Test Code : RE I / RE II (Short Answer type) 2006
(Junior Research Fellowship in Economics)

The candidates for Junior Research Fellowship in Economics are required to take two short answer type tests - RE I (Mathematics) in the forenoon session and RE II (Economics) in the afternoon session.

Syllabus for RE I

1. Permutations and combinations.
2. Elementary set theory; Functions and relations; Matrices, coordinate geometry.
3. Convergence of sequences and series.
4. Functions of one and several variables: limits, continuity, differentiation, applications, integration of elementary functions, definite integrals.
5. Constrained and unconstrained optimization, convexity of sets and concavity and convexity of functions.
6. Elements of probability theory, discrete and continuous random variables, expectation and variance, joint conditional and marginal distributions, distributions of functions of random variable.

Syllabus for RE II

1. Theory of consumer behaviour; theory of production; market structure; general equilibrium and welfare economics; international trade and finance; public economics.
2. Macroeconomic theories of income determination, Rational Expectations, Phillips Curve, Neo-classical Growth Model, Inequality.
3. Game Theory: Normal and extensive forms, Nash and sub-game perfect equilibrium.
4. Statistical inference, regression analysis, least squares and maximum likelihood estimation, specification bias, endogeneity, instrumental variables, elementary time-series analysis.

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2006

Sample Questions RE: I

No. 1 Solve the problem of maximizing $f(x, y) = x^2 + x + 4y^2$, subject to $2x + 2y \leq 1$, $x,\ y \geq 0$.
[16]

No. 2 Let Y be a non-negative random variable and k be any positive constant. Show that $\Pr(Y \geq k) \leq E(Y)/k$, where "Pr" denotes probability and $E(\cdot)$ denotes expectation.
[16]

No. 3 (a) Consider a system of three equations in three variables. Suppose $(1, 1, 0)$ and $(0, 0, 1)$ both solve this system. Is $(1/2, 1/2, 1/2)$ a solution of the system as well? In either case, justify your answer.
(b) "Consider two sets $A,\ B \subseteq \Re^2$, such that A and B are both closed and convex and $A \cap B = \emptyset$. Then there exists some straightline not touching either set, such that A lies on one side of it and B lies on the other." (This is a fact.) Would this statement be true if
(i) we drop the condition that A and B are convex?
(ii) we revert to condition that A and B are convex, but drop the condition that they are closed?
$[4 + 6 + 6]$

No. 4 (a) Consider the experiment of tossing a fair coin two times. The symmetry of the experiment indicates that the outcomes, $\{HH,\ HT,\ TH,\ TT\}$ are equally likely. Let three events be defined as follows:
A: Head in the first toss

B: Head in the second toss
C: Both tosses are heads or both are tails.
Find which pairs of events (among A, B and C) are independent and why.
(b) A box contains a white and b black balls; n balls are drawn at random with replacement. Find the expectation and variance of the number of white balls in the n balls drawn.
$[8+8]$

No. 5 For any set A which is a subset of the real line, let $\sim A$ denote the complement of A, i.e those elements of the real line which do not belong to A. Let $A_n = \{x \mid -n < x \leq \frac{1}{n}\}$, $n = 1, 2, 3, ...$ Find
(a) $\sim (\cup_{n=1}^{3} A_n)$.
(b)$\cup_{n=1}^{3} (\sim A_n)$.
(c)$\cap_{n=1}^{\infty} (\sim A_n)$.
(d)$\cup_{n=1}^{\infty} (\sim A_n)$.
$[4+4+4+4]$

No. 6 Construct sequences of real numbers a_n and b_n such that $\lim a_n$ and $\lim b_n$ do not exist but the limit of the product $a_n b_n$ exists.
[16]

No. 7 (a) Provide a definition of a strictly convex function $f : \Re \to \Re$.
(b) Consider such a function f. Let p, q, r, s be arbitrary real numbers satisfying $p + s = q + r$ and $p < q < r < s$. Which one is greater: (i) $f(p) + f(s)$, or (ii) $f(q) + f(r)$? Provide a proof for your answer.
$[3+13]$

No. 8 (a) Given that $f'(x) = f(x)$ and $f(0) = 1$, what is $f(x)$?
(b) Suppose $g(x) = \max\left(x, \frac{3}{x+2}\right)$. Find the value of $\int_0^2 g(x)dx$.
$[6+10]$

No. 9 (a) Consider the following statement providing a *sufficient* condition for a local maximum:
"Let $f : A \to \Re$ be a twice continuously differentiable function and $x^* \in A$ be such that $\frac{\partial f}{\partial x}(x^*) = 0$. If $\frac{\partial^2 f}{\partial x^2}(x^*) < 0$, then x^* is a point of local maximum of f."

Provide an example to show that a condition "$\frac{\partial^2 f}{\partial x^2}(x^*) < 0$" cannot

be replaced by "$\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$" in the above statement.
(b) Does the function $f(x) = -x^4$, $x \in (-\infty, \infty)$ have a maximum?
(c) Consider the following statement providing the *necessary* condition for a local maximum:
"Suppose $f : A \to \Re$ is a twice continuously differentiable function and $x^* \in A$ is a point of local maximum of f. Then $\frac{\partial f}{\partial x}(x^*) = 0$ and $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$."

Provide an example to argue that the condtion "$\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$" cannot be replaced by "$\frac{\partial^2 f}{\partial x^2}(x^*) < 0$" in the above statement.
$[8 + 3 + 5]$

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Sample Questions RE: II

No. 1 Consider a two-country (home and foreign), two-good (1 and 2) Ricardian economy. The variables for the foreign country are distinguished by *. The single factor of production, labour, is denoted by L. The per-unit labour requirements are ranked such that $a_{L1}/a_{L2} < a^*_{L1}/a^*_{L2}$. Apart from production costs, each country has to employ b units of own labour to transport one unit of import from the other country. Show that the necessary and sufficient condition for trade to take place between the two countries is: $a_{L1}/(a_{L2} - b) < (a^*_{L1} - b)/a^*_{L2}$.
[25]

No. 2 (a) Consider a Solow-type growth model with two factors: capital (K) and labour (L). For simplicity, the rate of capital depreciation is zero. A constant fraction s of income saved and invested, that is, $I_t = \dot{K}_t = sQ_t$, where Q_t is the output or income. Labour grows exogenously at the rate n. Let the production function be: $Q_t = K_t^{\alpha} L_t^{1-\alpha}$, $0 < \alpha < 1$. Derive the long-run growth rate of output and the capital stock.
(b) Modify the model by introducing a third factor, say land, denoted by R, whose supply is fixed over time. The technology is given by $Q_t = K_t^{\alpha} L_t^{\beta} R^{1-\alpha-\beta}$, $0 < \alpha,\ \beta,\ \alpha + \beta < 1$. What can you say about the long-run growth rate of output and the capital stock in comparison to your answer in part (a)?
[12 + 13]

No. 3 In the classical regression model $Y = X\beta + \epsilon$, where Y and ϵ are $n \times 1$ vectors, X is a $n \times k$ vector and β is a $k \times 1$ vector, we assume that $V(\epsilon) = \sigma^2 I$. In other words, the disturbances, ϵ, are homoskedastic and have zero covariance. However, show that the OLS residuals,

which estimate the disturbances, do not satisfy these conditions and yet the OLS estimator of β is minimum-variance-linear-unbiased.
[25]

No. 4 (a) Prove that the estimated slope parameter from the regression of Y on X (a scalar) will be equal to the reciprocal of the slope parameter from the regression of X on Y if $R^2 = 1$.
(b) Let m_i, f_i be dummy variables indicating whether individual i is male, female respectively. Let y_i denote log earnings and x_i a vector of demographic and education–related variables. There are two students. Student A estimates the coefficients of the model

$$y_i = f_i x_i' \beta_f + m_i x_i' \beta_m + u_i, i = 1, ..., n,$$

by using OLS. Student B runs separate regressions (again using OLS) for males and for females of y_i on x_i. To their surprise they discover that β_f in student A's regression is identical to the OLS estimate in the females regression of student B. Prove that this is not a coincidence.
$[12 + 13]$

No. 5 (a) Suppose you have data for 100 quarters on the average price and quantity sold of cars in India as well as data on the average household income. Is this sufficient to estimate the demand and supply functions for cars in India? Justify.
(b) Now suppose you are given the freedom to collect data on two more variables. What would you choose and how will you estimate the above functions?
$[7 + 18]$

No. 6 A monopolist faces a demand curve: $p = 2/q$. Her cost function is: $C(q) = 3q$. (a) What will be her optimal output decision?
(b) Suppose there are some competitive suppliers ready to sell the good at the price $p = 5$ in the same market. Show how the monopolist will change her output decision.
$[10 + 15]$

No. 7 (a) Two individuals, Mukesh and Anil are bargaining over Rs 100. Mukesh moves first and proposes a split of $(x, 100 - x)$ (i.e. Mukesh gets x and Anil $100-x$). Anil responds to Mukesh offer by saying either Yes or No. If he says Yes, then the split $(x, 100 - x)$ is implemented;

if he says No, both of them get zero.
(i) What are the strategies of Anil and Mukesh in this game? (ii) Can the split $(50, 50)$ be a Nash equilibrium in this game? (iii) What is the subgame-perfect equilibrium in this game?
(b) Suppose that the bargaining described in (a) carries on for another round. Once again if Anil says Yes to Mukesh's offer of $(x, 100 - x)$, the game ends. However if he says No to Mukesh's offer he gets to make a counteroffer (instead of the game ending). There is now only Rs 80 left to bargain over. Suppose Anil offers $(y, 80 - y)$ (i.e. Mukesh gets y and Anil $80 - y$). Mukesh now responds to this offer by saying Yes or No. If he says Yes, then the split $(y, 80 - y)$ is implemented. Otherwise the game ends with Mukesh and Anil getting payoffs of 50 and zero respectively. What are subgame-perfect Nash equilibrium payoffs and strategies in this game?
$[13 + 12]$

No. 8 There is one upstream firm and two downstream firms in an industry. (A downstream firm produces the final good for sale to the consumers and an upstream firm is one who supplies input to downstream firms.) The upstream firm produces the input with average cost = marginal cost = c. The downstream firms use this input and produce a homogeneous (final) good. The market demand for the (final) good is $p = a - Q$, where Q is the aggregate supply of output by the downstream firms. One unit of input is required to produce one unit of final good; there are no other costs involved in the production. Determine the equilibrium input and output prices and the corresponding profit levels of the firms when the downstream market is characterized by Cournot competition.
[25]

No. 9 There are three individuals $i = 1, 2, 3$ who have utility functions V_i, $i = 1, 2, 3$ defined over the unit interval $[0, 1]$ as follows: for all i and $x \in [0, 1]$ we have $V_i(x) = |x - \beta_i|$ where $\beta_i \in [0, 1]$. In other words, β_i is individual i's *worst* point in $[0, 1]$ and the farther she is from β_i, the better-off she is. Assume $\beta_1 < \beta_2 < \beta_3$. The utility functions of the three voters are aggregated into a social preference ordering as follows: for $x, y \in [0, 1]$, x is socially preferred to y if there exists at least two individuals i, j such that $V_i(x) > V_i(y)$ and $V_j(x) > V_j(y)$. That is, x is preferred to y socially if a majority of voters prefer x to y. Show that if $x < y \leq \beta_2$ or $\beta_2 \leq y < x$, x is socially preferred to y.

Multiple Choice-Macro

No. 10 **(a).** In the IS-LM framework, an increase in the price level leads to _____in the interest rate and _____in real income.

a. an increase, a decrease
b. an increase, an increase
c. a decrease, an increase
d. a decrease, a decrease.

No. 10 **(b).** Consider a Keynesian open economy in which exports are exogenous and imports increase with income. Let $s \equiv$ the marginal propensity to save, $c_d \equiv$ the marginal propensity to consume domestic goods and $c_m \equiv$ the marginal propensity to import. The multiplier in this economy equals:

a. $1/s$
b. $1/(s + c_m)$
c. $1/(s + c_d)$
d. $1/(c_d + c_m)$.

No. 10 **(c).** The real demand for money is a function of

a. the real income and the real interest rate
b. the real income, the real interest rate and the price level
c. the real income and the nominal interest rate
d. the nominal income, the nominal interest rate and the price level.

No. 10 **(d).** Under rational expectations and nominal rigidities,

a. anticipated and unanticipated monetary shocks can affect real GDP
b. only unanticipated monetary shocks can affect real GDP
c. only anticipated monetary shocks can affect real GDP
d. anticipated and unanticipated monetary shocks have no impact on real GDP.

No. 10 **(e).** In the basic Solow model of growth

a. an increase in the savings rate raises the steady-state growth rate
b. an increase in the growth rate of population lowers the steady-state growth rate
c. an increase in the growth rate of population has no impact on the steady-state growth rate

d. an increase in the savings rate has no impact on the steady-state growth rate.

No. 10 **(f).** Which of the following spells the most fundamental difference between the standard Solow model of growth and the standard optimal growth model?
a. The rate of technology progress is endogenous in the former but exogenous in the latter
b. The savings rate is exogenous in the former, but endogenous in the latter
c. Capital utilization is exogenous in the former, but endogenous in the latter
d. All of the above.

No. 10 **(g).** (I) In a (completely) fixed exchange rate system, the sum of current and capital account balances can be negative. (II) In a (completely) free i.e. floating exchange rate system, the sum of the two balances is zero.
a. (I) true; (II) false
b. (I) and (II) both true
c. (I) and (II) both false
d. (I) false; (II) true.

No. 10 **(h).** In an open economy with free capital mobility, an increase in the domestic interest rate would tend to
a. increase in the value of its currency
b. depreciate the value of its currency
c. increase capital outflow
d. has no impact on capital inflow.

No. 10 **(i).** For a small open economy with free capital mobility and flexible exchange rate system, in affecting real GDP,
a. both monetary and fiscal policy are effective
b. neither monetary policy nor fiscal policy is effective
c. monetary policy is effective, while fiscal policy is not
d. fiscal policy effective, while monetary policy is not.

No. 10 **(j).** In the IS-LM model, an increase in government spending financed through creation of new money would

a. increase real GDP and the interest rate

b. would increase real GDP and lower the interest rate

c. would increase real GDP, while the effect on the interest rate is ambiguous

d. would increase the interest rate, while the effect on the real GDP is ambiguous.