Physics

Q.1. An observer standing on a railway platform. Observes frequency of a whistling train to be 2.2 kHz and 1.8 kHz of the approaching and the receding train. Find speed of the train (given speed of sound = 300 m/s) **[2]**

Sol. Apparent frequency of approaching train

$$f = f\left(\frac{v}{v - v_s}\right) \text{ where } v_s \rightarrow \text{velocity of train}$$

$$2.2 = f_0\left(\frac{300}{300 - v_s}\right) \qquad \text{......(i)}$$

Apprarent frequency of receding train

$$1.8 = f_0\left(\frac{300}{300 + v_s}\right) \qquad \text{......(ii)}$$

(i) divided by (ii) gives

$$\Rightarrow \quad \frac{22}{18} = \frac{300 + v_s}{300 - v_s} \quad \therefore \text{ solving give; } v_s = 30 \text{ m/s}$$

Q.2. The potential energy of a particle of mass m is given by

$$V(x) = \{E_0 \quad 0 \le x \le 1\} \, \lambda_1$$
$$= \{0 \quad x > 1 \}\lambda_2$$

λ_1 and λ_2 are the de-Broglie wavelengths of particle. If the total energy of particle is $2E_0$. Find $\frac{\lambda_1}{\lambda_2}$. **[2]**

Sol. **For $0 \le x \le 1$**

$E = P.E. + K.E.$

$k_1 = KE = 2E_0 - P.E.$

$= 2E_0 - E_0 = E_0$

For $x > 1$

$k_2 = 2E_0 - 0$

$= 2E_0$

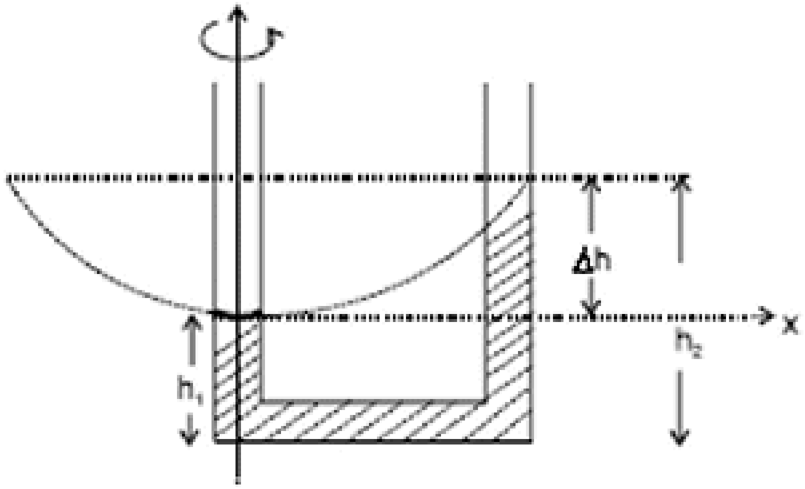
de-Broglie wavelength in above two cases -

$$\lambda_1 = \frac{h}{p_1} \qquad \lambda_2 = \frac{h}{p_2}$$

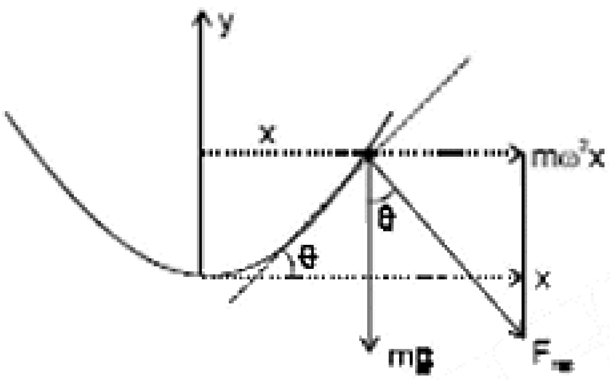
$$\lambda_1 = \frac{h}{\sqrt{2m(k_1)}} \qquad \lambda_2 = \frac{h}{\sqrt{2mk_2}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{k_2}{k_1}} = \sqrt{\frac{2E_0}{E_0}} = \sqrt{2}; \quad \text{Hence} \quad \frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

Q3. A U-tube is filled with liquid of density ρ. The separation between the limbs is L. U-tube is rotated about a vertical axis passing through one of the limbs. Find the difference 'Δh' in the level of the liquid in the two limbs. **[2]**



Sol. In container frame -

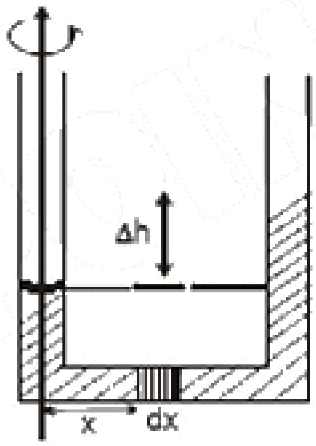


$$\frac{dy}{dx} = \tan\theta = \frac{m\omega^2 x}{mg} = \frac{\omega^2 x}{g}$$

$$\Delta h = \int_{h_1}^{h_2} dy = \int_0^L \frac{\omega^2 x}{g} dx$$

$$\Delta h = \frac{\omega^2 L^2}{2g}$$

OR

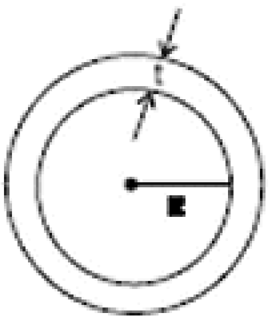


$$(\rho g\,\Delta h)A = \int_0^L (dm)\omega^2 x \qquad \{dm = (Adx)\rho\}$$

$$= \frac{\omega^2 L^2 \rho A}{2}$$

$$\Delta h = \frac{\omega^2 L^2}{2g}$$

Q4. A charged conducting liquid bubble of radius a and thickness t (t<<a) as shown in figure having potential V. If it collapse to droplet. Find the potential of the droplet. **[2]**



Sol. Let charge on the bubble is q.

Potential of the bubble

$$V = \frac{q}{4\pi\varepsilon_0 a}$$

$\therefore \quad q = (4\pi\varepsilon_0 a)v$ ....(i)

Potential of the droplet

$$V' = \frac{q}{4\pi\varepsilon_0 R} \quad ....(ii)$$

where R is radius of the droplet

Also $\quad 4\pi a^2 t = \frac{4}{3}\pi R^3$

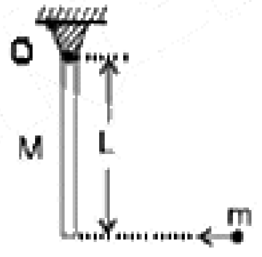
$\therefore \quad R = (3a^2t)^{1/3}$ ....(iii)

$$\therefore \quad V' = \frac{q}{4\pi\varepsilon_0 R} = \frac{4\pi\varepsilon_0 av}{4\pi\varepsilon_0 R} = \frac{a}{R}v$$

$$= \frac{a}{(3a^2t)^{1/3}}v$$

$$= \left(\frac{a}{3t}\right)^{1/3} v$$

Q.5. A wooden stick of mass m and length L is hinged at O. There is no friction at O. A particle of mass 'm' moving with velocity 'v' strikes the stick at its lower end and gets stuck with it as shown in figure. Find the angular velocity of the system about O just after the collision. **[2]**



Sol. Conserving angular momentum of system about O before and after collision.

$$(L_i)_O = (L_f)_O$$

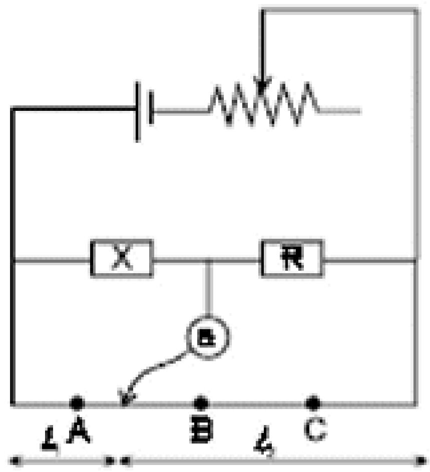
$$\Rightarrow \quad mvL = (I_{sys})\omega$$

$$\text{also - } I_{sys} = \left(mL^2 + \frac{ML^2}{3}\right)$$

$$\text{hence } \omega = \frac{mvL}{I_{sys}} = \frac{mvL}{mL^2 + \frac{ML^2}{3}} = \frac{3mv}{3mL + ML}$$

$$\omega = \frac{3mv}{(3m + M)L}$$

Q6. A unknown resistance is to be determined using resistance R_1, R_2, and R_3. If their corresponding null points are A, B and C. Which of the following will give most accurate reading. **[2]**



Sol. Since $\frac{X}{R} = \frac{\ell_1}{\ell_2} \Rightarrow x = \frac{\ell_1}{\ell_2}R$

$\Rightarrow \quad \ell nx = \ell n\ell_1 - \ell n\ell_2 + \ell nR$

So $\left|\frac{dx}{x}\right| = \left|\frac{d\ell_1}{\ell_1}\right| + \left|\frac{d\ell_2}{\ell_2}\right|$

$$\frac{\Delta x}{x} = \left(\frac{1}{\ell_1} + \frac{1}{\ell_2}\right)(\Delta\ell)$$; (since $|d\ell_1| = |d\ell_2| = \Delta\ell$ (say) i.e. error in measurement)

$$= \frac{\ell_1 + \ell_2}{\ell_1\ell_2}(\Delta\ell)$$; Here $\Delta\ell$ remain constant

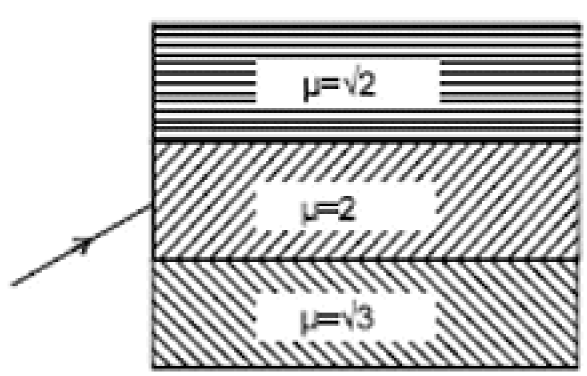
$$= \frac{\ell}{\ell_1\ell_2}(\Delta\ell)$$; Since $\ell_1 + \ell_2 = \ell$ (constant)

so $\frac{\Delta x}{x}$ is minimum when $\ell_1 \ell_2$ is maximum and if $\ell_1 + \ell_2$ is constant then $\ell_1 \ell_2$ will be maximum

if $\ell_1 = \ell_2$ (using A.M. ≥ G.M.)

Hence the null point at B will give most accurate reading.

Q7. What will be the minimum angle of incidence such that the total internal reflection taking place from both the surface ? **[2]**



Sol. For first surface

$2\sin i_1 = \sqrt{2}\sin 90°$

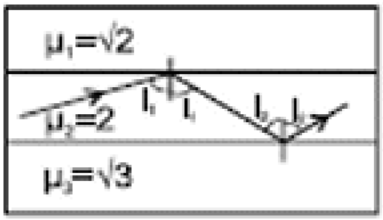
$\Rightarrow \quad i_1 = 45°$ $\quad \therefore$ Critical angle for first surface is 45°

For second surface

$$2 \sin i_2 = \sqrt{3} \sin 90°$$

$\Rightarrow \quad i_2 = 60°$ $\quad \therefore$ Critical angle for second surface is 60°

$\therefore$ minimum angle of incidence = 60°



Q.8. Side of a cube measured in vernier callipers (10 divisions of a vernier scale coincide with 9 division of main scale, where 1 division of main scale is 1mm) reading on the main scale is 10 mm and first division of vernier scale coincides with the main scale. If mass of the cube is 2.73 g. Find density of the cube with due regard to significant figure. **[2]**

Sol. Least count of the vernier callipers

$$= \left(1 - \frac{9}{10}\right)\text{mm} = 0.1 \text{ mm} = 0.01 \text{ cm}$$

Total length = 10 mm + 0.1 mm = 10.1 mm = 1.01 cm

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{2.736}{(1.01)^3} = 2.66 \text{ g/cm}^3$$

Q.9 A transverse wave is produced in a string the maximum particle velocity is 3 m/s and maximum particle acceleration is 9 m/s^2. If the wave velocity is 20 m/s find the wave equation. **[4]**

Sol. Let the wave equation be $y = A \sin(\omega t \pm kx + \phi)$

A be the amplitude of oscillation

$(v_p)_{max} = \omega A = 3$

$(a_p)_{max} = \omega^2 A = 90$

$$\omega = \frac{90}{3} = 30 \text{ rad/s}$$

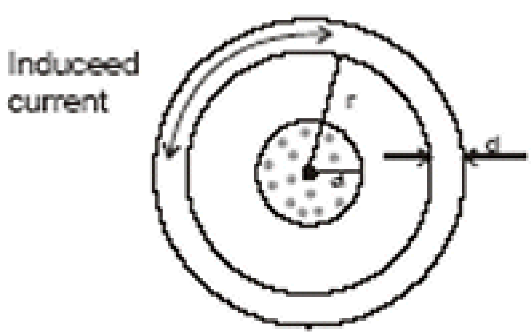
$$v = \frac{\omega}{K} \Rightarrow K = \frac{\omega}{v} = \frac{30}{20} = \frac{3}{2} \text{m}^{-1}$$

$$A = \frac{3}{\omega} = \frac{3}{30} = \frac{1}{10} \text{m} = 10 \text{ cm}$$

The wave form is $y = 0.1 \sin(30t \pm 1.5x + \phi)$

Q.10 Long solenoid of radius 'a' and number of turns per unit length 'n' is surrounded by a cylindrical shell of 'r', and thickness 't' ($t \ll r$) & length 'L'. A variable current $i = i_0 \sin \omega t$ flows through solenoid. The resistivity of the material of the cylindrical shell is δ. Find the induced current in the shell. **[4]**

Sol. Induced current = $\dfrac{\text{induced emf}}{\text{Resistance of shell}}$



$$\xi = \left|\frac{d\phi}{dt}\right|$$

Now $\phi = BA$

$B = \mu_0 ni = \mu_0 n\,(i_0 \sin \omega t)$

$A = \pi a^2$

$$\xi = \left|\frac{d}{dt}(BA)\right| = \left|\frac{AdB}{dt}\right|$$

$$\xi = A\,\mu_0 n i_0 \frac{d}{dt}(\sin \omega t)$$

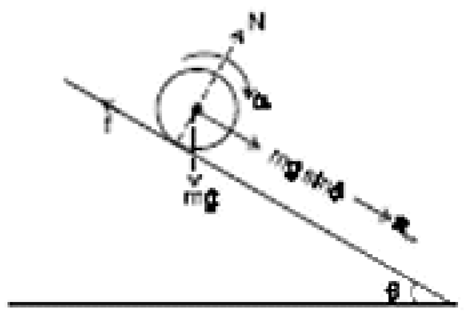
$\xi = A\,\mu_0 n\, i_0\, \omega \cos \omega t$

$\xi = \mu_0 n i_0\, \pi a^2 \omega \cos \omega t$

Resistance of shell $R = \frac{\rho \ell}{A} \Rightarrow R = \frac{\rho 2\pi r}{Ld}$ hence $i = \frac{\mu_0 n i_0 a^2 \omega L d}{2 r \rho} \cos\omega t$

Q.11 A cylinder of mass 'm' and Radius R rolls down an inclined plane of inclination θ. Calculate the acceleration of centre of mass of cylinder **[4]**

Sol.



$Mg \sin \theta - f = Ma_{cm}$ ......(i)

$$fR = \frac{MR^2}{2}\alpha$$ ......(ii)

For perfect rolling

$a_{cm} = \alpha R$ ......(iii)

By solving eqn. (i), (ii) & (iii)

$$a_{cm} = \frac{2}{3} g \sin \theta$$

Q.12 High energy electrons collide with a target of an element having 30 neutrons . The ratio of radii of nucleus of element to that of helium nucleus is $(14)^{1/3}$. Find

(a) The atomic number of nucleus

(b) Frequency of K_α line of the X – ray produced

(Rydberg const (R) = $1.1 \times 10^7\ m^{-1}$, C = 3×10^8 m/s) **[4]**

Sol. (a) Radius of nucleus is given as

$R = R_0 A^{1/3}$

where $A \rightarrow$ mass number of nucleus.

$R_0 \rightarrow$ Constant

$$(14)^{1/3} = \frac{R_{nuc}}{R_{He}}\left(\frac{A_{nuc}}{A_{He}}\right)^{1/3} = \left(\frac{A}{4}\right)^{1/3}$$

$A = 56$

Number of neutrons + No. of protons = mass number

$N + Z = A$

$Z = A - N = 56 - 30 = 26$

(b) From mosley's law

Frequency of K_α – lines is given as

$$v = \frac{3cR}{4}(z-1)^2$$

Z – atomic number of target element

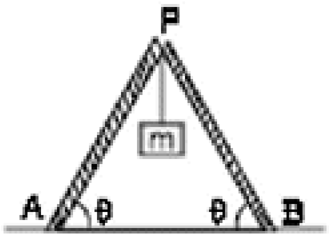
$$v = \frac{3\times3\times10^8\times1.1\times10^7}{4}(26-1)^2$$

$$v = \frac{9\times1.1\times10^{15}}{4}\times(25)^2$$

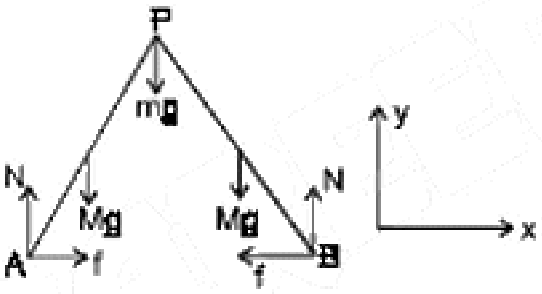
$$v = \frac{625\times9\times1.1}{4}\times10^{15}$$

$$v = \frac{6.25\times9\times1.1}{4}\times10^{17}\text{ Hz} = 1.546\times10^{18}\text{ Hz}$$

Q.13 Two ladder, each of mass 'M' and Length 'L' are resting on the rough horizontal surface as shown in the figure. If the system is in equilibrium, find the magnitude and direction of friction force at A and B. **[4]**



Sol. System = two ladders + mass



Translational equilibrium of system

$F_{net} = 0 \Rightarrow \Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma F_y = 0 \Rightarrow 2N - 2Mg - mg = 0$

$$N = \left(\frac{2M+m}{2}\right)g$$

Rotational equilibrium of either ladder about P

$\tau = 0$

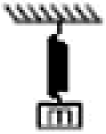
$$NL\cos\theta - Mg\frac{L}{2}\cos\theta - fL\sin\theta = 0$$

$$f = \frac{\left(\frac{N+Mg}{2}\right)L\cos\theta}{L\sin\theta}$$

$$f = \left[\left[\frac{2M+m}{2}\right]g + \frac{Mg}{2}\right]\cot\theta$$

$$f = (3M+m)\,g\frac{\cot\theta}{2}$$

Q.14 A small body attached to one end of a vertically hanging spring is performing SHM about it's mean position with angular frequency ω and amplitude a. If at a height 'y' from the mean position the body gets detached from the spring. Calculate the value of 'y' so that the height 'h' attained by the mass is maximum. The body does not interact with the spring during it's subsequenty motion after detachment. $(a\omega^2 > g)$ **[4]**

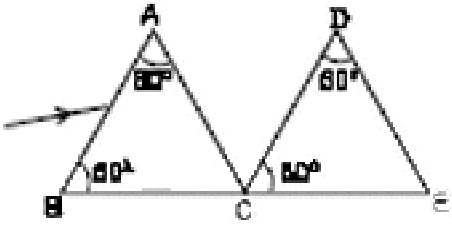


Sol. To attain maximum height, the block must detached from the spring at that moment when spring is in its natural length. At this stage, the potential energy of the spring will be zero. Hence the total energy of the block at this point will be maximum and therefore it will rise to a maximum height.

$mg = ky$

$\Rightarrow \quad y = \frac{mg}{k} = \frac{g}{\omega^2} < a$

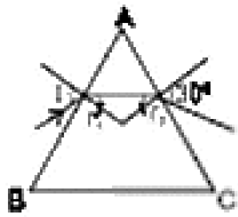
Q.15 Two equilateral prisms of vefractive index $\sqrt{3}$ are kept side by side as shown in figure A light ray strikes the first prism at face AB. Find



(a) The angle of incident for the minimum deviation of emergent ray from the first prism

(b) By what angle the prism DCE should be rotated about C to get the minimum deviation of final emergent ray from the face DE **[4]**

Sol. (a) For minimum deviation



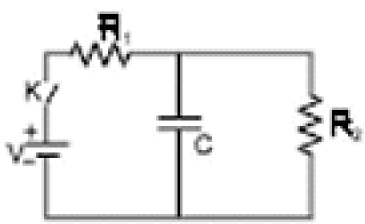
$r_1 = r_2 = \frac{A}{2} = 30°$

$\sin i = \sqrt{3}\ \sin 30$

$i = 60°$

(b) For minimum deviation, if the prism is rotated by an angle 60° to the anticlockwise, then both the prism will act as a glass slab and the deviation will be zero.

Q.16 In the given network Key k is closed at time t = 0. The charge q on the capacitor at any time t is given by $q(t) = q_0(1 - e^{-\beta t})$. Find the value of q_0 and β in terms of given parameters shown in the circuit **[4]**

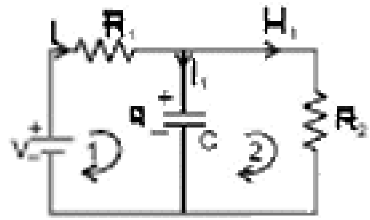


Sol. Applying KVL in loop 1 and 2

$V - iR_1 - \frac{q}{C} = 0$ ....(i)

$\frac{q}{C} - (i - i_1) R_2 = 0$ ....(ii)

$i_1 = \frac{dq}{dt}$ ....(iii)



From eq. (i), (ii) and (iii)

$$R_2V - \left(\frac{R_1}{C} + \frac{R_2}{C}\right)q - R_1R_2\frac{dq}{dt} = 0$$

on solving we get

$$q = \frac{CVR_2}{R_1 + R_2}\left\{1 - e^{-\left(\frac{R_1+R_2}{R_1R_2C}\right)t}\right\}$$

$\Rightarrow$ $q_0 = \frac{CVR_2}{R_1 + R_2}$ and $\beta = \frac{R_1 + R_2}{R_1R_2C}$

Q.17 A cylinder of mass 1 kg is kept at inital temperature 20°, now it is given heat of 20 kJ at atmospheric pressure. Find

(a) Final temperature of the cylinder.

(b) Work done by the cylinder to the surrounding

(c) change in internal energy of the cylinder

Given that specific heat of cylinder = 400 J kg^{-1} °C, co-efficient of volume expansion = 9 × 10^{-5} °C^{-1}, atmospheric pressure = 10^5 N/m^2 & density of cylinder = 9 × 10^3 kg/m^3.

[6]

Sol. (a) $\Delta Q = m\,S\Delta T$

$\Delta T = 50°C$

$\therefore$ final temp. = 70°C

(b) Work done

$W = P\Delta V$

where ΔV in change in volume.

$$\Delta V = V\gamma\Delta T = \left(\frac{m}{\rho}\right)\gamma\Delta T$$

$$W = 1.01 \times 10^5\ (N/m^2)\left(\frac{1}{9\times10^3}m^3\right)9\times10^{-5}(°C) \times 50(°C)$$

$= 5 \times 10^{-2}$ J

(c) $\Delta U = \Delta Q - W$

$= 20000 - 0.05 = 19999.95$ (J)

Q.18 Torque acting on a moving coil galvanometer can be given by equation $\tau = ki$, where i current through the galvanometer wire & k is a cont, coil of the galvanometer has number of turns N, area A, & moment of iertia I. If the coil is placed in a magnetic field B. Find.

(a) Constant K in terms of given parameter N, I, A & B

(b) The torsional constant of coil if current I_0 produces a deflection of $\frac{\pi}{2}$ in the coil

(c) Maximum angle through which coil is deflected. If charge Q is passed through the coil almost instaneously.

[6]

Sol.(a) $\therefore$ $\tau = NIAB \sin\alpha$

For moving coil galvanometer $\alpha = 90$ due to radial magnetic field ($\sin\alpha = 1$)

$ki = iNAB$

$k = NAB$

(b) Let torsional constant of the coil in C.

$\therefore$ torque $\tau = c\theta$

$$i_0 NAB = C\frac{\pi}{2}$$

$$\therefore \quad C = \frac{2I_0 NAB}{\pi}$$

(c) Angular impulse

$$L = \int \tau dt = \int NABidt = NABQ$$

Charge in angular momentum

$I\omega = NABQ$

$$\omega = \frac{NABQ}{I}$$

By conservation of Mechanical energy

$$\frac{1}{2}I\omega^2 = \frac{1}{2}C\theta^2 \quad \Rightarrow \theta = Q\sqrt{\frac{NAB\pi}{2I_0 I}}$$