Code : 100312

B.Tech 3rd Semester Exam., 2019
(New Course)

MATHEMATICS—III

(PDE, Probability and Statistics)

Time : 3 hours

Full Marks : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

(v) Relevant statistical data are given at the end of Question Paper.

1. Choose the correct answer (any *seven*) : 2×7=14

(a) If P_n is the Legendre polynomial of first kind, then the value of

$$\int_{-1}^{1} P_n(x)\,dx$$

is

(i) 0

(ii) $\frac{2}{(2n+1)}$

(iii) 2

(iv) 1

(Turn Over)

(b) If J_n is the Bessel's function of first kind, then the value of $2J'_n$, is

(i) $J_{n-1} + J_{n+1}$

(ii) $J_n - J_{n+1}$

(iii) $J_n + J_{n+1}$

(iv) $J_{n-1} - J_{n+1}$

(c) The particular integral of

$$(D^2 - D'^2)Z = x - y$$

is

(i) $\frac{1}{2}x^3 + yx^2$

(ii) $\frac{1}{3}x^3 - \frac{1}{2}yx^2$

(iii) $\frac{1}{6}x^3 - \frac{1}{2}yx^2$

(iv) $x^3 + \frac{1}{2}yx^2$

(d) The function $x^3 + x + 1$ in terms of Legendre polynomial is equal to

(i) $P_3 + 5P_1 - 5P_0$

(ii) $\frac{2}{5}P_3 + \frac{8}{5}P_1 + P_0$

(iii) $\frac{2}{3}P_3 + P_2 + P_1 - P_0$

(iv) $\frac{1}{5}P_3 + P_2 + 5P_1 - \frac{1}{5}P_0$

(Continued)

(e) Let the joint probability density function of the continuous random variables X and Y be

$$f(x, y) = \begin{cases} k(x^2 + y^2); & 0 < x < 1,\ 0 < y < 1 \\ 0 ; & \text{elsewhere} \end{cases}$$

Then the marginal density of X is

(i) $(3x^2 + 1)$ *(ii)* $\frac{3}{5}(2x^2 + 1)$

(iii) $\frac{1}{2}(3x^2 + 1)$ *(iv)* $\left(x^2 + \frac{2}{3}\right)$

(f) If

$$P(A \cap B) = \frac{1}{4},\ P(A \cup B) = \frac{3}{4},\ P(\bar{A}) = \frac{2}{3}$$

then $P(A / \bar{B})$ is equal to

(i) $\frac{1}{3}$ *(ii)* $\frac{1}{4}$

(iii) $\frac{1}{2}$ *(iv)* $\frac{3}{8}$

(g) Let A, B and C be any three mutually exclusive events. Which one of the following is incorrect?

(i) $P(A \cap B \cap C) = P(A) + P(B) + P(C)$

(ii) $P(A \cap B) = 0$

(iii) $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

(iv) $P(B \cap C) = 0$

(Turn Over)

(h) If μ is the mean and σ is the standard deviation of a set of measurements which are normally distributed, then percentage of measurements within the range $\mu \pm 2\sigma$ is

(i) 98 *(ii)* 95·44

(iii) 99·73 *(iv)* 95

(i) If the density function of gamma distribution is

$$f(x) = \begin{cases} \dfrac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma\alpha}, & x > 0 \\ 0 \quad , & x \le 0 \end{cases}$$

then variance is equal to

(i) $\alpha\beta$ *(ii)* β

(iii) $\alpha^2\beta$ *(iv)* $\alpha\beta^2$

(j) The moment generating function of a continuous random variable X be given as

$$M_X(t) = (1-t)^{-7} \text{ for } |t| < 1$$

Then its mean and variance is

(i) $\left(7, \frac{1}{7}\right)$ *(ii)* $\left(\frac{1}{7}, \frac{1}{7}\right)$

(iii) $\left(\frac{1}{7}, 7\right)$ *(iv)* $(7, 7)$

(Continued)

2. Solve : 7+7=14

(i) $x(y^2+z)\frac{\partial z}{\partial x}-y(x^2+z)\frac{\partial z}{\partial y}=(x^2-y^2)z$

(ii) $(D^2-6DD'+9D'^2)Z=$
$6x+2y+\tan(3x+y)+e^{(3x+2y)}$

3. Reduce the following equation into canonical form and hence solve it : 14

$$x^2\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}-x\frac{\partial z}{\partial x}+3y\frac{\partial z}{\partial y}=\frac{8y}{x}$$

4. State and prove Rodrigues formula. 14

5. (a) A coin is tossed. If it turns up H, two balls will be drawn from urn A, otherwise 2 balls will be drawn from urn B. Urn A contains 3 red and 5 blue balls, urn B contains 7 red and 5 blue balls. What is the probability that urn A is used, given that both balls are blue? (Find in both cases, when balls were chosen with replacement and without replacement.) 7

(Turn Over)

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(b) For events $A_1, A_2, A_3, A_4, A_5, \cdots, A_n$, assuming

$$P\left(\bigcup_{i=1}^{n} A_i\right)\le\sum_{i=1}^{n}P(A_i)$$

prove that

(i) $P\left(\bigcap_{i=1}^{n} A_i\right)\ge\sum_{i=1}^{n}P(A_i)-(n-1)$

(ii) $P\left(\bigcap_{i=1}^{n} A_i\right)\ge 1-\sum_{i=1}^{n}P(\overline{A}_i)$ 7

6. Let the continuous random variables X and Y have joint probability density function as

$$f(x,y)=\begin{cases}\frac{1}{y}; & 0<x<y,\ 0<y<1\\ 0; & \text{elsewhere}\end{cases}$$

(a) Determine whether X and Y are independent.

(b) Find $P\left(X>\frac{1}{2}\right)$.

(c) Find $P\left(X<\frac{1}{2}, Y>\frac{1}{3}\right)$.

(d) Find $P\left(X+Y>\frac{1}{2}\right)$. 14

(Continued)

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7. The following marks have been obtained by a class of students in Statistics (out of 100) :

Paper X :	80	45	55	56	58	60	65	68	70	75
Paper Y :	82	56	50	48	60	62	64	65	70	74

Compute the coefficient of correlation for the above data. Also, find the line of regression of Y on X. 14

8. *(a)* State and prove Bayes theorem. 7

(b) A random variable X follows Binomial distribution with parameters $n = 40$ and $p = \frac{1}{4}$. Use Chebyshev's inequality to find bounds for

(i) $P(|X - 10| < 8)$

(ii) $P(|X - 10| > 10)$

9. *(a)* It has been found from experience that the mean breaking strength of a particular brand of thread is 275·6 gram with a standard deviation of 39·7 gram. Recently a sample of 36 pieces of thread showed a mean breaking strength of 253·2 gram. Can one conclude at a significance level of (i) 0·05 and (ii) 0·01 that the thread has become inferior? 7

(Turn Over)

(b) On an elementary school examination in spelling, the mean grade of 32 boys was 72 with a standard deviation of 8, while the mean grade of 36 girls was 75 with a standard deviation of 6. Test the hypothesis at (i) 0·05 and (ii) 0·01 level of significance that the girls are better in spelling than the boys. 7

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Statistical Data : Given that

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{x^2}{2}} dx$$

then $f(1\cdot0) = 0\cdot3413$, $f(1\cdot230) = 0\cdot39$, $f(1\cdot645) = 0\cdot45$, $f(1\cdot96) = 0\cdot4750$, $f(2\cdot0) = 0\cdot4772$, $f(2\cdot33) = 0\cdot49$, $f(2\cdot58) = 0\cdot4950$, $f(3\cdot0) = 0\cdot4987$.

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