Directorate of Correspondence Course
Second Year B.Sc. Degree Examinations
August /September 2010

(New Scheme)

MATHEMATICS

Paper - II

Time: 3 hrs.] [Max.Marks : 90

Note: *Answer any SIX full questions of the following choosing atleast ONE from each Part.*

PART - A

A. *Answer the following.* 5×1=5 Marks

1. a) i) Find the order and degree of the differential equation.

$$\left[x + \left(\frac{d^2y}{dx^2}\right)^3\right]^{\frac{3}{4}} = 6\frac{d^3y}{dx^3}$$ 2 Marks

ii) Solve : $(1 + x^2)\, dy + \sqrt{1 - y^2}\, dx = 0$ 2 Marks

b) Solve $x.\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$ 5 Marks

c) Solve $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ by testing the exactness of the equation.

6 Marks

2. a) i) Solve $px^2 - xyp - y^2 = 0$ 2 Marks

ii) Solve $\cos^3 x \frac{dy}{dx} + y \cos x = \sin x$ 2 Marks

b) Find the general and singular solution of $\sin px \cdot \cos y = \cos px \cdot \sin y + p$ 5 Marks

c) Find the orthogonal trajectories of the family of straight lines with slope and y - intercept equal. 6 Marks

PART - B

3. a) i) Solve : $\frac{d^3y}{dx^3} + 6.\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$ 2 Marks

ii) Solve : $(D^2 + 4)y = 2 \sin^2 x$ where $D = \frac{d}{dx}$ 2 Marks

b) Solve $\frac{d^2y}{dx^2} + 9y = \cos 2x \cdot \cos x$ 5 Marks

c) Solve $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = \sin[2\log(1 + x)]$ 6 Marks

Contd.... 2

4. a) i) Evaluate $\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)}$ 2 Marks

ii) Evaluate $\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{e^x - 1}\right]$ 2 Marks

b) State and prove Rolle's theorem. 5 Marks

c) Expand $\tan^{-1} x$ upto the term containing x^5 using Maclaurin's expansion. 6 Marks

PART - C

5. a) i) If every element of a group G is its own inverse then prove that G is abelian. 2 Marks

ii) Define order of an element of a group and find the order of an element w in the multiplicative group $\{1, w, w^2\}$. 2 Marks

b) Prove that the order of an element of a group is always equal to the order of its inverse. 5 Marks

c) State and Prove Euler's theorem. 6 Marks

6. a) i) If x and y are any two real numbers then prove that $|x+y| \leq |x| + |y|$. 2 Marks

ii) Solve the inequality $4x - 6 < 5x - 4 < 4x + 1$ 2 Marks

b) Find the order of the permutation

$$\psi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 5 & 4 & 8 & 2 & 7 & 1 \end{pmatrix}$$

Also find whether ψ is even or odd. 5 Marks

c) Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a and b are connected by the relation $ab = c^2$. 6 Marks

PART - D

7. a) i) Define monotonically increasing sequence with an example. 2 Marks

ii) Test the sequence $\left\{\left(1 + \frac{1}{n}\right)^{\frac{3n^2}{n+1}}\right\}$ 2 Marks

b) If $\lim_{n \to \infty} (x_n) = l$ and $\lim_{n \to \infty} (y_n) = m$ then prove that $\lim_{n \to \infty} (x_n\, y_n) = lm$ 5 Marks

c) In $\{x_n\}$ is a sequence such that $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ then Prove that the sequence converges to the positive root of the equation $x^2 - x - 2 = a$. 6 Marks

8. a) i) Test the series $\sum \left\{\frac{1}{an^2+b}\right\}$ for convergence. 2 Marks

ii) Test the series $\frac{\sqrt{1}}{5} + \frac{\sqrt{2}}{7} + \frac{\sqrt{3}}{9} + \ldots\ldots$ for convergence. 2 Marks

b) Discuss behaviour of the series $\sum \frac{2^n}{n^2+1}$ 5 Marks

c) Find sum to infinity of

$$\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}$$

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