1 and ψ_2 on the interval. **[3]**

c) Prove or disprove : Every fourier series differentiable. **[2]**

Q6) a) Let λ be an eigenvalue of the regular sturm - liouville problem $(r\,X^1)^1 +$

$(q + \lambda p) X = 0$ $(a < x < b)$ under the condition $a_1 X(a) + a_2 X^1(a) = 0$ $b_1 X(b) + b_2 X^1(b) = 0$. If the conditions. $q(x) \leq o$ $(a \leq x \leq b)$ and $a_1 a_2 \leq o$, $b_1 b_2 \geq o$, are satisfied, then prove that $\lambda \geq o$. **[5]**

b) Find the eigenvalues and normalized eigten - function of sturm - liouville problem. $X^{||} + \lambda X = o$, $X(o) = o$, $X(1) - X^1(1) = o$. **[3]**

c) If $\phi_o(x) = \frac{1}{\sqrt{\pi}}$, $\phi_n(x) = \sqrt{\frac{2}{\pi}} \cos nx$ $(n = 1, 2, 3, \ldots)$ then show that the set $\{\phi_n(x)\}$ $(n = 1, 2, 3, \ldots)$ is orthonormal on the interval $o < x < \pi$. **[2]**

Q7) a) For the singular sturm - liouville problem consisting of the differential equation $\frac{x d^2X}{dx^2} + \frac{dX}{dx} + \lambda x X = o$ $(o < x < C)$, prove that.

i) the eigenvalues are $\lambda j = \alpha_j^2$ and corresponding eigenfunctions are $X_j = j_o(\alpha_j x)$ when $X(c) = o$ $(j = 1,2,3, \ldots)$

ii) the eigenvalues are $\lambda j = \alpha_j^2$ and corresponding eigenfunctions are $Xj = j_o(\alpha_j x)$ where α_j $(j = 1, 2, 3 \ldots)$ are positive roots of equation $hJ_o(\alpha c) + (\alpha c) J^1_o(\alpha c) = o$. **[6]**

b) Establish the recurrence relations :

i) $\frac{d}{dx}[x^{-n} J_{n(x)}] = -x^{-n} J_{n+1}(x)$ $(n = 1, 2, 3 \ldots)$

ii) $\frac{d}{dx}[x^n j_n(x)] = x^n J_{n-1}(x)$ $(n = 1, 2, 3 \ldots)$ **[4]**

Q8) a) Solve the legendre's differential equation $(1 - x^2) y^{||} - 2xy^{|} + \lambda y = 0$. **[5]**

b) Obtain the integration formula $\int_a^1 P_n(x) dx = \frac{1}{(2n+1)} [P_{n-1}(a) - P_{n+1}(a)]$ **[3]**

c) Expand the function $f(x) = 1$ $(o < x < 1)$ in a series of Legendre polynomial of odd degree. **[2]**

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