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MDE/M-16 **4031**

COMPLEX ANALYSIS

Paper : MM-404

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all, selecting at least *one* question from each section. All questions carry equal marks.

SECTION–I

1. (a) Define Complex line integral, and evaluate $\int_L z\,dz$.

(b) State and prove Cauchy's intergral formula for higher order derivatives.

2. (a) State and prove converse of Cauchy's integral theorem.

(b) State and prove Minimum modulus theorem.

3. State and prove Taylor's theorem. Also find the Taylor's series for the function $f(z) = \dfrac{2z^3+1}{z^2+z}$ valid in the neighbourhood of the point $z = i$.

SECTION–II

4. (a) State and prove Schwarz's lemma.

(b) State Rouche's theorem, and use it to prove that the expansion $e^z = az^n$ has n roots inside the circle $|z| = 1$ for $a > e$.

5. (a) Using Contour integration, evaluate

$$\int_0^{\pi} \frac{d\theta}{a + \sin^2\theta}, \ a > 0.$$

(b) If $w = f(z)$ represents a conformal transformation of a domain D in the z-plane into a domain D′ of the w-plane then show that $f(z)$ is an analytic function of z in D′.

SECTION–III

6. State and prove Riemann Mapping theorem.

7. (a) Prove that $\Gamma(2z)\sqrt{\pi} = 2^{2z-1}\Gamma(z)\Gamma\left(z + \frac{1}{2}\right)$.

(b) Prove that $\zeta^2(z) = \sum_{n=1}^{\infty} \frac{d(n)}{n^z}$ for $\text{Re } z > 1$, where $d(n)$ is the number of divisor of n.

SECTION–IV

8. (a) State and prove Schwarz's Reflection principle.

(b) Define Natural boundary. Also explain the consequences of Monodromy theorem.

9. (a) State Harnack's inequality. State and prove Harnack's theorem.

(b) State and prove Poisson–Jensen formula.

10. (a) State Hadamard's factorization theorem and use it to show that $\sin \pi z = \pi z \prod_{n=1}^{\infty}\left(1 - \frac{z^2}{n^2}\right)$.

(b) State Bieberbach's Conjecture. State and prove Schottky's theorem.