AFC–10852

Sub. Code
1MMA1C4

M.Sc. DEGREE EXAMINATION, APRIL 2018

First Semester

Mathematics

DIFFERENTIAL EQUATIONS

(CBCS – 2011 and 2012 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(10 \times 2 = 20)$

Answer **all** questions.

1. If $\phi_1(x) = x^2$ is a solution of $y'' - \frac{2}{x^2}y = 0$, find a second independent solution.

2. Are the solutions e^{2x}, xe^{2x} of $y'' - 4y' + 4y = 0$. Linearly independent on any interval? Justify.

3. Define regular singular point.

4. Compute the indicial polynomial and its roots for the differential equation $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$.

5. Eliminate the arbitrary constants a and b from $z = (x+a)(y+b)$.

6. Explain the difference between particular solution and singular solution of a partial differential equation.

7. If $u = f(x+iy) + g(x-iy)$, where the functions f and g are arbitrary, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

8. Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$.

9. State exterior Dirichlet problem.

10. Write down the one dimensional wave equation and d'Alemberts solution of the one dimensional wave equation.

Section B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) One solution of $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. Find a basis for the solutions for $x > 0$.

Or

(b) Show that $\int_{-1}^{1} P_n(x) P_m(x)\, dx = 0$, when $n \neq m$.

12. (a) Show that

(i) $J_0'(x) = -J_1(x)$;

(ii) $K_0'(x) = -K_1(x)$.

Or

(b) Show that –1 and +1 are regular singular points for the Legendre equation

$(1-x^2) y'' - 2xy' + \alpha(\alpha+1) y = 0$.

13. (a) Find the general solution of the differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z$.

Or

(b) Show that the equations $f(x,y,p,q)=0$, $g(x,y,p,q)=0$ are compatible if $\frac{\partial(f,g)}{\partial(x,p)} \neq \frac{\partial(f,g)}{\partial(y,q)} = 0$.

14. (a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and solve it.

Or

(b) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$.

15. (a) Show that in two-dimensional case it is possible to reduce Neumann problem to the Dirichlet problem.

Or

(b) A uniform insulated sphere of dielectric constant k and radius a carries on its surface a change of density $\lambda P_n(\cos\theta)$. Prove that interior of the sphere contributes an amount $\frac{8\pi^2 \lambda^2 a^3 K_n}{(2n+1)(Kn+n+1)^2}$ to the electrostatic energy.

Section C (3 × 10 = 30)

Answer any **three** questions.

16. If $\phi_1, \phi_2, \ldots \phi_n$ are n solutions of $L(y)=0$ on an interval *I*, show that they are linearly independent if and only if $W(\phi_1, \phi_2, \ldots \phi_n)(x) \neq 0$ for all x in *I*.

17. Derive Bessel's function of zero order of second kind $K_0(x)$.

18. Find the complete integral of the equation $px^5 - 4q^3x^2 + 6x^2z - 2 = 0$ using Jacobi's method.

19. Show that in spherical polar co-ordinates r, a, ϕ Laplace's equation possess solutions of the form $\left(Ar^n + \frac{B}{r^{n+1}}\right) \odot$ $(\cos\theta)e^{\pm im\phi}$ where A, B, m and n are constants are $\theta(\mu)$ satisfies the ordinary differential equation $(1-\mu^2)\frac{d\odot}{d\mu^2} - 2\mu\frac{d\odot}{d\mu} + \left(n(n+1) - \frac{m^2}{1-\mu^2}\right)\odot = 0$.

20. Find the potential function $\psi(x, y, z)$ in the region $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ satisfying the conditions.

    (a) $\psi = 0$ on $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$;

    (b) $\psi = f(x, y)$, on $z = c$, $0 \le x \le a$, $0 \le y \le b$.

AFC–10853

Sub. Code
1MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2018

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2011 & 2012 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Prove that the real part of an analytic function is harmonic.

2. Find the radius of convergence of $\sum \frac{z^n}{n!}$.

3. Evaluate $\int_C x\,dz$ where C is the line segement from 0 to $1+i$.

4. Write down the Cauchy's representation formula.

5. Show that the function $\sin z$ has an essential singularity at ∞.

6. State the Schwarz Lemma.

7. Calculate the residue of $f(z)=\frac{ze^z}{(z-1)^3}$ at its pole.

8. Evaluate $\int_C \frac{e^{-z}}{z^2} dz$, where C is the circle $|z| = 1$.

9. Write down the power series expansion of are $\tan z$.

10. State the Taylor series.

Part B (5 × 5 = 25)

Answer ALL questions choosing either (a) or (b).

11. (a) Prove that the limit function of a uniformly convergent sequence of continuous functions is itself continuous.

Or

(b) If $u(x)$ is harmonic in Ω, then prove that $\frac{\partial u}{\partial x} = -i \frac{\partial u}{\partial y}$ is analytic in Ω.

12. (a) With the usual notations, prove that $\left| \int_a^b f(t) dt \right| \leq \int_a^b |f(t)| dt$.

Or

(b) State and prove Morera's theorem.

13. (a) State and prove the Taylor's theorem.

Or

(b) Show that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

14. (a) State and prove the residue theorem.

Or

(b) Evaluate : $\int_0^\infty \frac{x^2\,dx}{\left(x^2+a^2\right)^3}$, a real.

15. (a) State and prove the Weierstrass's theorem.

Or

(b) Derive the Jensen's formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove Luca's theorem.

(b) Show that every reflection carries circles into circles.

17. State and prove Cauchy's theorem for a rectangle.

18. State and prove local mapping theorem.

19. (a) State and prove the argument principle.

(b) Show that $\int_0^\pi \log\sin x\,dx = -\pi\log 2$.

20. Obtain the Laurent expansion $\sum_{n=-\infty}^{\infty} An(z-a)^n$ for the function $f(z)$ analytic in $R_1 < |z-a| < R_2$.

AFC–10854

Sub. Code
1MMA3C2

M.Sc. DEGREE EXAMINATION, APRIL 2018.

Third Semester

Mathematics

TOPOLOGY — I

(CBCS – 2011 & 2012 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a topological space. Give an example.
2. Define a limit point with an example.
3. What is meant by the product topology?
4. Define the uniform metric.
5. Define a linear continuum.
6. Define the locally path connected. Give an example.
7. When will you say a space is compact?
8. State the Lebesgue number lemma.
9. Define second countable space. Give an example.
10. State the Urysohn lemma.

Part B (5 × 5 = 25)

Answer **all** questions by choosing either (a) or (b).

11\. (a) Let X be a set and $\mathcal{B}$ Be a basis for a topology J on X. Prove that J equals the collection of all unions of elements of $\mathcal{B}$.

Or

(b) Let Y be a subspace of X. If U is open in Y and Y is open in X. Prove that U is open in X.

12\. (a) State and prove the pasting lemma.

Or

(b) State and prove uniform limit theorem.

13\. (a) Let A be a connected subspace of X. If $A \subset B \subset \overline{A}$, then prove that B is also connected.

Or

(b) State and prove intermediate value theorem.

14\. (a) Prove that every closed subspace of a compact space is compact.

Or

(b) Let X be a metrizable space. If X is compact. then prove that X is limit point compact.

15\. (a) Show that a subspace of a regular space is regular.

Or

(b) State and prove the imbedding theorem.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Let X be a topological space. prove that arbitrary intersections of closed sets are closed.

(b) Let Y be a subspace of X and let A be a subset of Y. Let $\overline{A}$ denote the closure of A in X. Prove that the closure of A in Y equals $\overline{A} \cap Y$.

17. Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metrix d and the squarte metrix ρ are the same as the product topology on $\mathbb{R}^n$.

18. If L is a linear continuum in the order topology, then prove that L is connected, and so are interval and rays in L.

19. (a) State and prove the tube lemma.

(b) State and prove uniform continuity theorem.

20. Prove that every regular space X with a countable basis is metrizable.

AFC–10855

Sub. Code
1MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2018

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2011 & 2012 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Derive normed space with an example.

2. State Jensen's inequality.

3. Define a Hamel basis for a normed space X with an example.

4. State Hahn-Banach extension theorem.

5. State uniform roundedness principle.

6. Define graph of $F, Gr(F)$.

7. Define weak * convergence of a sequence.

8. Define the term normed dual.

9. Write parseval formula.

10. Define orthonormal basis.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let Y be a subspace of a normed space X. Prove that Y and its closure $\overline{Y}$ are normed space with the induced norm.

Or

(b) Let X and Y be normed space and $F : X \to Y$ be a linear map such that the range $R(F)$ of F is finite dimensional. Prove that F is continuous if, and only if the zero space $Z(F)$ of F is closed in X.

12. (a) Let X be a normed space. Prove that X is Banach iff every absolutely summable series of elememnts in X is summable in X.

Or

(b) Prove that any finite dimensional space is a Banach space.

13. (a) Let X and Y be Banach spaces and $FeBL(X,Y)$. Prove that $R(F) = Y$,if and only if F' is bounded below.

Or

(b) State and prove resonance theorem.

14. (a) State and prove closed range theorem of Banach.

Or

(b) State and prove Bessel's inequality.

15. (a) State and prove Schwarz inequality.

Or

(b) State and prove Parallelogram law.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Prove that in a finite normed linear space of $\mathbb{R}$ (or), $\mathbb{C}$ norms are equivalent.

17. State and prove Hahn-Banach seperation theorem.

18. State and prove open mapping theorem.

19. State and prove Riesz representation theorem for $C[a,b]$.

20. Let H be a non-zero Hilbert space over K. Prove that following are equivalent.

(a) H has a countable orthonormal basis

(b) H is linearly isometric to K^n for some n, or to l^2.

(c) H is separable.

AFC–10834

Sub. Code
1MMA3C4

M.Sc. DEGREE EXAMINATION, APRIL 2018

Third Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2011 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define a Spanning tree.

2. Write down any two rules for constructing the network.

3. Define a holding cost.

4. Define periodic review with an example.

5. Identity the customer and the server of the following :
   "Planes arriving at an airport".

6. Illustrate completely random by an example.

7. Define state dependent.

8. Write the formula for finding λ_{eff} in machine servicing model – $(M|M|R):(GD/K/K)$, $R < K$.

9. Write down two algorithms for the unconstrained problem.

10. When we say that a function $f(x_1, x_2, ...x_n)$ is separable.

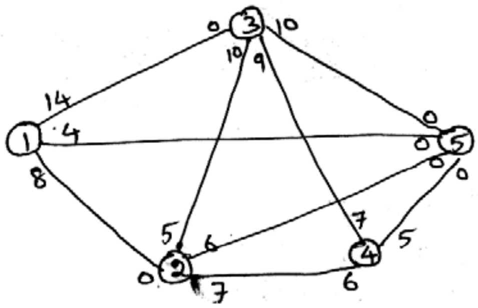
Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Explain the procedure for minimal spanning tree algorithm.

Or

(b) Determine the maximal flow in each arc for the network.



12. (a) Explain classic EOQ model.

Or

(b) Explain about NO–Setup model.

13. (a) Explain your understanding of the relationship between the arrival rate λ and the average inter arrival time.

Or

(b) An art collector travels to art auctions once a month on the average. Each trip is guaranteed to produce one purchase. The time between trips is exponentially distributed. Determine the following :

(i) The probability that no purchase is made in a 3–month period.

(ii) The probability that no more than 8 purchases are made per year.

14. (a) Explain about single – server model $(M|M|1):(GD/\infty/\infty)$.

Or

(b) Determine the minimum number of parallel server needed in each of the following : (Poisson arrival/ departure) situations to guarantee that the operation of the queuing situation will be stable :

(i) Customers arrive every 5 minutes and are served at the rate of 10 customers per hour.

(ii) The average inter arrival time is 2 minutes, and the average service time is 6 minutes.

15. (a) Write down the general step of the dichotomous method.

Or

(b) Approximate the following problem as a mixed integer program

$$Maximize\ z = e^{-x_1} + x_1 + (x_2 + 1)^2$$

Subject to:

$$x_1^2 + x_2 + \leq 3$$

$$x_1, x_2 \geq 0.$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Construct the project network comprised of activities A to L with the following precedence relationships.

(a) A,B and C, the first activities of the project, Can be executed concurrently.

(b) A and B precede D.

(c) B precedes E, F and H.

(d) F and C precede G.

(e) E and H precede I and K

(f) C, D, F and J precede K

(g) K precedes L.

(h) I, G and L are the terminal activities of the project.

17. Find the optimal inventory policy for the following six–period inventory situation.

Period i	D_i (units)	Ki($)	hi($)
1	10	20	1
2	15	17	1
3	7	10	1
4	20	18	3
5	13	5	1
6	25	50	1

The unit production cost is $2 for all the periods.

18. Discuss about pure birth model.

19. B and K Groceries operates with three check–out counters. The manager uses the following schedule to determine the number of counters in operation, depending on the number of customers in store.

No. of customers	No. of counters in operation
1 to 3	1
4 to 6	2
More than 6	3

Customers arrive in the counters area according to a Poisson distribution with a mean rate of 10 customers per hour. The average check–out time per customer is exponential with mean 12 minutes. Determine the Steady – state probability P_n of n customers in the check–out area.

20. Solve the following problem using restricted basis method.

Maximize $Z = x_1 + x_2^4$

Subject to :

$3x_1 + 2x_2^2 \leq 9$

$x_1, x_2 \geq 0.$