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Reg. No. : ....

D 2090 **Q.P. Code : [D 07 PMA 01]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2013.

First Year

Mathematics

ALGEBRA

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

($5 \times 20 = 100$)

1. (a) If p is a prime number and $P/O(G)$, then prove that G has an element of order p.

   (b) Prove that a group of order 28 has a normal subgroup of order 7.

2. (a) State and prove third part of gylow's theorem.

   (b) Let G be a group, $K_1, K_2, ..., K_n$ normal subgroups of G. Suppose that $K_1 \cap K_2 \cap ... AK_n = (e)$. Let $V_i = G/K_i$. Prove that there is an isomorphism of G into $V_1 \times V_2 \times .... \times V_n$.

3. (a) Define Euclidean rings. Let R be a Euclidean ring. Prove that any two elements a and b in R have a greatest common divisor d and also prove that $d=\lambda a+\mu b$ for some $\lambda,\mu\in R$.

(b) State and prove unique factorization theorem.

4. (a) Prove that the ideal $A=(P(x))$ in $F(x)$ is a maximal ideal if and only if $P(x)$ is irreducible over F.

(b) Let $f(x)=a_0+a_1x+a_2x^2+....+a_nx^n$ be a polynomial with integer co-efficients. Suppose that for some prime number p, $pXa_n, p/a_1, p/a_2,...p/a_0, p^2/a_0$. Prove that $f(x)$ is irreducible over the rationals.

5. (a) If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F, and $[L:F]=[L:K][K:F]$.

(b) If $p(x)$ is irreducible in $F[x]$ and if v is a root of $p(x)$, then prove that $F(v)$ is isomorphic to $F'(w)$ where w is a root of $p'(t)$; moreover; this isomorphism σ can so be chosen that $v\sigma=w$ and $\alpha\sigma=\alpha^1$ for every $\alpha\in F$.

6. (a) Prove that the polynomial $f(x) \in F(x)$ has a multiple root iff $f(x)$ and $f'(x)$ have a nontrivial common factor.

(b) If K is a finite extension of F, then prove that $G(K,F)$ is a finite group and its order $O(G(K,F))$ satisfies $O(G(K,F)) \leq [K:F]$.

7. (a) Prove that G is solvable if and only if $G^{(K)} = (e)$ for some integer k.

(b) If $T \in A(v)$ has all its characteristic roots in F, then prove that there is a basis of v in which the matrix of T is triangular.

8. (a) Prove that two nilpotent linear transformations are similar if and only if they have the some invariants.

(b) The prove that normal transformation N is Hermititan if and only if its characteristic roots are real and prove that the normal transformation N is a Unitary iff its characteristic roots are all of absolute value.

Reg. No. : ....

D 2091 **Q.P. Code : [D 07 PMA 02]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2013.

First Year

Mathematics

REAL ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

$(5 \times 20 = 100)$

1. (a) Prove that $f \in \mathbf{R}(\alpha)$ on $[a,b]$ if and only if for every $\in > 0$ there exists a partition P such that $(P, f, \alpha) - L(P, f, \alpha) < \in$.

   (b) Assume α increases monotonically and $\alpha' \in \mathbf{R}$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Prove that $f \in \mathbf{R}(\alpha)$ if $f\alpha' \in \mathbf{R}$. In that case Prove that

   $$\int_a^b f\, d\alpha = \int_a^b f(x)\, \alpha'(x)\, dx.$$

2. (a) Suppose F and G are differentiable functions on $[a,b]$, $F' = f \in \mathbf{R}$ and $G' = g \in \mathbf{R}$. Prove that $\int_a^b F(x)\, g(x)\, dx =$

$$F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)\, dx.$$

(b) If f maps $[a,b]$ into R^k and if $f \in \mathbf{R}(\alpha)$ for some monotonically increasing function α on $[a,b]$, then prove that $|f| \in \mathbf{R}(\alpha)$, and

$$\left|\int_a^b f\, d\alpha\right| \le \int_a^b |f|\, d\alpha.$$

3. (a) Suppose $f_n \to f$ uniformly on set E in a metric space. Let x be a limit point of E, and suppose that $\lim_{f \to x} f_n(t) = A_n\,(n = 1, 2....)$.

Then prove that $\{A_n\}$ converges and $\lim_{t \to x} f(t) = \lim_{n \to a} A_n$.

(b) Prove that there exists a real continuous function on the real line which is now where differentiable.

4. (a) Let A be an algebra of real continuous functions on a compact set K. If A separates point on K and if A vanishes at no point of K , then prove that the uniform closure **B** of A consists of all real continuous function on K.

(b) Suppose E is an open set in R^n, f maps E into R^m, f is differentiable at $x_0 \in E$, g maps an open set containing $f(E)$ into R^k, and g is differentiable at $f(x_0)$. Then prove that the mapping F of E into R^k defined by $F(x) = g(f(x))$ is differentiable at x_0, and $F'(x_0) = g'(f(x_0))\, f'(x_0)$.

5. (a) If X is a complete metric space, and if ϕ is a contraction of X into X then prove that there exists one and only one $x \in X$ such that $\phi(x) = x$.

(b) Put $f(0, 0) = 0$ and $f(x, y) = \dfrac{xy(x^2 - y^2)}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$. Prove that $f, D_1 f, D_2 f$ are continuous in R^2 and $(D_{12} f)(0, 0) = 1$, and $(D_{21}, f)(0, 0) = -1$.

6. (a) Prove that every Borel set is measurable.

(b) Let f be an extended real-valued function whose domain is measurable. Prove that the following are equivalent.

(i) For each real number α the set $\{x : f(x) > \alpha\}$ is measurable.

(ii) For each real number α the set $\{x : f(x) \geq \alpha\}$ is measurable.

(iii) For each real number α the set $\{x : f(x) < \alpha\}$ is measurable.

7. (a) If $\langle f_n \rangle$ is sequence of nonnegative measurable function and $f_n(x) \to f(x)$ almost every where on a set E, then prove that $\int_E f \leq \lim \int_E f_n$.

(b) Prove that a function f is of bounded variation on $[a,b]$ if and only if f is the difference on two monotone real-valued functions on $[a,b]$.

8. (a) Let f be an integrable function on $[a,b]$ and suppose that $F(x) = F(a) + \int_a^x f(t)\, dt$. Prove that $F'(x) = f(x)$ for almost all x is $[a,b]$.

(b) Let F be a bounded linear functional on $L^p, 1 \leq P < \infty$. Prove that there is a function g in L^p such that $F(f) = \int fg.$ and $\|F\| = \|g\|_q$.

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Reg. No. : ....

D 2092 **Q.P. Code : [D 07 PMA 03]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2013.

First Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

(5 × 20 = 100)

1. (a) State and prove the uniqueness theorem for the solutions of systems of first order equation.

   (b) If ϕ is the fundamental matrix for linear homogeneous equation prove that the function ϕ defined by

   $\phi(t) = \Phi(t)\int_{r}^{t} \Phi^{-1}(s)b(s)ds (t \in I)$ is a solution

   to the non homogeneous equation satisfying $\phi(r) = 0, r \in \text{I}$.

2. (a) Find the first four approximation for the equation $y' = 1 + xy;\ y(0) = 0$.

(b) State and prove the Picard's theorem.

3. (a) A tightly stretched homogeneous string of length L, with its fixed ends at x=0 and x=L executes tranverse vibration. Motion is started with zero initial velocity by displacing the string into the form $f(x) = k(x^2 - x^3)$. Find the deflection $u(n,t)$ at any time t.

(b) State and prove Cauchy-Kowsalewsky theorem.

4. (a) Solve the wave equation.

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2};\ 0 \le n \le \pi, t \ge 0$$

Subject to $u = 0$ when $x = 0$ and $x = \pi$

$u_t = 0$ when $t = 0$ and $u(x,o) = x\ ;\ 0 < x < \pi$

(b) Solve the initial value problem,

$$\frac{\partial^2 u}{\partial t^2} - C^2 \frac{\partial^2 u}{\partial x^2} = e^x \text{ since that}$$

$$u(x,0) = 5, \frac{\partial u(x,0)}{\partial t} = x^2$$

5. (a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ satisfying the condition $u(0, y) = 0, u(l, y) = 0,\ u(x,0) = 0$ and $u(x, a) = Sin\left(\frac{Sin x}{l}\right)$.

(b) Obtain the Fourier series solution to the problem $\frac{\partial^2 u}{\partial t^2} = \frac{d^2 u}{\partial x^2} = 0 < x < \pi, t > 0$

subject to the condition

$u(x,0) = 0, \frac{\partial u}{\partial t}(x,0) = 0 \text{ for } 0 < x < \pi$

$\frac{\partial u}{\partial x}(0,t) = 0 \text{ and } \frac{\partial u}{\partial x}(x,t) = 0 \text{ for } t \geq 0$

6. (a) Solve

$$r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0;\ 0 \leq r \leq a,\ 0 \leq \theta \leq 2\pi.$$

with boundary condition

$u(a, \theta) = f(\theta); 0 \leq \theta \leq 2\pi.$

(b) Solve

$$\frac{\partial^2 u}{\partial n^2} + \frac{\partial^2 u}{rs^2} = 0 \text{ satisfying the condition.}$$

$$\frac{\partial u}{\partial y}(x,0) = 0,\ u(x,b) = 0 \text{ for } 0 \le x \le a,$$

$$u(0,y) = 0,\ u(a,y) = T \text{ for } 0 < y < b.$$

7. (a) State and prove the minimum principle.

(b) State the Newton problem for a rectangle an obtain its solution.

8. (a) Solve the Dirichlet problem for a circle.

(b) Solve the Dirichlet problem for a circular annulus.

Reg. No. : ....

D 2093 **Q.P. Code : [D 07 PMA 04]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2013.

First Year

Mathematics

NUMERICAL METHODS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

$(5 \times 20 = 100)$

1. (a) Solve the equation $\cos x - xe^x = 0$ by iteration method.

   (b) Obtain the second derivative of y at $x = 0.96$ from the data.

x:	0.96	0.98	1.00	1.02	1.04
y:	0.7825	0.7739	0.7651	0.7563	0.7473

2. (a) Evaluate $\int_0^1 e^{-x} dx$ dividing in to ten equal parts using Simpson's rule and trapezoidal rule.

(b) From the following table, using Stirling's formula, estimated the value of tan 16°.

x :	0°	5°	10°	15°	20°	25°	30°
$y = \tan x$:	0.0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

3. (a) By Gaussian elimination, find A^{-1} if

$$A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}.$$

(b) Solve the following system of equation by using Gauss-Jacobi methods (correct to 3 decimals places)

$$8x - 3y + 2z = 20$$

$$4x + 11y - z = 33$$

$$6x + 3y + 12z = 35$$

4. Solve, by Triangularization method, the following system :

$$x + 5y + z = 14;$$

$$2x + y + 3z = 13;$$

$$3x + y + 4z = 17.$$

5. (a) Using Taylor's series method, find correct to four decimal places, the value of $y(0.1)$ given $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$.

(b) Using R.K. method of fourth order find $y(0.8)$ correct to 4 decimal places if $y' = y - x^2$, $y(0.7) = 1.8763$.

6. (a) Solve the equation $\frac{dy}{dx} = 1 - y$, given $y(0) = 0$ using modified Euler's method and tabulate the solutions at $x = 0.1, 0.2$.

(b) Find $y(2)$ if $y(x)$ is the solution of $\frac{dy}{dx} = \frac{1}{2}(x + y)$ given
$y(0) = 2$, $y(0.5) = 2.636$, $y(1) = 3.595$
and $y(1.5) = 4.968$ using Milne's method.

7. Using power method, find all the eigen values of

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}.$$

8. (a) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0$, $y = 0$, $x = 3$, $y = 3$ with $u = 0$ on the boundary and mesh length 1 unit.

(b) Using Crack-Nicholson's scheme solve $u_{xx} = 164_t$, $0 < x < 1$, $t > 0$ given $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 100t$. Compute u for one step in t direction taking $h = \frac{1}{4}$.

Reg. No. : ....

D 2094 **Q.P. Code : [D 07 PMA 05]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2013.

First Year

Mathematics

COMPLEX ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each questions carries equal marks.

$(5 \times 20 = 100)$

1. (a) If $f(z)$ is an analytic function, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial s^2}\right) |f(z)|^2 = 4|f'(z)|^2$$

(b) Define bilinear transformation and cross ratio. Prove that the cross ratio of four points is invariant under bilinear transformation.

2. (a) State and prove that the Cauchy's theorem for a Rectangle.

(b) If the piecewise differentiable smooth close curve γ does not pass through the point a, prove that the value of the integral $\int_{\gamma} \frac{dz}{z-n}$ is an integral multiple of 2π.

3. (a) State and prove Lioville's theorem and deduce the fundamental theorem of algebra.

(b) State and prove the Taylor's theorem.

4. (a) State and prove the Residue theorem.

(b) Evulate $\int_{a}^{\pi} \frac{d\theta}{a^2 + \sin^2\theta};(a>0)$

5. (a) State and prove the Poisson Formula satisfied by a harmonic function.

(b) Evaluate $\int_{-a}^{\infty} \frac{x^2 dx}{(x^2+a^2)^3}$; a is real and a > 0.

6. (a) State and prove the Laurent's theorem. Prove also that the Laurent series is unique.

(b) Prove that $\frac{\pi^2}{\sin^2 \pi 2} = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$ deduce that $\pi \cot \pi z = \frac{1}{z} + 2\sum_{n=1}^{\infty} \frac{z}{z^2 - n^2}$.

7. (a) State and prove the Schwarz–Christoffel formula.

(b) Prove that every function which is meromorphic in the whole plane is the quotient of two entire function.

8. (a) Prove that the zeroes $a_1, a_2, \ldots\ldots a_n$ and the poles $b_1, b_2, \ldots\ldots b_n$ of an elliptic function satisfy the relation $\sum_{k=1}^{n} a_n \equiv \sum_{k=1}^{n} b_k \pmod{m}$.

(b) Prove that any two bases of the period module are connected by a unimodular transformation.
